

Control System

6th semester

Electrical Engg

Fundamentals of Control System.

Defination of Control system.

→ It is a combinations of various elements, arranged in such a manner that it control and provide a desired objective.

Indipendent Variable.

These are the signals or reference points provided to the system. Generally these are input to a system. The inputs are called as Reference inputs and denoted by $r(t)$.

Dependent Variable

These are the signals generated by the system. These signals may be some intermediate signals or the final output of the systems. The intermediate signals are called as Actuating signals $u(t)$ and the output is called as Controlled output $c(t)$.

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Classification of Control System.

Based on feedback

1. open loop. Control System.
2. Closed loop Control System

Based on continuity

1. Continuous Control System.
2. Discrete Control System.

Based on Linearity

1. Linear Control System
2. Non-Linear Control System.

Based on Number of Input/output.

1. Single Input single Output System.

(SISO)

2. Multiple Input Multiple output Control system (MIMO)

If the input and outputs are Bounded then it is called as

Bounded Input Bounded Output Control system (BIBO).

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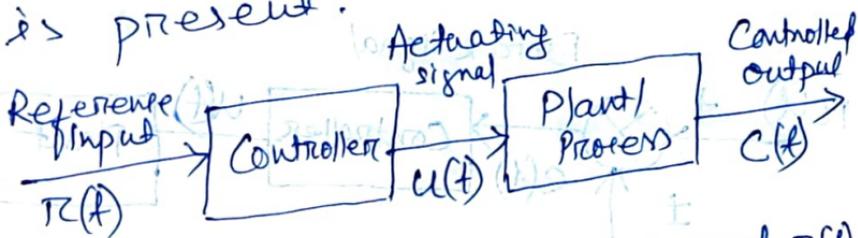
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Open loop Control System

→ In an open loop control system the control action is independent of the desired output. Here there is no feedback signal is present.



→ Here a reference input $r(t)$ is provided to a controller which controls the signal to provide an actuating signal. The actuating signal is fed to the plant which produces the desired output.

- * Controller - It controls the signal to be provided to the plant or process. It conducts the main controlling action.
- * Plant - It produces the final output of the system.

Examples of open loop control system.
Fan with Regulator, Washing machine, Traffic light control.

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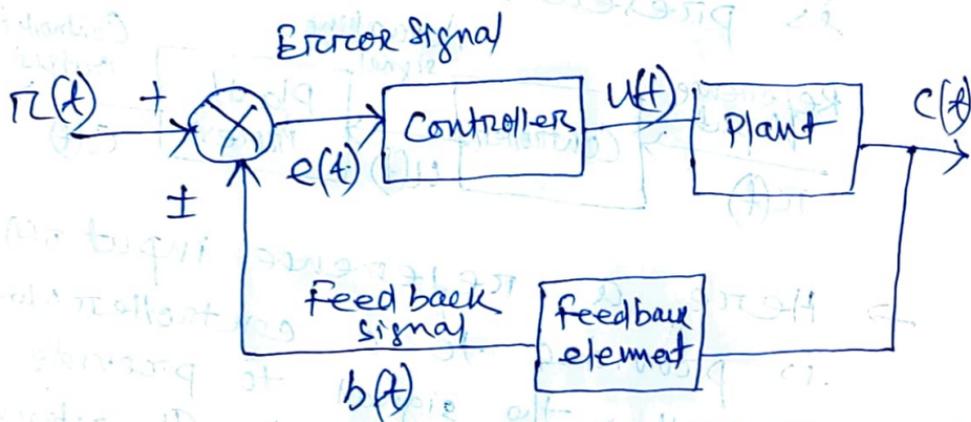
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Closed loop Control System.

→ In a closed loop control system the control action is dependent on the final output of the system.

There is a feedback element present in the system.



→ Here the output is sensed and detected and feed backed to the input section through a feedback element.

→ The feedback signal is compared with the reference input to produce a error signal.

$$e(t) = r(t) \pm b(t)$$

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→ The error signal drives the controller in such a manner that the ~~plant/process~~ error signal itself becomes zero over time.

→ There are two types of feedback control.

1. Negative Feedback CS.
2. Positive Feedback CS.

* If the feedback signal is compared ~~or added~~ with the reference input in a negative way (subtracted) then it is called Negative feedback control system.

* If the feedback signal is compared with the reference input in a positive way (Added) then it is called positive feedback control system.

Examples of Closed loop Control system

Automatic Access Iron

Air Conditioner

Geyser.

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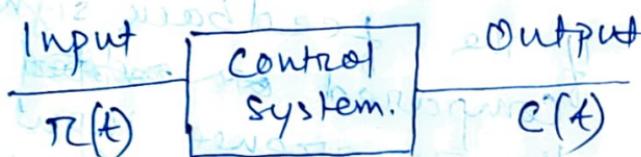
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Transfer Function $T(s)$

→ The transfer function of a Control System is defined as the ratio of Laplace transformation of the output variable to the Laplace transformation of the input variable assuming the initial condition to be zero.



Let

$$\begin{array}{ccc} r(t) & \xleftrightarrow{L} & R(s) \\ c(t) & \xleftrightarrow{L} & C(s) \end{array} \left\{ \begin{array}{l} \text{Laplace} \\ \text{transform.} \end{array} \right.$$

Then Transfer function denoted by $T(s)$ can be expressed as.

$$T(s) = \frac{C(s)}{R(s)} \quad \left| \begin{array}{l} \text{at } t=0 \\ c(t)=0 \end{array} \right.$$

→ When we provide an unit impulse signal to a control system then the output of the system is equals to the Transfer function of CS.

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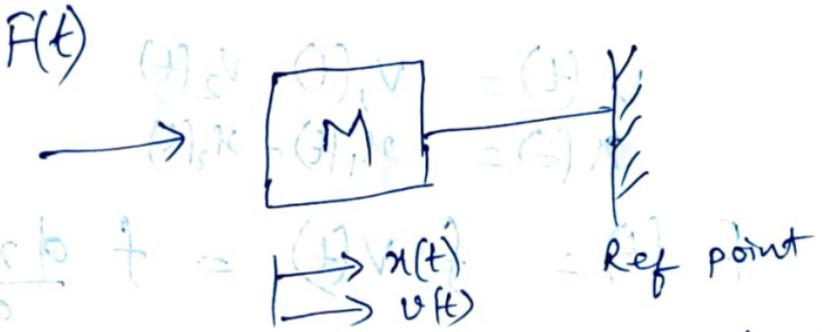
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Mathematical Modelling of CS

Mechanical System. Linear System.

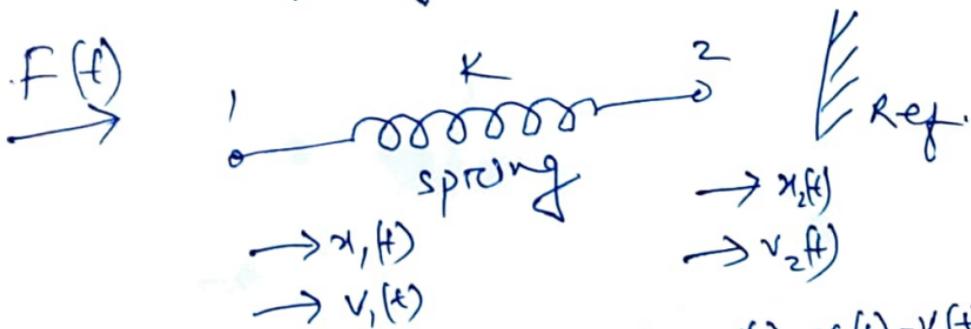
1. The Mass Element



$$F(t) = M \frac{d^2 x(t)}{dt^2} = M \frac{d v(t)}{dt}$$

$$F(s) = M s^2 X(s) = M s V(s)$$

2. The spring Element



$$x(t) = x_1(t) - x_2(t), \quad v(t) = v_1(t) - v_2(t)$$

$$F(t) = K x(t) = K \int v(t) dt$$

$$F(s) = K X(s) = \frac{K V(s)}{s}$$

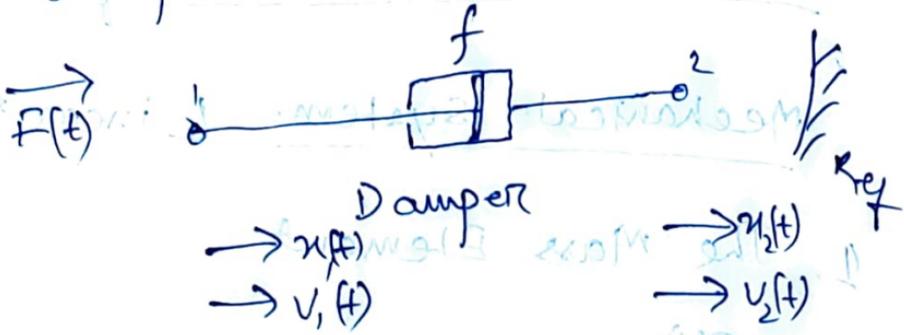
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3. Damper Element



$$v(t) = v_1(t) - v_2(t)$$

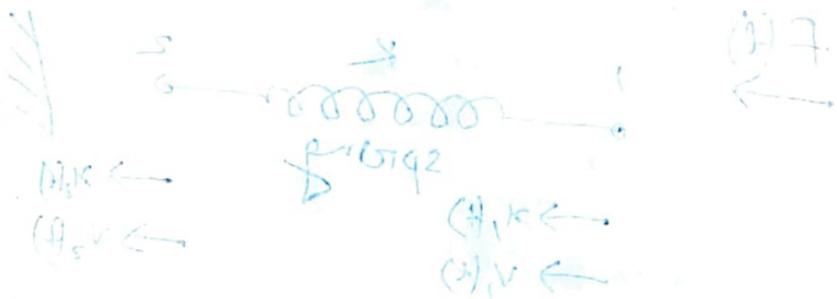
$$x(t) = x_1(t) - x_2(t)$$

$$F(t) = f \cdot v(t) = f \frac{dx(t)}{dt}$$

$$F(s) = f s X(s) = f s X(s)$$

$$F(s) = M s^2 X(s) = M s^2 X(s)$$

The spring element



$$F(s) = K X(s) = K X(s)$$

$$F(s) = K X(s) = K X(s)$$

$$F(s) = K X(s) = K X(s)$$

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Poles & Zeros of Transfer Function

Let the transfer function of a control system be represented as an expression of two polynomials.

$$T(s) = \frac{A(s)}{B(s)}$$

$$= \frac{a_0 s^m + a_1 s^{m-1} + a_2 s^{m-2} + \dots + a_m}{b_0 s^n + b_1 s^{n-1} + b_2 s^{n-2} + \dots + a_n}$$

where $n > m$

As the $A(s)$ polynomial has 'm' number of roots and $B(s)$ polynomial has 'n' number of roots, $A(s)$ and $B(s)$ can be expressed as its roots form

$$T(s) = \frac{(s-s_a)(s-s_b)\dots(s-s_m)}{(s-s_1)(s-s_2)\dots(s-s_n)}$$

where s_a, s_b, \dots are the roots of the $A(s)$ numerator polynomial and s_1, s_2, \dots are the roots of the denominator polynomial.

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- The roots of the denominator polynomial of a transfer function is called as "Poles" of a control system.
- The roots of the numerator polynomial of a transfer function is called as "Zeros" of a control system.
- The highest power of the denominator polynomial is called as the "Order" of the control system.
- The number of roots of the denominator polynomial lies on the origin is called as "type" of control system.
- The equation represented by the denominator polynomial of a transfer function is called as the "characteristic equation" of the control system.

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Time response of 1st order control system.

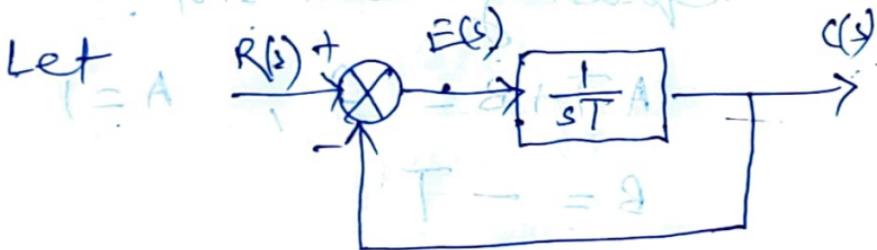
→ subject to unit step input.

for unit step input.

$$r(t) = 1$$

$$R(s) = \frac{1}{s}$$

The control system is



$$\frac{C(s)}{R(s)} = \frac{1/sT}{1 + 1/sT} = \frac{1}{1 + sT}$$

$$\Rightarrow C(s) = R(s) \cdot \frac{1}{1 + sT}$$

$$C(s) = \frac{1}{s} \cdot \frac{1}{1 + sT}$$

Using partial fraction.

$$\frac{1}{s(1 + sT)} = \frac{A}{s} + \frac{B}{1 + sT}$$

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$$\frac{1}{s} \cdot \frac{1}{1+sT} = \frac{A}{s} + \frac{B}{1+sT}$$
$$= \frac{A + AsT + Bs}{s \cdot (1+sT)}$$

$$\Rightarrow X = \frac{A + AsT + Bs}{s(1+sT)}$$

$$\Rightarrow 1 = A + AsT + Bs$$

$$\Rightarrow 1 = s(AT+B) + A$$

equating both side.

$$AT+B = 0 \quad A=1$$

$$B = -T$$

$$C(s) = \frac{1}{s} - \frac{T}{1+sT}$$

$$\Rightarrow C(s) = \frac{1}{s} - \frac{1}{s + (1/T)}$$

applying inverse laplace transform to both side of the eqⁿ.

$$C(t) = 1 - e^{-\frac{t}{T}}$$

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$$e \xleftrightarrow{h} \frac{1}{s+a}$$

$$e(t) = r(t) - c(t) \\ = 1 - (1 - e^{-t/T})$$

$$e(t) = e^{-t/T}$$

(T, t)	$e(t)$
T	0.36 36%
$2T$	0.135 13.5%
$3T$	0.0498 4.98%
$4T$	0.018 1.80%
$5T$	0.0067 0.067%

For 5% error tolerance system
Setting time $= t_s = 3T$

For 2% error " "
 $t_s = 4T$

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→ For unit ramp input:

$$r(t) = t$$

$$R(s) = \frac{1}{s^2}$$

$$C(s) = R(s) \cdot \frac{1}{1+sT}$$
$$= \frac{1}{s^2} \cdot \frac{1}{(1+sT)}$$

Using Partial Fraction:

$$\frac{1}{s^2(1+sT)} = \frac{As+B}{s^2} + \frac{c}{(1+sT)}$$

$$= \frac{As + A^2T + B + BsT + cs^2}{s^2(1+sT)}$$

By equating:

$$\Rightarrow 1 = (AT+1)s^2 + (A+BT)s + B$$

$$AT + c = 0, \quad A + BT = 0$$

$$B = 1, \quad A = -T$$

$$c = T^2$$

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$$C(s) = \frac{-Ts + 1}{s^2} + \frac{T^2}{1 + sT}$$

$$= (-T) \frac{1}{s} + \frac{1}{s^2} + (T) \frac{1}{s + (1/T)}$$

applying inverse laplace transform to the eqⁿ.

$$C(t) = -T + t + T e^{-t/T}$$

$$e(t) = r(t) - C(t)$$

$$= t - (-T + t + T e^{-t/T})$$

$$e(t) = -T - T e^{-t/T} = T(1 - e^{-t/T})$$

t	e(t)
T	
2T	
3T	
4T	.2
5T	

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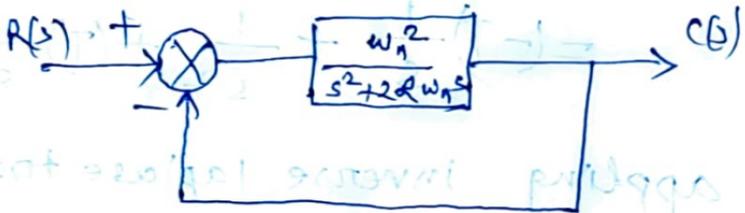
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Time response of 2nd control system subject to unit step input.



$\omega_n \rightarrow$ natural frequency of oscillation
 $\zeta \rightarrow$ Damping ratio

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2 / s^2 + 2\zeta\omega_n s}{-1 + \omega_n^2 / s^2 + 2\zeta\omega_n s}$$

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

For unit step input

$$R(s) = \frac{1}{s}$$

$$C(s) = \frac{1}{s} \cdot \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

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Using partial fraction.

$$\frac{1}{s} \cdot \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{A}{s} + \frac{Bs + C}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$\omega_n^2 = \underline{As^2} + \underline{2\zeta\omega_n As} + \underline{\omega_n^2 A} + \underline{Bs^2} + \underline{Cs}$$

$$\omega_n^2 = (A+B)s^2 + (2\zeta\omega_n A + C)s + \omega_n^2 A$$

$$A+B=0, \quad 2\zeta\omega_n A + C=0$$

$$\omega_n^2 = \omega_n^2 A \Rightarrow A=1, \quad B=-1$$

$$C = -2\zeta\omega_n$$

$$C(s) = \frac{1}{s} - \frac{s + 2\zeta\omega_n}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$e^{-at} \cos \omega t \longleftrightarrow \frac{s+a}{(s+a)^2 + \omega^2}$$

$$e^{-at} \sin \omega t \longleftrightarrow \frac{\omega}{(s+a)^2 + \omega^2}$$

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$$C(s) = \frac{1}{s} - \frac{s + 2\zeta\omega_n}{s^2 + 2\zeta\omega_n s + \zeta^2\omega_n^2 + \omega_n^2(1-\zeta^2)}$$
$$= \frac{1}{s} - \frac{s + 2\zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_n^2(1-\zeta^2)}$$

$$\text{Let } \omega_d = \omega_n \sqrt{1-\zeta^2}$$

ω_d = Damped frequency of oscillation.

$$C(s) = \frac{1}{s} - \left[\frac{s + \zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2} - \frac{\zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2} \right]$$

$$C(s) = \frac{1}{s} - \frac{s + \zeta\omega_n}{(s + \zeta\omega_n)^2 + \omega_d^2} + \frac{\zeta\omega_n}{\omega_d} \frac{\omega_d}{(s + \zeta\omega_n)^2 + \omega_d^2}$$

applying inverse laplace transform to both side of the eqⁿ

$$C(t) = 1 - e^{-\zeta\omega_n t} \cos \omega_d t - \frac{\zeta\omega_n}{\omega_d} e^{-\zeta\omega_n t} \sin \omega_d t$$

$$\underline{a = \zeta\omega_n}$$

$$\omega_d = \omega_n \sqrt{1-\zeta^2}$$

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$$c(t) = 1 - e^{-at} \cos \omega_d t - \frac{a \omega_d}{\omega_n \sqrt{1-z^2}} e^{-at} \sin \omega_d t$$

$$= 1 - \frac{e^{-at}}{\sqrt{1-z^2}} \left\{ \sqrt{1-z^2} \cos \omega_d t + z \sin \omega_d t \right\}$$

Let $z = \cos \phi$

$$\sqrt{1-z^2} = \sin \phi$$

$$\phi = \tan^{-1} \left(\frac{\sqrt{1-z^2}}{z} \right)$$

$$c(t) = 1 - \frac{e^{-at}}{\sqrt{1-z^2}} \left\{ \sin \phi \cos \omega_d t + \cos \phi \sin \omega_d t \right\}$$

$$c(t) = 1 - \frac{e^{-at}}{1-z^2} \sin(\omega_d t + \phi)$$

$$a = z \omega_n$$

where $\omega_d = \omega_n \sqrt{1-z^2}$, $\phi = \tan^{-1} \left(\frac{\sqrt{1-z^2}}{z} \right)$

$$e(t) = r(t) - c(t)$$

$$e(t) = \frac{e^{-at}}{1-z^2} \sin(\omega_d t + \phi)$$

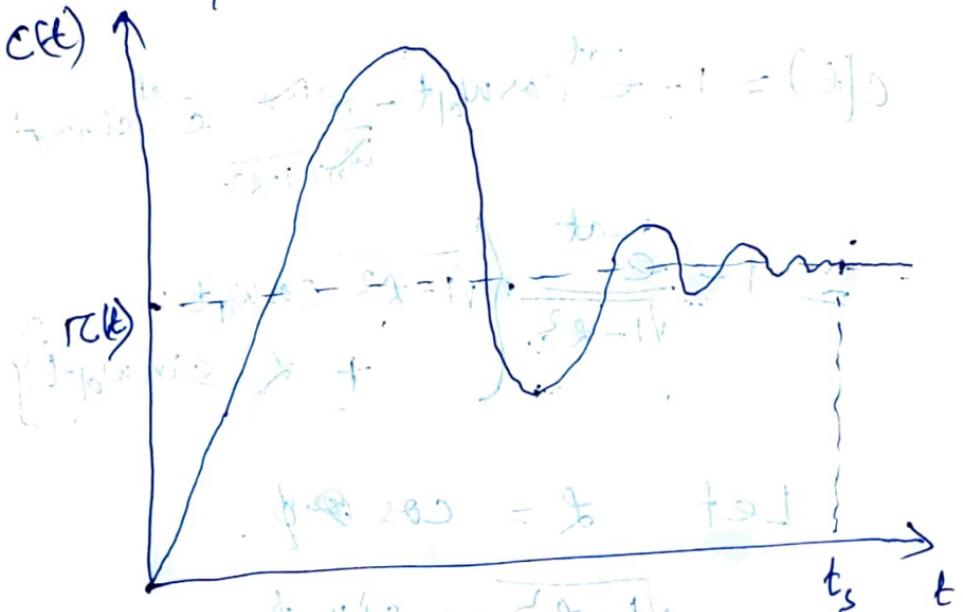
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Graph for response of control system



Above graph

is for the condition

$$0 < \zeta < 1$$

Some of other condition are:

$$\zeta = 0, \quad \zeta = 1, \quad \underline{\zeta > 1}$$

Case 1 $\zeta = 0$

$$C(t) = 1 - \frac{e^{-\zeta \omega_n t} \sin(\omega_d t + \phi)}{\sqrt{1 - \zeta^2}}$$

$$= 1 - \frac{e^{-\zeta \omega_n t} \sin\left(\omega_n \sqrt{1 - \zeta^2} t + \tan^{-1} \frac{\sqrt{1 - \zeta^2}}{\zeta}\right)}{\sqrt{1 - \zeta^2}}$$

$$\left(\phi + \tan^{-1} \frac{\sqrt{1 - \zeta^2}}{\zeta} \right) \sin(\omega_n t + \pi/2)$$

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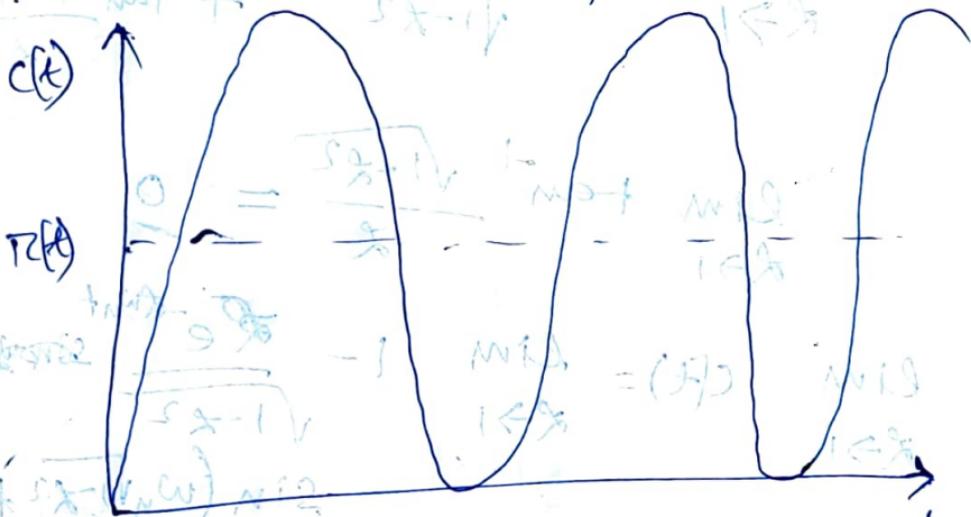
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$$\Rightarrow c(t) = 1 - \cos \omega_n t$$

$$\omega_n = 2\pi f_n$$



The response will sustain the oscillation at ω_n

Case - 2 $0 < \zeta < 1$ Under Damped condition

$$c(t) = 1 - \frac{e^{-\zeta \omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \phi)$$

Case - 3 $\zeta = 1$

$$\lim_{\zeta \rightarrow 1} c(t) = \lim_{\zeta \rightarrow 1} 1 - \frac{e^{-\zeta \omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_d t + \phi)$$

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$$\lim_{z \rightarrow 1} 1 - \frac{e^{-z\omega_n t}}{\sqrt{1-z^2}} \sin(\omega_n \sqrt{1-z^2} t + \phi)$$

$$= \lim_{z \rightarrow 1} 1 - \frac{z^{-z\omega_n t}}{\sqrt{1-z^2}} \sin(\omega_n \sqrt{1-z^2} t + \tan^{-1} \frac{\sqrt{1-z^2}}{z})$$

$$\lim_{z \rightarrow 1} \tan^{-1} \frac{\sqrt{1-z^2}}{z} = 0$$

$$\lim_{z \rightarrow 1} c(t) = \lim_{z \rightarrow 1} 1 - \frac{e^{-z\omega_n t}}{\sqrt{1-z^2}} \sin(\omega_n \sqrt{1-z^2} t)$$

$$\lim_{\theta \rightarrow 0} \sin \theta = \theta$$

$$\lim_{z \rightarrow 1} c(t) = \lim_{z \rightarrow 1} 1 - \frac{e^{-z\omega_n t}}{\sqrt{1-z^2}} \sin(\omega_n \sqrt{1-z^2} t)$$

$$= 1 - \frac{e^{-\omega_n t}}{\omega_n t} \sin(\omega_n t)$$

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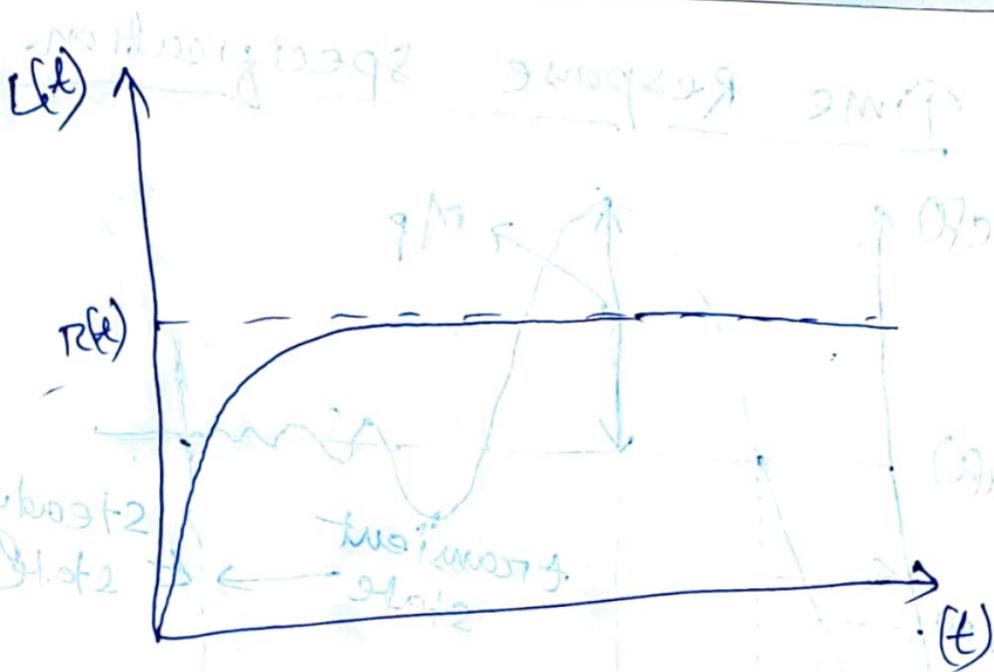
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Critically damped condition.

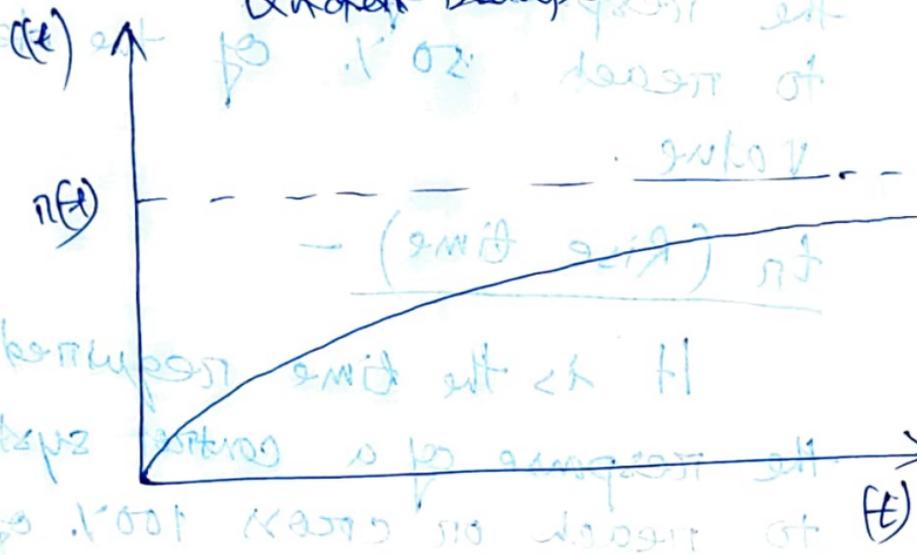
$$\zeta = 1$$

This ideal condition to run any control system.

Case-4

$\zeta > 1$ Over Damped condition.

Under Damped



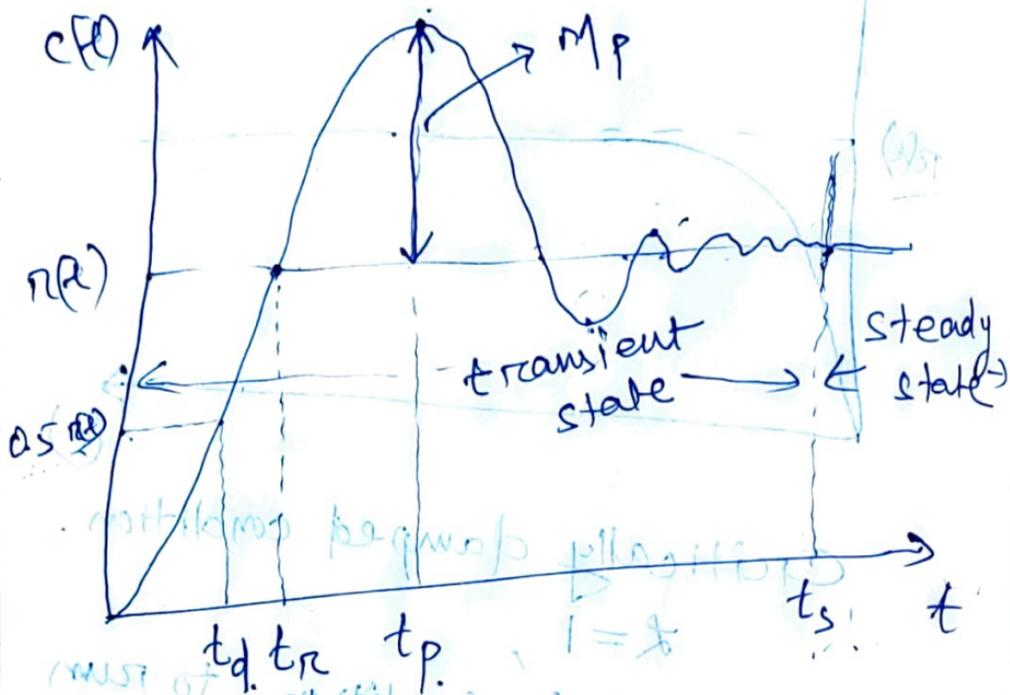
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Time Response Specification



t_d (Delay time)

It is the time required by the response of a control system to reach 50% of the (desired) value.

t_r (Rise time)

It is the time required by the response of a control system to reach or cross 100% of the

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desired value.

M_p - Maximum overshoot.

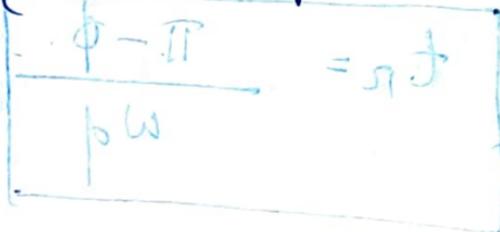
It is the maximum positive deviation of the response of a control system from its desired value. It is expressed in percentage term.

t_p - Peak Time

It is the time required by the response of a control system to reach its maximum value.

t_s - Settling time

It is the time required by the response of a control system to reach within 2% or 5% of the desired value.



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Rise time t_r .

At rise time. $t = t_r$

$$C(t_r) = r(t) = 1$$

$$\Rightarrow 1 - \frac{e^{-\zeta \omega_n t_r} \sin(\omega_d t_r + \phi)}{\sqrt{1 - \zeta^2}} = 1$$

$$\Rightarrow \frac{e^{-\zeta \omega_n t_r} \sin(\omega_d t_r + \phi)}{\sqrt{1 - \zeta^2}} = 0$$

$$\frac{e^{-\zeta \omega_n t_r}}{\sqrt{1 - \zeta^2}} \rightarrow \text{finite and nonzero.}$$

$$\Rightarrow \sin(\omega_d t_r + \phi) = 0$$

The ^{General} solution

$$\omega_d t_r + \phi = n\pi$$

$$\text{for } t = t_r, n = 1$$

$$\Rightarrow \boxed{t_r = \frac{\pi - \phi}{\omega_d}} \checkmark$$

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where $\phi = \tan^{-1} \left(\frac{\sqrt{1-\zeta^2}}{\zeta} \right)$

$$\omega_d = \omega_n \sqrt{1-\zeta^2}$$

peak time t_p

At maximum point $\frac{d(c(t))}{dt} = 0$.

$$d \left\{ \frac{1 - e^{-\zeta \omega_n t} \sin(\omega_d t + \phi)}{\sqrt{1-\zeta^2}} \right\} = 0$$

$$\frac{d}{dt} e^{-\zeta \omega_n t}$$

$$\Rightarrow \frac{+\zeta \omega_n e^{-\zeta \omega_n t} \sin(\omega_d t + \phi)}{\sqrt{1-\zeta^2}} - \frac{\omega_d e^{-\zeta \omega_n t} \cos(\omega_d t + \phi)}{\sqrt{1-\zeta^2}} = 0$$

$$\Rightarrow \frac{\zeta \omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \sin(\omega_d t + \phi) - \frac{\omega_n \sqrt{1-\zeta^2}}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \cos(\omega_d t + \phi) = 0$$

$$\Rightarrow \frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \left[\zeta \sin(\omega_d t + \phi) - \sqrt{1-\zeta^2} \cos(\omega_d t + \phi) \right] = 0$$

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$$\Rightarrow \cos \phi \sin(\omega_d t + \phi) - \sin \phi \cos(\omega_d t + \phi) = 0$$

$$\Rightarrow \mathcal{L} \sin(\omega_d t + \phi) - \sqrt{1 - \mathcal{L}^2} \cos(\omega_d t + \phi) = 0$$

$$\Rightarrow \mathcal{L} \sin(\omega_d t + \phi) = \sqrt{1 - \mathcal{L}^2} \cos(\omega_d t + \phi)$$

$$\Rightarrow \tan(\omega_d t + \phi) = \frac{\sqrt{1 - \mathcal{L}^2}}{\mathcal{L}}$$

$$\phi = \tan^{-1} \left(\frac{\sqrt{1 - \mathcal{L}^2}}{\mathcal{L}} \right)$$

The general solution

$$\omega_d t = n\pi \quad \left\{ \begin{array}{l} \text{where} \\ n = 1, 2, 3, \dots \end{array} \right.$$

at $t = t_p$, $n = 1$

$$t_p = \frac{\pi}{\omega_d}$$

For first minima $t_{\min} = \frac{2\pi}{\omega_d}$

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Maximum overshoot M_p

at - $t = t_p$, ~~$(t_p = \pi/\omega_d)$~~ $(t_p = \pi/\omega_d)$

$$M_p = c(t_p) - 1$$

$$M_p = 1 - \frac{e^{-\zeta\omega_n t_p} \sin(\omega_d t_p + \phi)}{\sqrt{1-\zeta^2}} - 1$$

$$= - \frac{e^{-\zeta\omega_n \pi/\omega_d} \sin\left(\omega_d \frac{\pi}{\omega_d} + \phi\right)}{\sqrt{1-\zeta^2}}$$

$$= - \frac{e^{-\frac{\zeta\omega_n \pi}{\omega_n \sqrt{1-\zeta^2}}} \sin(\pi + \phi)}{\sqrt{1-\zeta^2}}$$

$$= + \frac{e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \sin \phi}{\sqrt{1-\zeta^2}}$$

$$= \frac{e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \sqrt{1-\zeta^2}}{\sqrt{1-\zeta^2}}$$

$$\Rightarrow M_p = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \quad \checkmark$$

$$M_{p\%} = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100 \quad \checkmark$$

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Problem for Practice

Example-1

A control system has its open loop transfer function: $\frac{16}{s^2 + 2s}$ having unity negative feedback. Find its rise time, Peak time, Maximum overshoot, damping ratio, natural frequency of oscillation, Damped frequency of oscillation and its output?

Note

open loop transfer function is $G(s)H(s)$. It is different from open loop control system. The name comes open loop transfer function because in most of cases $H(s) = 1$ and $G(s)H(s) = G(s)$ which is equal to the transfer function of an open loop control system without any feedback.

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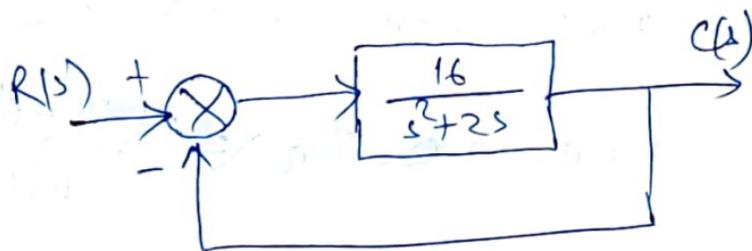
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Answer



$$\frac{C(s)}{R(s)} = \frac{16/s^2 + 2s}{1 + 16/s^2 + 2s} = \frac{16}{s^2 + 2s + 16}$$

As in the denominator the highest power of s is 2, it is a second order control system.

The standard second order control system is represented as,

$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Comparing the given transfer function with the standard one

$$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{16}{s^2 + 2s + 16}$$

We have $\omega_n^2 = 16 \Rightarrow \omega_n = 4$
natural frequency

$$2\zeta\omega_n = 2 \Rightarrow \zeta = \frac{2}{2 \times 4} = 0.5$$

Damping ratio

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Damped frequency of oscillation,

$$\omega_d = \omega_n \sqrt{1 - \zeta^2} = 0.5 \times 4 = 2$$

$$\text{Rise time } t_r = \frac{\pi - \phi}{\omega_d}$$

$$\begin{aligned}\omega_d &= \omega_n \sqrt{1 - \zeta^2} = 4 \sqrt{1 - (0.5)^2} \\ &= 4 \sqrt{0.75} = 3.46 \text{ rad/sec.}\end{aligned}$$

$$\phi = \tan^{-1} \left(\frac{\sqrt{1 - \zeta^2}}{\zeta} \right) = 1.047 \text{ rad.}$$

$$t_r = \frac{\pi - \phi}{\omega_d} = \frac{\pi - 1.047}{3.46} = 0.6 \text{ sec.}$$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{3.46} = 0.9 \text{ sec.}$$

$$\begin{aligned}M_p &= e^{-\frac{\zeta \pi}{\sqrt{1 - \zeta^2}}} = e^{-\frac{0.5 \pi}{\sqrt{0.75}}} \\ &= 0.163\end{aligned}$$

$$M_p \% = 16.3 \%$$

$$\begin{aligned}C(t) &= 1 - \frac{e^{-\zeta \omega_n t}}{\sqrt{1 - \zeta^2}} \sin(\omega_d t + \phi) \\ &= 1 - \frac{e^{-0.5 \times 4 \times t}}{\sqrt{1 - 0.25}} \sin(3.46t + 1.047)\end{aligned}$$

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$$\Rightarrow c(t) = 1 - \frac{e^{-2t}}{0.866} \sin(3.46t + 1.047)$$

$$c(t) = 1 - 1.15 e^{-2t} \sin(3.46t + 1.047)$$

Answers are

Damping ratio $\zeta = 0.5$

Natural frequency $\omega_n = 4 \text{ rad/sec}$

Damped " $\omega_d = 3.46 \text{ rad/sec}$

$\phi = 1.047 \text{ rad}$

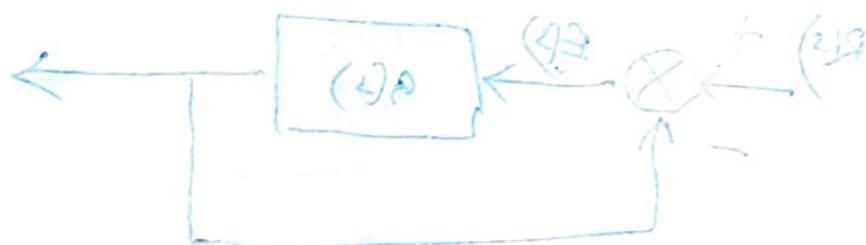
Time $t_n = 0.6 \text{ sec}$

Peak time $t_p = 0.9 \text{ sec}$

Max. overshoot $M_{p\%} = 16.3\%$

Output eqⁿ

$$c(t) = 1 - 1.15 e^{-2t} \sin(3.46t + 1.047)$$



$$(2) - (1) = (4)$$

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PID Controller

P - Proportional Controller.

I - Integral Controller.

D - Derivative Controller.

x

Steady state Error (e_{ss})

Theoretical

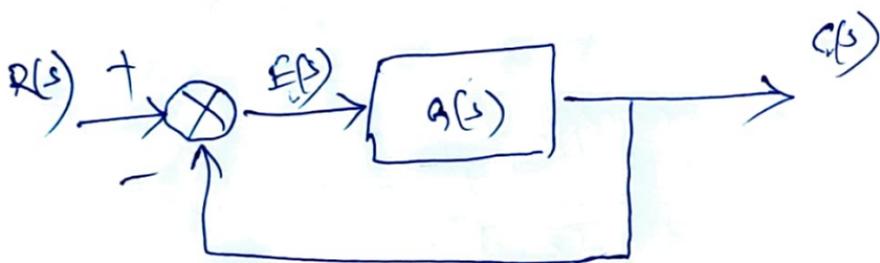
→ It is error of the system.

where the time approaches infinity.

Practical

→ It is the error of the system.

where the control system reaches its steady state condition.



$$E(s) = R(s) - C(s)$$

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$$e(t) = r(t) - c(t)$$

$$e_{ss} = \lim_{t \rightarrow \infty} e(t)$$

$$e_{ss} = \lim_{s \rightarrow 0} s \cdot E(s)$$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

$$\frac{C(s)}{E(s)} = G(s)$$

$$\begin{aligned} \frac{E(s)}{R(s)} &= \frac{C(s)}{R(s)} \cdot \frac{E(s)}{C(s)} \\ &= \frac{G(s)}{1 + G(s)H(s)} \cdot \frac{1}{G(s)} \end{aligned}$$

$$\Rightarrow \frac{E(s)}{R(s)} = \frac{1}{1 + G(s)H(s)}$$

Error transfer function.

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$$e_{ss} = \lim_{s \rightarrow 0} s \cdot R(s) \cdot \frac{1}{1 + G(s)H(s)}$$

$G(s)H(s) \rightarrow$ open loop transfer function.

Static Error Coefficient

Let the control system is a type '0' system, and represented

by

$$\frac{C(s)}{R(s)} = \frac{K}{1 + G(s)H(s)}$$

$$\frac{C(s)}{R(s)} = \frac{K}{1 + G(s)H(s)}$$

$$\frac{1}{1 + G(s)H(s)}$$

$$\frac{E(s)}{R(s)} = \frac{1}{1 + G(s)H(s)}$$

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Steady Error Co-efficient

for step input $R(s) = \frac{1}{s}$

$$E(s) = R(s) \cdot \frac{1}{1 + G(s)H(s)}$$

$$E(s) = \frac{1}{s} \cdot \frac{1}{1 + G(s)H(s)}$$

$$e_{ss} = \lim_{s \rightarrow 0} s \cdot E(s)$$

$$= \lim_{s \rightarrow 0} s \cdot \frac{1}{s} \cdot \frac{1}{1 + G(s)H(s)}$$

$$= \lim_{s \rightarrow 0} \frac{1}{1 + G(s)H(s)}$$

$$e_{ss} = \frac{1}{1 + \lim_{s \rightarrow 0} G(s)H(s)}$$

$$\text{Let } \lim_{s \rightarrow 0} G(s)H(s) = K_p$$

K_p is called Positional error Co-efficient.

$$e_{ss} = \frac{1}{1 + K_p}$$

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For Ramp Input

$$R(s) = \frac{1}{s^2}$$

$$E(s) = \frac{1}{s^2} \cdot \frac{1}{1 + G(s)H(s)}$$

$$e_{ss} = \lim_{s \rightarrow 0} s \cdot \frac{1}{s^2} \cdot \frac{1}{1 + G(s)H(s)}$$

$$= \lim_{s \rightarrow 0} \frac{1}{s(1 + G(s)H(s))}$$

$$= \lim_{s \rightarrow 0} \frac{10^2}{s + sG(s)H(s)}$$

Normally $G(s)H(s) \gg 1$

$$e_{ss} = \frac{1}{\lim_{s \rightarrow 0} sG(s)H(s)}$$

Let $\lim_{s \rightarrow 0} s \cdot G(s)H(s) = K_v$

K_v is called as Velocity error coefficient

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$$e_{ss} = \frac{1}{K_v}$$

Similarly for Ramp input

$$R(s) = \frac{1}{s^2}$$

$$e_{ss} = \lim_{s \rightarrow 0} s^2 G(s) H(s)$$

Let

$$\lim_{s \rightarrow 0} s^2 G(s) H(s) = K_a$$

$$e_{ss} = \frac{1}{K_a}$$

K_a is called as acceleration error co-efficient.

$$\left(\frac{s}{\omega_n} + 1\right)^{p_2} = (s^2 + 2)$$

$$\frac{p_2}{s^2} = \omega_n^2 (s^2 + 1)^{p_2} =$$

$$\frac{1}{s^2} = \frac{1}{s^2 + 1}$$

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For different 'type' of transfer function,

Type 0

$$G(s)H(s) = \frac{(s-s_a)(s-s_b)\dots(s-s_m)}{(s-s_1)(s-s_2)\dots(s-s_n)}$$

$$K_p = \lim_{s \rightarrow 0} G(s)H(s)$$

$$= \lim_{s \rightarrow 0} \frac{K(s-s_a)(s-s_b)\dots(s-s_m)}{(s-s_1)(s-s_2)\dots(s-s_n)}$$

$$= \frac{K \cdot s_a \cdot s_b \cdot \dots \cdot s_m}{s_1 \cdot s_2 \cdot \dots \cdot s_n}$$

$$= K$$

Note $(s+s_a) = s_a \left(1 + \frac{s}{s_a}\right)$
 $= s_a (1 + sT_a) \quad \underline{T_a = 1/s_a}$

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1+K} \quad \checkmark$$

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$$\underline{K_V} = \lim_{s \rightarrow 0} s \cdot G(s) H(s)$$

$$= \lim_{s \rightarrow 0} s \cdot \frac{(s-s_a)(s-s_b) \dots}{(s-s_1)(s-s_2) \dots}$$

$$\underline{K_V} = 0$$

$$e_{ss} = \frac{1}{K_V} = \infty$$

$$\underline{K_a} = 0, \quad e_{ss} = \infty$$

Type 1

$$G(s)H(s) = \frac{K_0 (s-s_a)(s-s_b) \dots}{s \cdot (s-s_1)(s-s_2) \dots}$$

$$K_P = \lim_{s \rightarrow 0} G(s)H(s) \\ = \lim_{s \rightarrow 0} \frac{K (s-s_a)(s-s_b) \dots}{s \cdot (s-s_1)(s-s_2) \dots}$$

$$\underline{K_P} = \infty$$

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$$e_{ss} = \frac{1}{K_p} = \frac{1}{\infty} = 0$$

$$K_v = \lim_{s \rightarrow 0} s \cdot G(s)H(s)$$

$$= \lim_{s \rightarrow 0} \frac{K (s-s_1)(s-s_2)}{(s-s_1)(s-s_2)}$$

$$K_v = K'$$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{K'}$$

$$K_a = \lim_{s \rightarrow 0} s^2 \cdot G(s)H(s)$$

$$K_a = 0$$

$$e_{ss} = \frac{1}{K_a} = \infty$$

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For Type 2

$$K_p = \infty, \quad e_{ss} = 0$$

$$K_v = \infty, \quad e_{ss} = 0$$

$$K_a = K, \quad e_{ss} = \frac{1}{K}$$

type \rightarrow	0	1	2	3
	e_{ss}	e_{ss}	e_{ss}	e_{ss}
step	$K_p = K$	$\frac{1}{K}$	0	0
Ramp	$K_v = 0$	∞	0	∞
Parabolic	$K_a = 0$	∞	$\frac{1}{K}$	0

— X —

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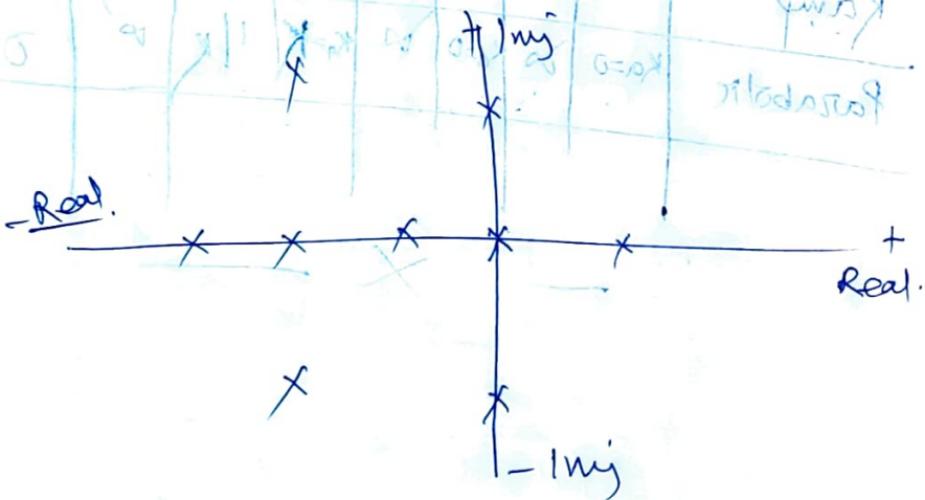
Stability

- ① Stable system.
- ② Marginal stable system.
- ③ Unstable system.

$$T(s) = \frac{A(s)}{B(s)}$$

$B(s) = 0$

→ characteristics eq.



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- ① If all roots of the characteristic eqⁿ lies on left half of the s-plane then the system is said to be a stable system.
- ② If one or more roots of the chⁿ eqⁿ lies on the imaginary axis and all other roots lies to the left of the img. axis the system is said to be marginally stable system.
- ③ If one or more roots of the chⁿ eqⁿ lies to the right of the img axis, then system is an unstable system.

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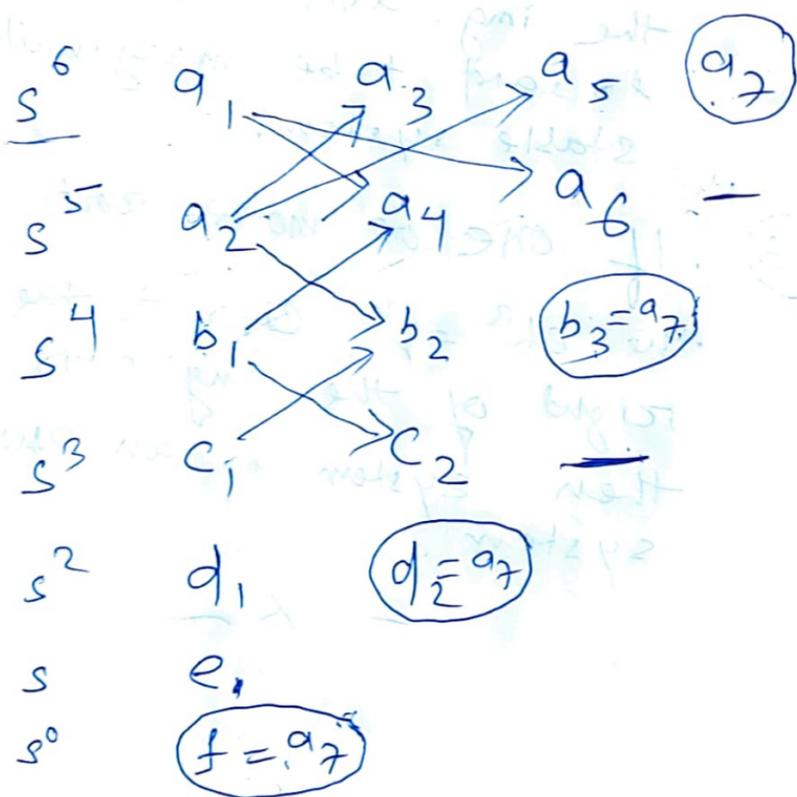
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Routh - Hurwitz Criterion.

Routh array

Let the char eqⁿ of a CS is given by.

$$\underline{a_1} s^6 + \underline{a_2} s^5 + \underline{a_3} s^4 + \underline{a_4} s^3 + \underline{a_5} s^2 + \underline{a_6} s + \underline{a_7} = 0.$$



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$$b_1 = \frac{a_2 a_3 - a_1 a_4}{a_2}$$

$$b_2 = \frac{a_2 a_5 - a_1 a_6}{a_2}$$

$$b_3 = a_7$$

$$c_1 = \frac{b_1 a_4 - a_2 b_2}{b_1}$$

$$c_2 = \frac{b_1 a_6 - a_2 a_7}{b_1}$$

$$d_1 = - \frac{c_1 b_2 - b_1 c_2}{c_1}$$

$$d_2 = a_7$$

$$e = \frac{d_1 c_2 - c_1 a_7}{d_1}$$

$$f = a_7$$

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Example

A char eqⁿ of a ~~closed~~ CS is given by.

$$s^5 + \underline{2s^4} + \underline{3s^3} + \underline{10s^2} + \underline{16s} + \underline{25} = 0$$

Determine its stability?

Routh array

	s^5	1	3	16	
	s^4	2	10	35	
sgn change	s^3	-2	3.5		$\frac{16-10}{2} = \frac{-4}{2} = -2$
sgn change	s^2	+13.5	25		$\frac{32-25}{2} = 3.5$
	s^1	7.2	-		$\frac{-20-7}{-2} = +13.5$
	s^0	25			$\frac{-2}{-2} = 1$
					$\frac{13.5 \times 3.5 + 25}{-2} = -7.2$
					13.5

As ~~there~~ there are two number of times the sign of the coefficient of the first column of the Routh array.

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changes, hence there ^{are} two roots present which lies on the right half of the s-plane.

And Hence the system is unstable.

— x —

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Root Locus

Root locus is the plot of the roots of the characteristic equation in the s -plane while varying gain term K from 0 to ∞ .

- Using root locus we can judge the relative stability of a control system.
- We can also able to find the range of gain value with which we can run the control system in a stable condition.
- The root locus always starts at $K=0$ and open loop poles and terminates at $K=\infty$ and open loop zeros.
- The number of root locus plot is equals to the order of the control system, or the number

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→ The root locus is symmetrical with respect to real axis.

Procedure for root locus and
→ root locus technique.

1. Starting Point (P)

Root locus starts at open loop poles "P" & $K=0$.

Find open loop poles from $G(s)H(s)$.

2. Ending Point (Z)

Root locus terminates at open loop zeros 'Z' & $K=\infty$.

Find open loop zeros from $G(s)H(s)$.

$N =$ Number of Branches (N).

The number of root locus branches is equal to higher of P or Z.

$$N = \begin{cases} P & \text{if } P > Z \\ Z & \text{if } Z > P \end{cases}$$

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4. Existence on Real axis

The root locus exist on a section of real axis if the number of poles & zeros to the right of the section is odd.

5. Break Point

→ The root locus branches break away from the real axis into the complex zone at Break Point.

→ At Break point two or more root locus branches meet and depart from each other.

→ It is find out by rewriting the characteristic equation in terms of K and finding for the solution $\frac{dK}{ds} = 0$.

→ The valid break points are those solution of $\frac{dK}{ds} = 0$, which are lies on the root locus branch existed on the real axis.

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6. Centroid Angle of asymptotes.

For higher values of K the root locus branches are approximate by asymptotic lines. The asymptotic lines are drawn from the centroid with an angle θ_k from positive real axis given by

$$\theta_k = \frac{(2k+1)180^\circ}{P-Z} \quad \left\{ \begin{array}{l} k = 0, 1, 2, \dots \\ (P-Z-1) \end{array} \right.$$

P = number of poles.

Z = number of zeros.

7. Centroid

The asymptotes are drawn from centroid. Centroid is given by

$$\sigma = \frac{\sum \text{Poles} - \sum \text{Zeros}}{P-Z}$$

8. Imaginary axis cross-over point.

The root locus branches crosses the imaginary axis at points determined by finding the value of K from root array applied to char eqⁿ.

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9 Angle of departure from complex poles: ϕ_d

The root locus branch depart from a complex poles making an angle to the positive real axis determined by

$$\phi_d = 180^\circ - (\phi_p - \phi_z)$$

ϕ_p = sum of all angles subtended by remaining poles.

ϕ_z = sum of all angles subtended by zeros.

10 Angle of arrival at complex zero: ϕ_a

The root locus branch terminates at a complex zero making an angle to the positive real axis determined by

$$\phi_a = 180^\circ - (\phi_z - \phi_p)$$

ϕ_z = sum of all angles subtended by remaining zero.

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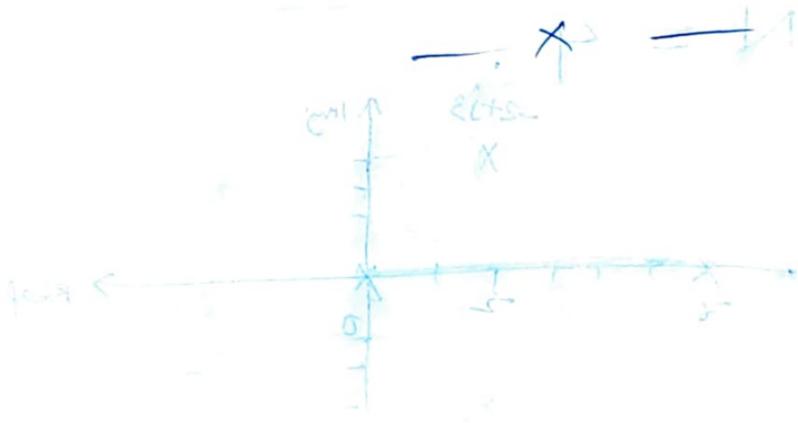
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$\phi_p =$ Sum of all angle subtended by poles.

⇒ All the point found from other step are marked in a s -plane. The asymptote lines are drawn from centroid in dashed lines. All the points are joined in free hand such that root locus branch starts at open loop poles and terminates at open loop zeros and or matches the asymptote lines. Mark all the values at appropriate position. Give direction of the root locus branch from $K=0$ to $K=\infty$.

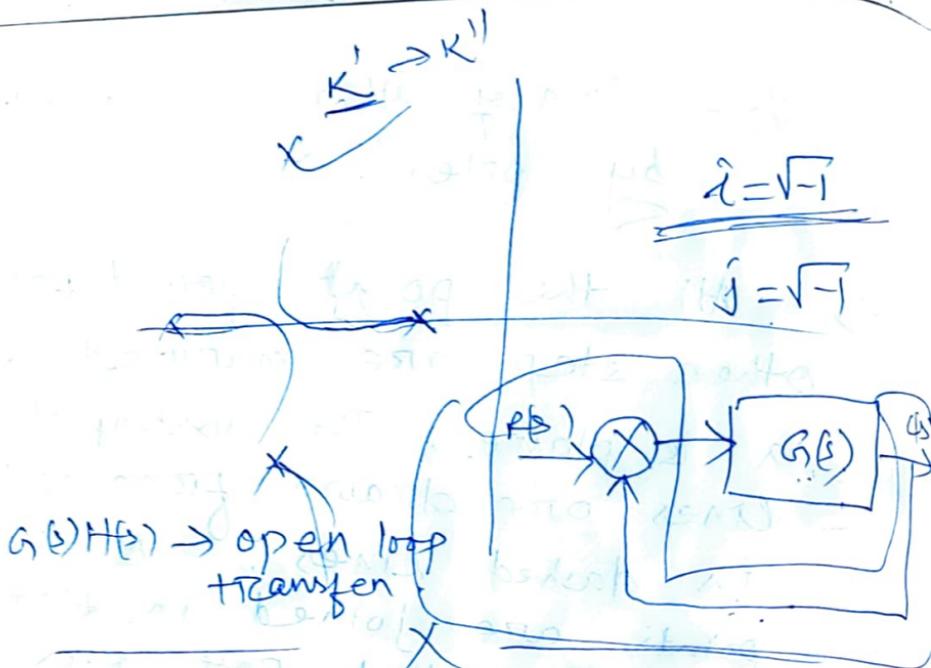


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Example } Plot the root locus of the following control system.

$$G(s)H(s) = \frac{s(s+6)(s^2+4s+3)}{s^2+4s+3}$$

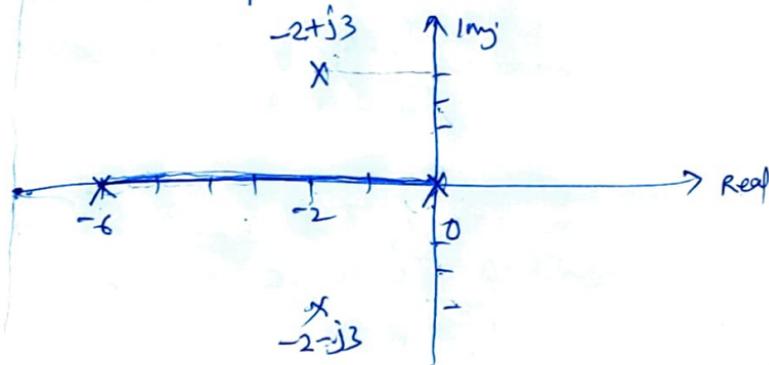
Solⁿ

1. Poles $p = 4$ $(0, -6, -2 \pm j3)$

2. Zeros $z = 0$

3. $N = 4$

4.



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The root locus exist on the real axis from -6 to 0.

s.

$$1 + G(s)H(s) = 0$$

$$\Rightarrow 1 + \frac{K}{s(s+6)(s^2+4s+13)} = 0$$

$$\Rightarrow \frac{(s^2+6s)(s^2+4s+13) + K}{s(s+6)(s^2+4s+13)} = 0$$

$$\Rightarrow s^4 + 4s^3 + 78s^2 + 78s + K = 0$$

$$\Rightarrow s^4 + 10s^3 + 37s^2 + 78s + K = 0$$

char eqⁿ.

$$K = -(s^4 + 10s^3 + 37s^2 + 78s)$$

$$\frac{dK}{ds} = 0 \Rightarrow 4s^3 + 30s^2 + 74s + 78 = 0$$

-4.2, a ± jb

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6 Angle of Asymptotes

$$\theta_k = \frac{(2k+1)180^\circ}{P-2}$$

$k = 0, 1, 2, \dots, (P-2)$
 $k = 0, 1, 2, 3$

$$\theta_0 = \frac{180^\circ}{4} = 45^\circ$$

$$\theta_1 = \frac{3 \times 180^\circ}{4} = 135^\circ$$

$$\theta_2 = \frac{5 \times 180^\circ}{4} = 225^\circ$$

$$\theta_3 = \frac{7 \times 180^\circ}{4} = 315^\circ$$

7 Centroid

$$\sigma = \frac{\sum(\text{Poles}) - \sum(\text{Zeros})}{P-2}$$

$$= \frac{0 - 6 - 2 + j3 - 2 - j3}{4}$$

$$= \frac{-10}{4} = \underline{\underline{-2.5}}$$

8 Imaginary axis cross over point.

$$(2s^2 - 2s + 20) + (s^2 + 2s + 20) = 0$$

$$3s^2 + 2s + 40 = 0 \quad \leftarrow 0 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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$$s^4 \quad 1 \quad 37 \quad K$$

$$s^3 \quad 10 \quad 78$$

Aux
eqⁿ.

$$s^2 \quad 29.2 \quad K$$

$$s^1 \quad \left(78 - \frac{10K}{29.2}\right) \quad -$$

$$s^0 \quad K$$

$$\frac{370 - 78}{10} = \frac{292}{10}$$

$$= 29.2$$

$$\frac{29.2 \times 78 - 10K}{29.2}$$

For stable system

$$K > 0$$

$$78 - \frac{10K}{29.2} > 0$$

$$\Rightarrow 78 > \frac{10K}{29.2}$$

$$\Rightarrow K < \frac{78 \times 29.2}{10} = \underline{\underline{228}}$$

$$\underline{\underline{0 < K < 228}}$$

$$\text{Aux eq}^n \Rightarrow 29.2s^2 + K = 0$$

$$\Rightarrow 29.2s^2 + 228 = 0$$

$\Rightarrow \Rightarrow$

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$$s^2 = \frac{-228}{29.2} = -7.80$$

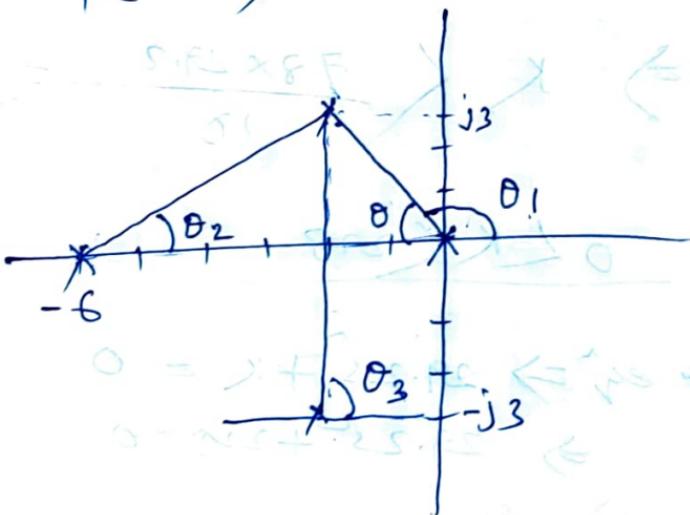
$$s = \pm j 2.79$$

At $s = \pm j 2.79$ and $K = 228$
the root locus branch crosses the
imaginary axis.

Q . Angle of departure from
complex pole.

$$\phi_1(-2+j3) \quad ; \quad \phi_2(-2-j3)$$

$$\phi_1(-2+j3) = 180^\circ - (\phi_1 - \phi_2)$$



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$$\phi_p = \theta_1 + \theta_2 + \theta_3$$

$$\tan \theta = \frac{3}{2} \Rightarrow \theta = \tan^{-1} \left(\frac{3}{2} \right)$$

$$\Rightarrow \theta = 56.3^\circ$$

$$\theta_1 = 180^\circ - \theta = 180^\circ - 56.3^\circ \\ = 123.7^\circ = 124^\circ$$

$$\theta_2 = \tan^{-1} \left(\frac{3}{4} \right) = 37^\circ$$

$$\theta_3 = 90^\circ$$

$$\phi_p = 124^\circ + 37^\circ + 90^\circ \\ = 251^\circ$$

$$\phi_z = \omega \theta$$

$$\therefore \phi_p(-2+j3) = 180^\circ - 251^\circ \\ = -71^\circ$$

$$\phi_p(-2-j3) = +71^\circ \quad \parallel$$

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Bode

$$G(s)H(s) = \frac{K(s+a)}{(s+b)(s+c)}$$

$$\left(T_a = \frac{1}{a}, T_b = \frac{1}{b} \right)$$

$$\left(s^2 + 2\zeta\omega_n s + \omega_n^2 \right)$$

$$G(s)H(s) = \frac{K(1+sT_a)}{(1+sT_b)(1+sT_c)}$$

$$\left(s^2 + 2\zeta\omega_n s + \omega_n^2 \right)$$

~~Replace~~ Replace
 $s = j\omega$

$$G(j\omega)H(j\omega) = \frac{K(1+j\omega T_a)}{(1+j\omega T_b)(1+j\omega T_c)}$$

$$\left(-\omega^2 + 2\zeta\omega_n \omega + \omega_n^2 \right)$$

$$M = \left| G(j\omega)H(j\omega) \right|$$

$$\phi = \angle G(j\omega)H(j\omega)$$

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$$M = |a + jb| = \sqrt{a^2 + b^2}$$

Magnitude M is decibal form.

$$20 \log_{10} |M|$$

$$= 20 \log_{10} \left[\frac{K (1 + j\omega T_a)}{(1 + j\omega T_b)(1 + j\omega T_c)} \right]$$

$$\left[\frac{K (1 + j\omega T_a)}{(1 + j\omega T_b)(1 + j\omega T_c)} \right]$$

$$= 20 \log_{10} K + 20 \log_{10} |1 + j\omega T_a|$$

$$- 20 \log_{10} |1 + j\omega T_b|$$

$$- 20 \log_{10} |1 + j\omega T_c|$$

$$- 20 \log_{10} \sqrt{\omega_b^2 - \omega^2 + j2\omega\omega_b}$$

$$= 20 \log_{10} K + 20 \log_{10} \sqrt{1 + \omega^2 T_a^2}$$

$$- 20 \log_{10} \sqrt{1 + \omega^2 T_b^2}$$

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$$M_{dB} = 20 \log_{10} K + \dots$$

Bode Plot (Logarithmic Plot)

$$\frac{G(s)H(s)}{s = j\omega}$$

open loop transfer function

$$G(j\omega)H(j\omega) = \frac{R(s)}{G(s)}$$

$$\omega = 2\pi f$$

$f \rightarrow$ frequency of the operation
input & output

(In linear system the output always same as input)

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$$s = j\omega$$

$$G(s)H(s) = G(j\omega)H(j\omega)$$

$$M(\omega) = |G(j\omega)H(j\omega)|$$

$$\phi(\omega) = \angle G(j\omega)H(j\omega) \rightarrow \text{radian}$$

$$\frac{\ln |M(j\omega)|}{\omega} \rightarrow \text{neper}$$

Mag_i in decibal form.

$$\frac{20 \log_{10} |M(j\omega)|}{\omega} \quad \text{in dB}$$

$$0 < \omega < \infty$$

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In General,

$$G(s)H(s) = \frac{\omega_n^2 (1+sT_a)(1+sT_b) \dots}{s^N (1+sT_1)(1+sT_2) \dots}$$

$$s = j\omega$$

$$G(j\omega)H(j\omega) = \frac{\omega_n^2 (1+j\omega T_a)(1+j\omega T_b) \dots}{(j\omega)^N (1+j\omega T_1)(1+j\omega T_2) \dots}$$

$(j\omega)^2 + 2\zeta\omega_n j\omega + \omega_n^2$

$$M_{dB} = 20 \log_{10} |G(j\omega)H(j\omega)|$$

$$= 20 \log_{10} \omega_n^2 - 20 \log_{10} (j\omega)^N + 20 \log_{10} (1+j\omega T_a) + \dots - 20 \log_{10} (1+j\omega T_1) - \dots - 20 \log_{10} (\omega_n^2 - \omega^2 + j 2\zeta\omega\omega_n)$$

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For (ω_n^2)

$$M_{dB} = 20 \log_{10} \omega_n^2 \\ = \underline{40 \log_{10} \omega_n}$$

Plot for ω_n^2 term is a fixed value. This decided the starting point of the plot

Plot $\left(\frac{1}{s^N} \right)$

$$M_{dB} = -20 \log_{10} |(j\omega)^N| \\ = -20N \log_{10} |j\omega| \\ = -20N \cdot \frac{\log_{10} \omega}{\log_{10} 10} \text{ dB}$$

$\log_{10} \omega \rightarrow$ The slope of the term.

for $N=0$, slope = 0 dB/decade
 $N=1$, slope = -20 dB/decade
 $N=2$, slope = -40 dB/decade

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decade slope

$\log_{10} \omega$ (ω_1) initial frequency ω_1
 $\log_{10} \omega$ (ω_2) final " " $10\omega_1$

$$\boxed{\omega_2 = 10\omega_1} \quad \checkmark$$

Initial

$$\log_{10} \omega_1$$

Final

$$\log_{10} 10\omega_1 = \log_{10} 10 + \log_{10} \omega_1$$
$$= \log_{10} 10 + \log_{10} \omega_1$$

$$\log_{10} \omega_2 - \log_{10} \omega_1$$

$$\log_{10} \omega_2 - \log_{10} \omega_1 = \log_{10} 10$$

$$\Rightarrow \log_{10} \left(\frac{\omega_2}{\omega_1} \right) = \log_{10} 10 = 1$$

octave slope

$$\boxed{\omega_2 = 2\omega_1}$$

$$\boxed{20 \text{ dB/decade} = 6 \text{ dB/octave}}$$

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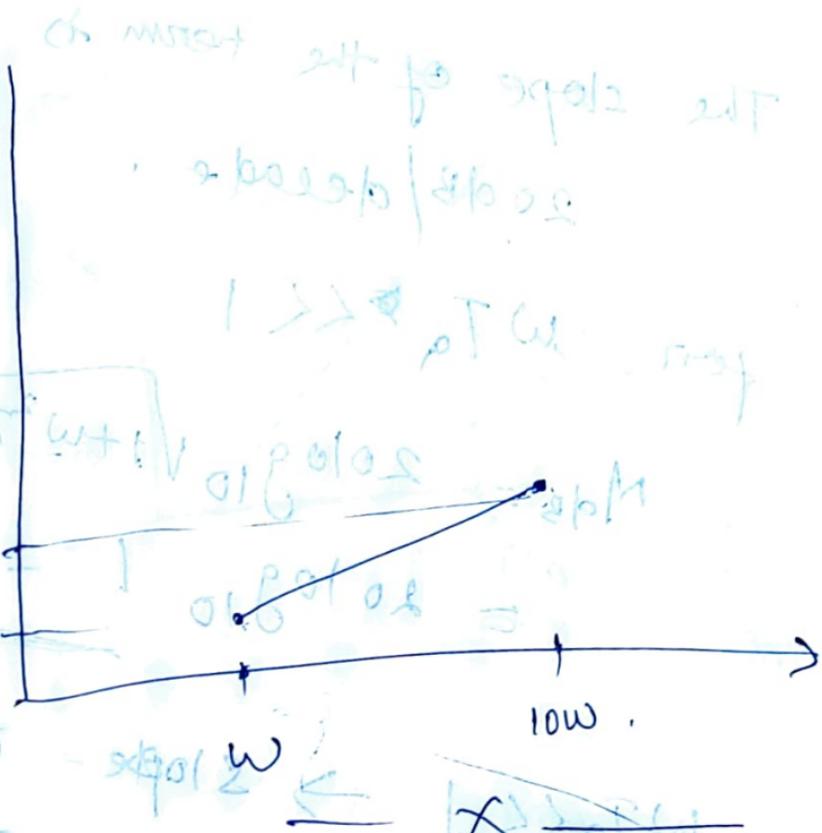
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For Term $(1 + sT_a)$:

$$M_{dB} = 20 \log_{10} |1 + j\omega T_a|$$

for $\omega T_a \gg 1$

$$M_{dB} = 20 \log_{10} \sqrt{1 + \omega^2 T_a^2}$$

$$\approx 20 \log_{10} \sqrt{\omega^2 T_a^2}$$

$$= 20 \log_{10} \omega T_a$$

$$= \underline{20 \log_{10} \omega} + \underline{20 \log_{10} T_a}$$

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The slope of the term is
20 dB/decade.

for $\omega T_a \ll 1$

$$M_{dB} = 20 \log_{10} \sqrt{1 + \omega^2 T_a^2}$$
$$= 20 \log_{10} 1 = 0$$

$\omega T_a \ll 1 \rightarrow$ slope = 0

$\omega T_a \gg 1 \rightarrow$ slope = ~~20 dB/dec~~
20 dB/dec

at $\omega T_a = 1 \Rightarrow \omega = 1/T_a$

$$M = 20 \log_{10} \sqrt{1 + \omega^2 T_a^2}$$
$$= 20 \log_{10} \sqrt{2} = \underline{\underline{3.01 \text{ dB}}}$$

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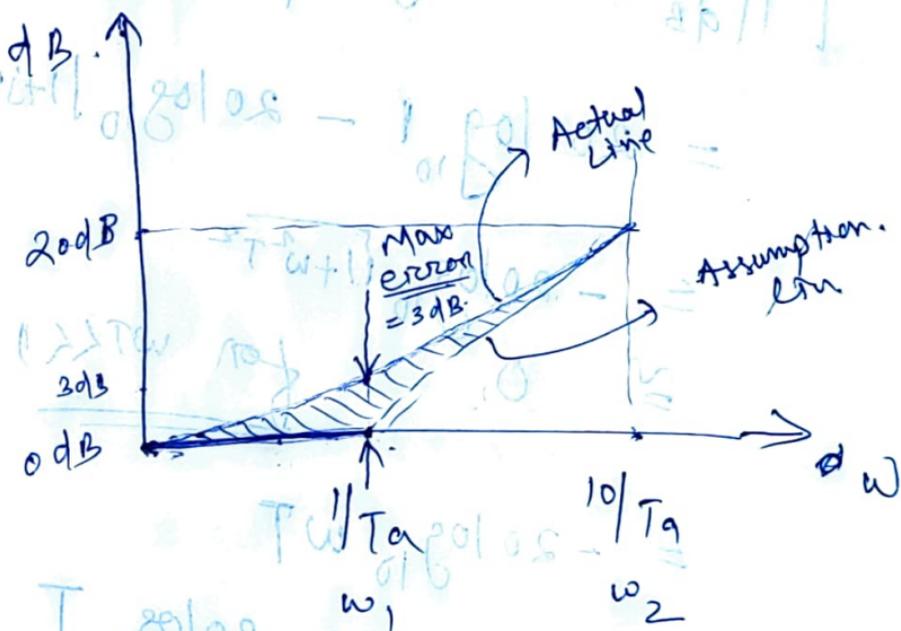
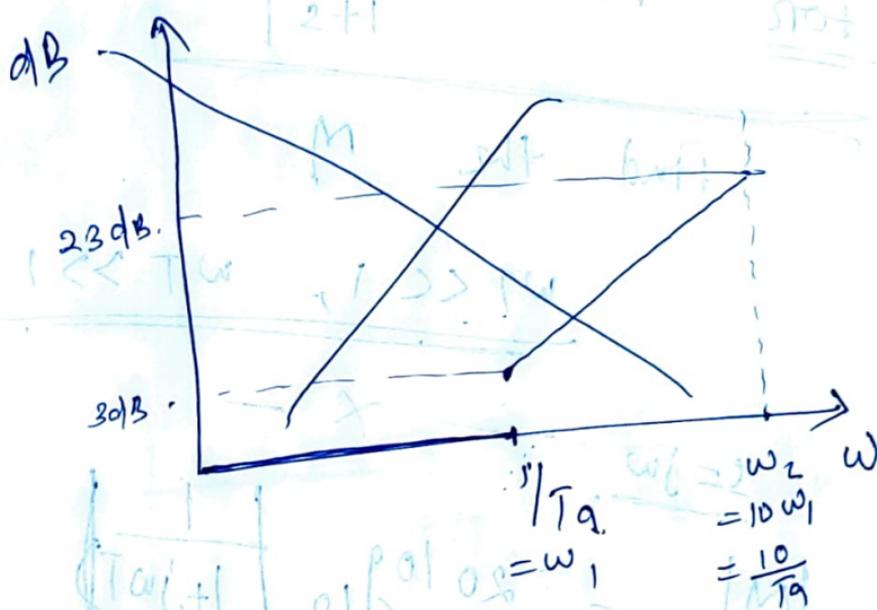
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Since the maximum error.

at $\omega = 1/T_a$ is 3 dB, we can ignore the error and continue with our asymptotic line.

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For Term $\frac{1}{1+sT}$

Find the M

$\omega T \ll 1, \omega T \gg 1$

$s = j\omega$

$$|M|_{dB} = 20 \log_{10} \left| \frac{1}{1+j\omega T} \right|$$

$$= 20 \log_{10} 1 - 20 \log_{10} |1+j\omega T|$$

$$= -20 \log_{10} \sqrt{1+\omega^2 T^2}$$

$$\approx 0$$

for $\omega T \ll 1$

$$\approx -20 \log_{10} \omega T$$

$$= -20 \log_{10} \omega - 20 \log_{10} T$$

slope of the term is

-20 dB/decade

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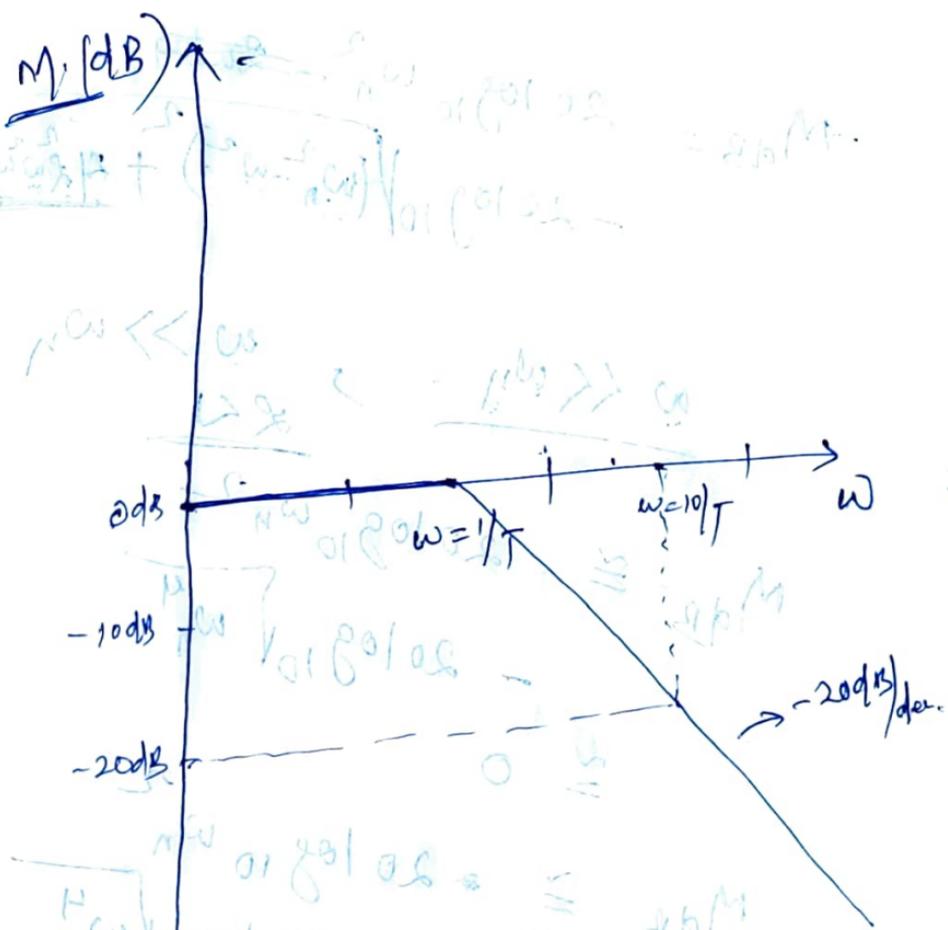
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Forc. Term $\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$s = j\omega$

$$\frac{\omega_n^2}{\omega^2 + 2j\zeta\omega_n\omega + \omega_n^2} = \frac{\omega_n^2}{\omega_n^2 - \omega^2 + j(2\zeta\omega_n\omega)}$$

$$M_{dB} = 20 \log_{10} \left| \frac{\omega_n^2}{(\omega_n^2 - \omega^2) + j(2\zeta\omega_n\omega)} \right|$$

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$$M_{dB} = 20 \log_{10} \omega_n^2 - 20 \log_{10} \sqrt{(\omega_n^2 - \omega^2)^2 + 2\zeta\omega\omega_n^2}$$

$$\omega \ll \omega_n \quad \omega \gg \omega_n$$

$$M_{dB} \approx 20 \log_{10} \omega_n^2 - 20 \log_{10} \sqrt{\omega_n^4}$$
$$\approx 0$$

$$M_{dB} \approx 20 \log_{10} \omega_n^2 - 20 \log_{10} \sqrt{\omega^4}$$
$$\approx 20 \log_{10} \omega_n^2 - 20 \log_{10} \omega^2$$

$$\approx -40 \log_{10} \omega$$

slope of the term is
(-40 dB/decade)

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at $\omega = \omega_n$.

$$\begin{aligned}M_{dB} &= 20 \log_{10} \omega_n^2 \\ &\quad - 20 \log_{10} \sqrt{4\zeta^2 \omega_n^4} \\ &= 20 \log_{10} \left(\frac{\omega_n^2}{4\zeta^2 \omega_n^2} \right) \\ &= -20 \log_{10} (2\zeta)\end{aligned}$$

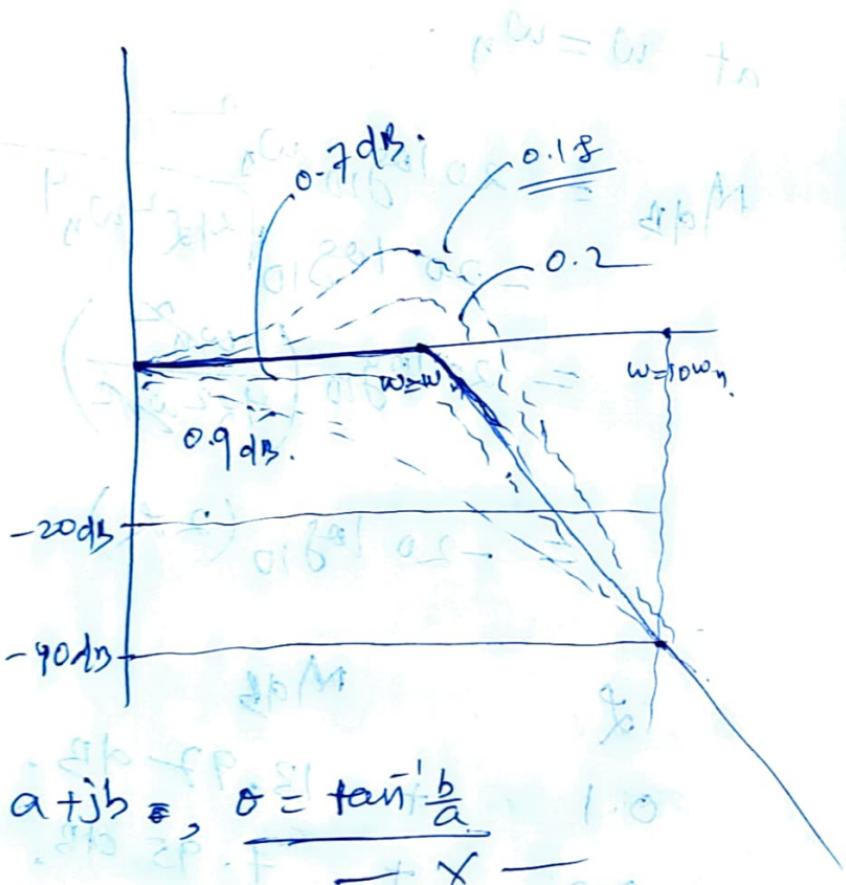
ζ	M_{dB}
0.1	+ 13.97 dB.
0.2	+ 7.95 dB.
0.3	+ 4.43 dB.
0.4	+ 1.93 dB.
0.7	- 2.92 dB.
0.9	- 5.10 dB.
1	- 6.02 dB.

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$$a + jb \Rightarrow \phi = \tan^{-1} \frac{b}{a}$$

$$\phi = \angle M(j\omega) = \tan^{-1} \omega T_a + \dots - N \frac{\pi}{2} - \tan^{-1} \omega T_1 - \dots - \tan^{-1} \left(\frac{2\xi\omega\omega_n}{\omega_n^2 - \omega^2} \right)$$

ω	ϕ
$1/T_a$	—
$1/T_1$	—
ω_n	✓

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Sketch the Bode plot for the transfer function

$$G(s) = \frac{10}{s(1+0.5s)(1+0.01s)}$$

The corner frequencies are.

$$\omega_1 = \frac{1}{T} = \frac{1}{0.5} = 2 \text{ rad/sec}$$

$$\omega_2 = \frac{1}{0.01} = 100 \text{ rad/sec.}$$

For Magnitude plot.

<u>Terms</u>	<u>Corner frequency</u>	<u>slope of term</u>	<u>Total slope</u>
$\frac{10}{s}$	—	-20dB/decade	-20dB/decade
$\frac{1}{1+0.5s}$	2	-20dB/decade	-40dB/decade
$\frac{1}{1+0.01s}$	100	-20dB/decade	-60dB/decade

Starting

for $\omega = 1$

$$20 \log_{10} \left| \frac{10}{j} \right| = 20 \log_{10} 10 - 20 \log_{10} |j|$$
$$= 20 - 0 = \underline{20 \text{ dB}}$$

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For the term $\frac{K}{s}$ slope -20 dB/dec

at $\omega_1 = 1$, $M_1 = 20 \text{ dB}$.

$\omega_2 = 10$, $M_2 = 20 - 20 \text{ dB} = 0 \text{ dB}$.

For ~~the~~ the term $\frac{1}{1+0.5s}$

starting point $\omega_1 = 2$, slope -40 dB/dec .

$\omega_1 = 2$, $M_1 = 14 \text{ dB}$.

$\omega_2 = 20$, $M_2 = 14 \text{ dB} - 40 \text{ dB} = -26 \text{ dB}$

For the term $\frac{1}{1+0.01s}$

starting point $\omega_1 = 100$, slope -60 dB/dec

$\omega_1 = 100$, $M_1 = -53 \text{ dB}$

$\omega_2 = 1000$, $M_2 = -53 - 60 = -113 \text{ dB}$

The values are out of the graph area.

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Converting the decade slope to octave slope.

$$20 \text{ dB/decade} = 6 \text{ dB/octave}$$

$$-60 \text{ dB/decade} = -18 \text{ dB/octave}$$

$$\text{now } \omega' = 200 \quad M' = -33 - 18 = -71 \text{ dB}$$

$$\phi = \cancel{\pi/2} - \pi/2 - \tan^{-1} 0.5\omega - \tan^{-1} 0.01\omega$$

ω	ϕ
0	0
1	-28°
2	-47°
20	-87°
100	-135°
200	-154°
1000	-220°
∞	
∞	

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