Lecture notes on

STRENGTH OF MATERIALS

3rd semester Mechanical engineering

Prepaired by

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① Strength of Materials
(3rd Sem) Mechanical Engg.
(01) Simple stress and strain

Types of Load

Generally a load may act in 3-ways.

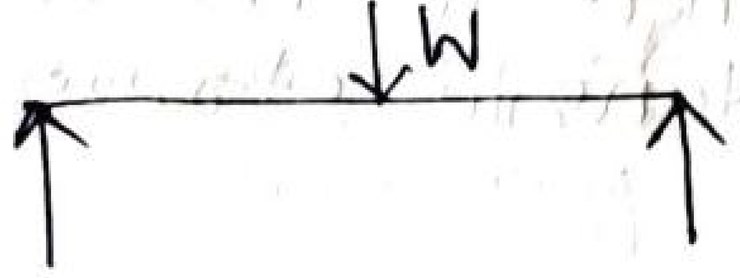
a) Gradually applied Load – In which the load starts from zero & increases gradually till the body is fully loaded.

b) Suddenly applied load – load applied suddenly on to the body.

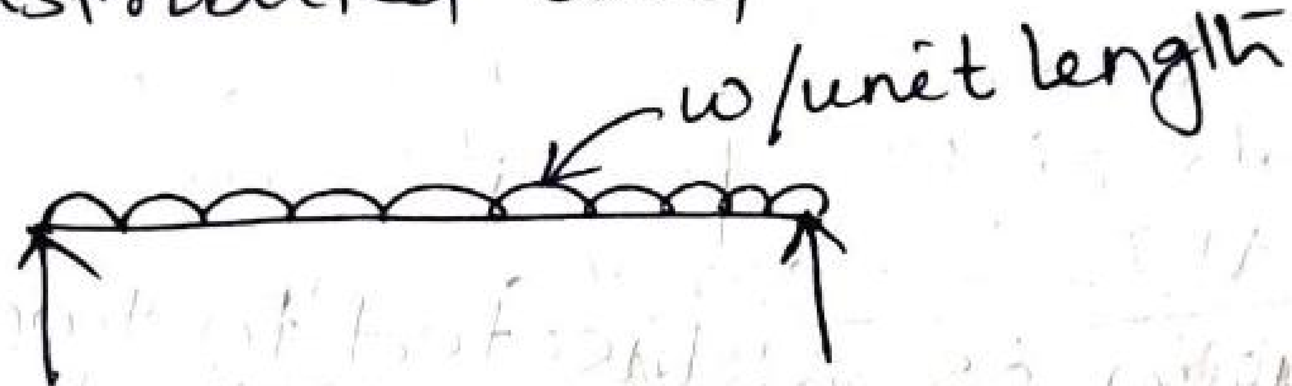
c) Impact Load – In which load applied with some impact.

For beams as in SF & BM

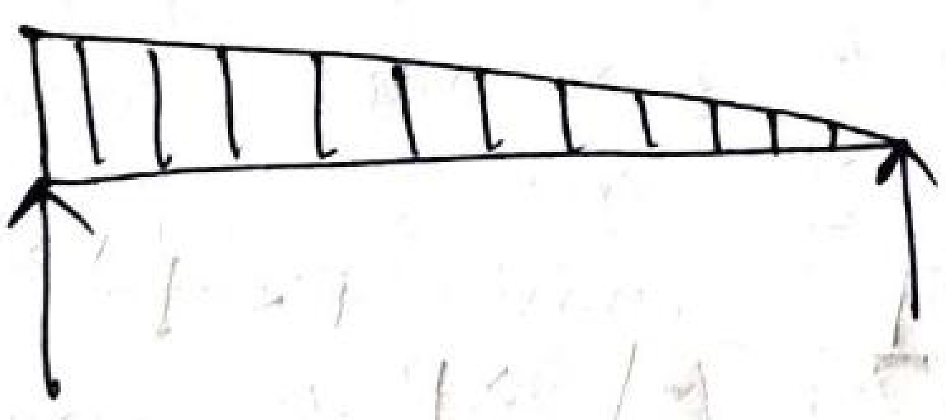
a) Point load / concentrated load



b) Uniformly distributed load



c) Uniformly varying load



Stress

Every material is elastic in nature. So, when some force (external) acts on a body, it undergoes some deformation. Under this its molecules setup some resistance to deformation. This resistance to deformation per unit area is known as "Stress".

* The force of resistance offered by a body against deformation is called as "Stress".

Mathematically,

$\sigma = P/A$, P = Load, A = Cross-sectional area of the body

Unit – Pa = $1 N/m^2$
MPa = $1 N/mm^2$, GPa = $1 KN/mm^2$

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Strain

Strain – Whenever a system of force acts on a body it undergoes some deformation. This deformation per unit length is called "Strain".

Mathematically,

$$\epsilon = \frac{\delta l}{l}, \quad \delta l = \text{change in length}, \quad l = \text{original length}$$

Unit – no unit

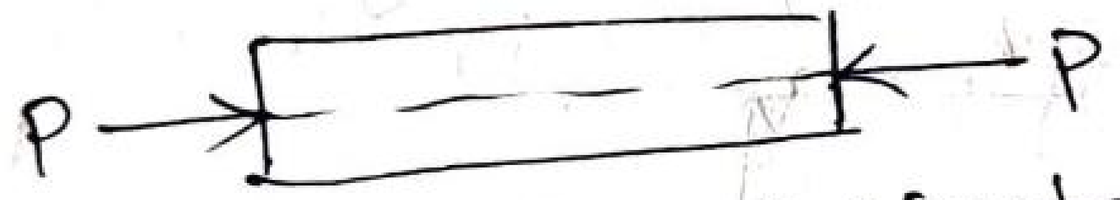
Types of stress

a) Tensile & b) Compressive

Tensile

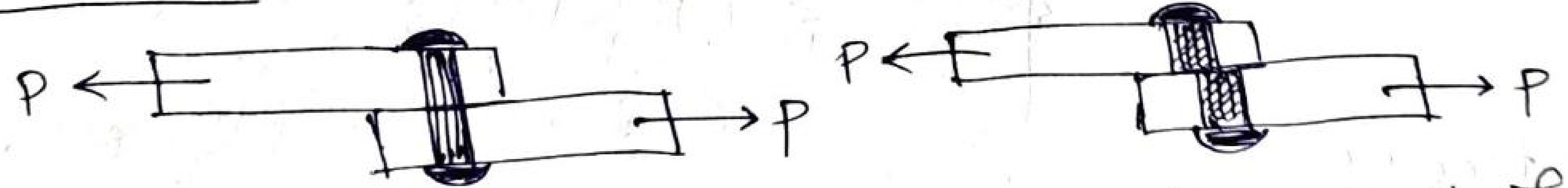
When a section is subjected to two equal & opposite pulls then body tends to increase its length. The stress induced is called tensile stress & corresponding strain is tensile strain.

Compressive

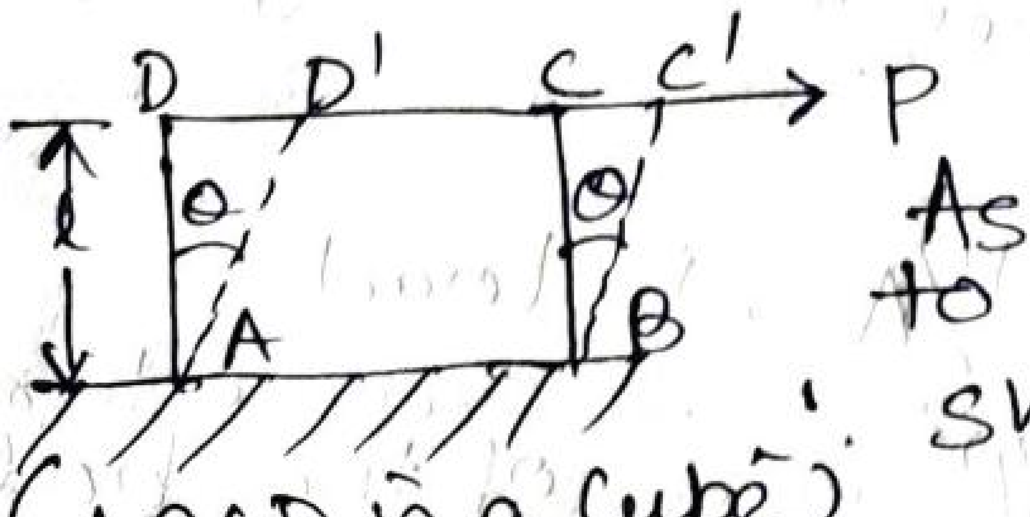


When a section is subjected to two equal & opposite pushes then the body tends to shorten its length, then the stress induced is called compressive stress & the corresponding strain is compressive strain.

Shear stress



When a section is subjected to two equal & opposite forces acting tangentially across the resting section as a result of which the body tends to shear off across the section as shown above. Then the stress induced is called shear stress. The corresponding strain is shear strain.



(ABCD is a cube)

face AB = fixed

P = force applied tangentially

shear stress $(\tau) = P/AB$

As a result of 'P' ABCD distorted to ABC_1D_1 through an angle 'ϕ'.

shear strain $= CC_1/l = \phi$

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Elastic Constants

Elastic Limit – If a load on the body is increased the deformation will also increase. The limit upto which the material behaves as perfectly elastic is called as elastic limit.

Poission's ratio – Within elastic limit, it is the ratio between lateral strain to linear strain.

$$\therefore \text{Poission's ratio} = \frac{\text{Lateral strain}}{\text{Linear strain}} = \frac{\delta d/d}{\delta l/l}$$

It is denoted by $\frac{1}{m}$ or μ. It has no unit.

Linear strain – The deformation of a bar per unit length in the direction of the force i.e. $\frac{\delta l}{l}$ is called as Primary or Linear strain.

Lateral strain – Every direct stress is always accompanied by a strain in its own direction & an opposite strain in every direction at 90° to it. Such a strain called as Secondary or Lateral strain.

Young's modulus or Modulus of elasticity (E) –
It is the ratio of linear stress to the corresponding strain. It is denoted by E.

$$\therefore E = \frac{\text{linear stress}}{\text{linear strain}} = \frac{P/A}{\delta l/l} = \frac{Pl}{A(\delta l)}$$

Modulus of rigidity or shear modulus (N or C) –
It is the ratio of shear stress to the corresponding shear strain. It is denoted by C, G or N.

$$C \text{ or } G \text{ or } N = \frac{\text{Shear stress}}{\text{Shear strain}} = \frac{\tau}{\phi}$$

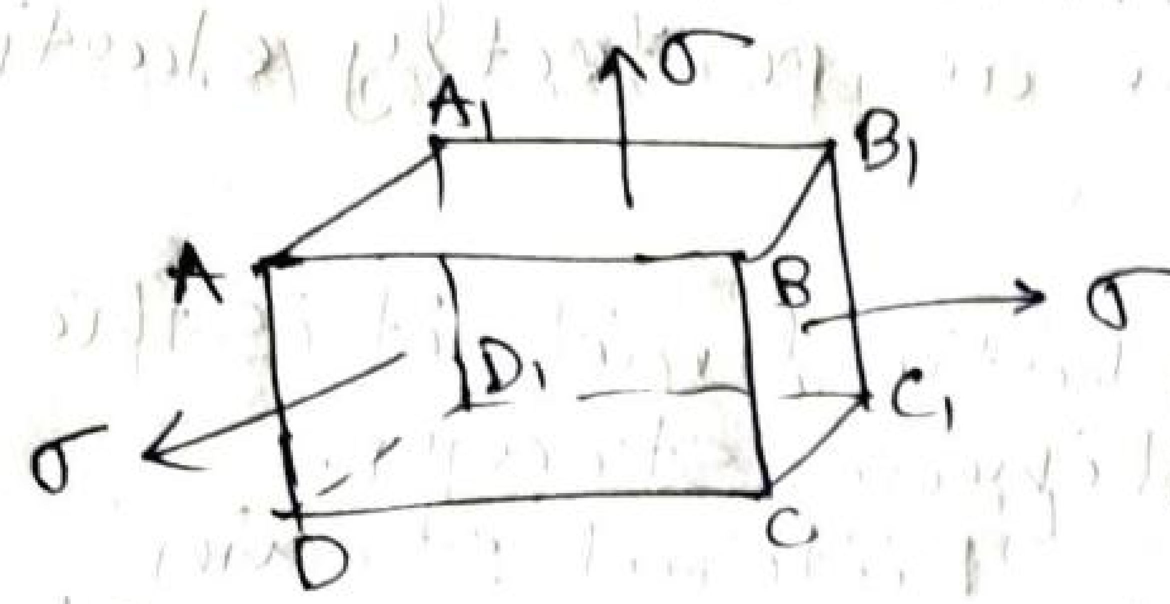
where, τ = shear stress, ϕ = shear strain

Bulk modulus (K) – It is the ratio of direct stress to corresponding volumetric strain, when a body is subjected to stresses in more than one direction.

$$\therefore K = \frac{\text{Direct stress}}{\text{volumetric strain}} = \frac{\sigma}{\delta V/V}$$

Relation between Elastic Constants

Relation between E & K



Take a cube $ABCDA_1B_1C_1D_1$ as shown & subjected to 3-mutually perpendicular tensile stresses of equal intensity.

Let σ = Stress on the faces
l = length of the cube
E = Young's modulus
K = Bulk modulus

Now consider the deformation of side AB. Then tensile strain occurs on faces BB_1CC_1 & AA_1DD_1. compressive lateral strain occurs on faces AA_1BB_1 & DD_1CC_1 & faces ABCD and $A_1B_1C_1D_1$.

$$\therefore \text{Net tensile strain} = \frac{\delta l}{l} = \frac{\sigma}{E} - \left(\frac{1}{m}\times\frac{\sigma}{E}\right) - \left(\frac{1}{m}\times\frac{\sigma}{E}\right)$$

$$= \frac{\sigma}{E}\left(1-\frac{2}{m}\right) \quad \text{—(1)}$$

$\therefore$ Original volume of the cube is $V = l^3$

Now differentiating the above equation w.r.t. l $\longrightarrow$

$$\frac{\delta V}{\delta l} = 3l^2$$

$$\text{or } \delta V = 3l^2 \cdot \delta l = 3l^3 \times \frac{\delta l}{l}$$

Now put the value of $\frac{\delta l}{l}$ in eqn (1) $\longrightarrow$

$$\delta V = 3l^3 \times \frac{\sigma}{E}\left(1-\frac{2}{m}\right)$$

$$\text{or } \frac{\delta V}{V} = \frac{3l^3}{l^3} \times \frac{\sigma}{E}\left(1-\frac{2}{m}\right) = \frac{3\sigma}{E}\left(1-\frac{2}{m}\right)$$

$$\therefore \frac{\delta V}{V} = \frac{3\sigma}{E}\left(1-\frac{2}{m}\right)$$

$$\therefore \frac{\sigma}{\frac{\delta V}{V}} = \frac{E}{3}\left(\frac{1}{1-\frac{2}{m}}\right) = \frac{E}{3}\times\frac{1}{\frac{m-2}{m}}$$

$$\text{or } \boxed{K = \frac{mE}{3(m-2)}}$$

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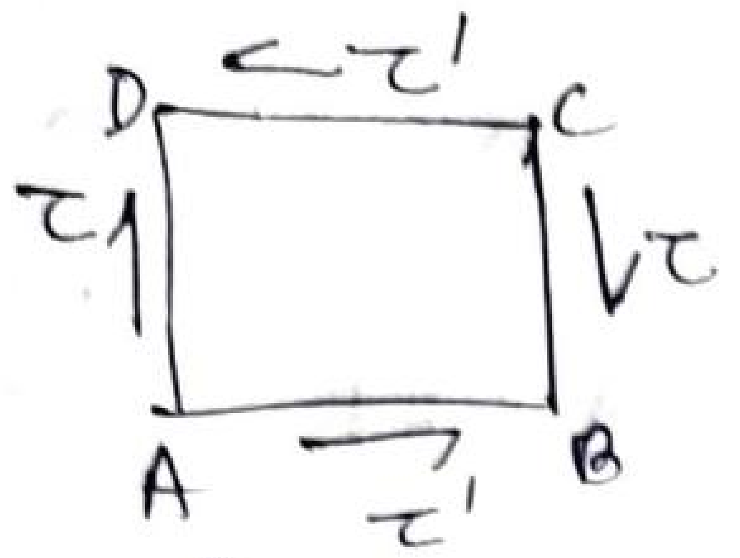
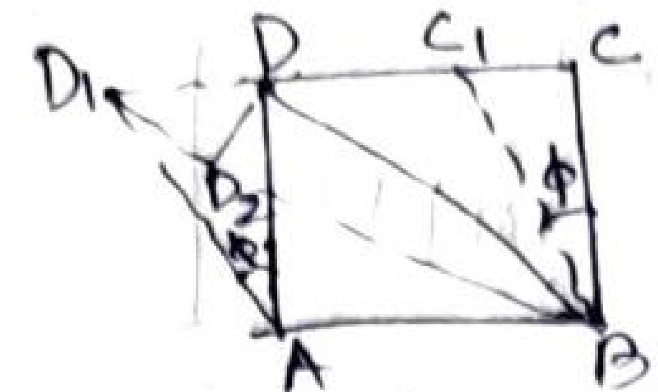
Relation between E & C



(Before distortion)



(After distortion)

ABCD is a cube having length 'l' & is subjected to shear stress τ. Due to these stresses ABCD is subjected to some distortion such that BD will be elongated & AC will shortened.

Let this τ cause shear strain 'ϕ'.

From fig., BD is distorted to BD_1.

$$\therefore \text{ strain of } BD = \frac{BD_1 - BD}{BD} = \frac{D_1D_2}{BD} = \frac{DD_1 \cos 45^\circ}{AD\sqrt{2}}$$

$$= \frac{DD_1}{2AD} = \frac{\phi}{2}$$

$\therefore$ Linear strain of BD $= \frac{1}{2}$ of the shear strain (tensile in nature)

Similarly, linear strain of AC $= \frac{1}{2}$ of shear strain (compressive in nature)

$$\therefore \text{ Linear strain of } BD = \frac{\phi}{2} = \frac{\tau}{2C}$$

τ = shear stress, C = modulus of rigidity $= \frac{\tau}{\phi}$

$$\text{or } \frac{\phi}{2} = \frac{\tau}{2C} \quad \text{—(1)}$$

Let consider the shear stress τ now acting on AB, CD, CB & AD.

We know that because of this stress – BD subjected to tensile & AC subjected to compressive.

$\therefore$ Tensile strain on BD due to tensile stress $= \frac{\tau}{E}$ —(2)

$\therefore$ Tensile strain on BD due to compressive stress on AC $= \frac{1}{m} \times \frac{\tau}{E}$ —(3)

$\therefore$ Combined effect of two stresses on BD $= \frac{\tau}{E} + \frac{1}{m} \times \frac{\tau}{E}$

$$= \frac{\tau}{E}\left(1 + \frac{1}{m}\right)$$

$$= \frac{\tau}{E}\left(\frac{m+1}{m}\right) \quad \text{—(4)}$$

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Equating equation (1) & (4) →

$$\frac{\tau}{2C} = \frac{\tau}{E}\left(\frac{m+1}{m}\right)$$

or $$\boxed{C = \frac{mE}{2(m+1)}}$$

Relationship between E, C & K

$$E = \frac{9KC}{3K+C}$$, E = Young's modulus
K = Bulk modulus
C = Shear modulus

Hooke's law

The variation of stress in direct proportion to strain is called Hooke's law.

When a material is loaded within elastic limit, the stress is proportional to the strain.

Mathematically,

$$\frac{\text{Stress}}{\text{Strain}} = E = \text{Constant}$$

No1 An alloy specimen has a modulus of elasticity of 120 GPa & modulus of rigidity of 45 GPa. Find Poission's ratio of the material.

Soln. Given, E = 120 GPa
C = 45 GPa.

We know that, $C = \frac{mE}{2(m+1)}$

or $45 = \frac{m \times 120}{2(m+1)} = \frac{120m}{2m+2}$

or, $90m + 90 = 120m$ or $30m = 90$

or $m = 3$

or $\frac{1}{m} = \frac{1}{3} = 0.3$

where, $\frac{1}{m}$ = Poission's ratio

Ans.

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No2 A steel rod 1m long & 20mm x 20mm in cross-section is subjected to a tensile force of 40KN. Find the elongation of the rod, if modulus of elasticity for the rod material is 200GPa.

Soln

Given

$l = 1m = 1 \times 10^3 mm$

$A = 20 \times 20 = 400 mm^2$

$force (P) = 40KN = 40 \times 10^3 N$

$E = 200 GPa = 200 \times 10^3 N/mm^2$

$\therefore$ elongation of the rod,

$$\delta l = \frac{Pl}{AE} = \frac{(40 \times 10^3) \times (1 \times 10^3)}{400 \times (20 \times 10^3)}$$

or, $\delta l = 0.5 mm$ Ans

Deformation of a body due to force acting on it —

Take a body subjected to a tensile stress.

Let P = Load acting on the body
l = Length of the body
A = cross-sectional area of the body
σ = stress induced in the body
E = Modulus of elasticity of the body material
ϵ = Strain
δl = Deformation of body

We know that,

$$stress(\sigma) = \frac{P}{A}, \quad Strain(\epsilon) = \frac{\sigma}{E} = \frac{P}{AE}$$

& $$deformation(\delta l) = \epsilon \cdot l = \frac{Pl}{AE}$$

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No1 A steel rod 1m long & 20mm X 20mm in cross-sectional area is subjected to a tensile force of 40kN. Find the deformation of the rod. Take E = 200GPa.

Soln Given

$l = 1m = 1\times10^3 mm$

$A = 20\times20 = 400 mm^2$

$P = 40kN = 40\times10^3 N$

& $E = 200 GPa = 200\times10^3 N/mm^2$

∴ We know that,

deformation of the rod $(\delta l) = \frac{Pl}{AE}$

or $\delta l = \frac{(40\times10^3)\times(1\times10^3)}{400\times200\times10^3} = 0.5 mm$ Ans

No2 A hollow cylinder 2m long has an outside dia. 50mm & inside dia 30mm. If the cylinder is carrying a load of 25kN, find the stress. Also find deformation of the cylinder. Take E = 100 GPa.

Soln Given

$l = 2m = 2\times10^3 mm$

Outside dia. $(D) = 50mm$

Inside dia $(d) = 30mm$

$P = 25kN = 25\times10^3 N$

$E = 100 GPa = 100\times10^3 N/mm^2$

∴ Area $(A) = \frac{\pi}{4}(D^2 - d^2) = \frac{\pi}{4}(50^2 - 30^2) = 1257 mm^2$

Then $\sigma = \frac{P}{A} = \frac{25\times10^3}{1257} = 19.9 N/mm^2 = 19.9 MPa$ Ans

Also, we know that,

$\delta l = \frac{Pl}{AE} = \frac{(25\times10^3)\times(2\times10^3)}{1257\times(100\times10^3)} = 0.4mm$

Ans

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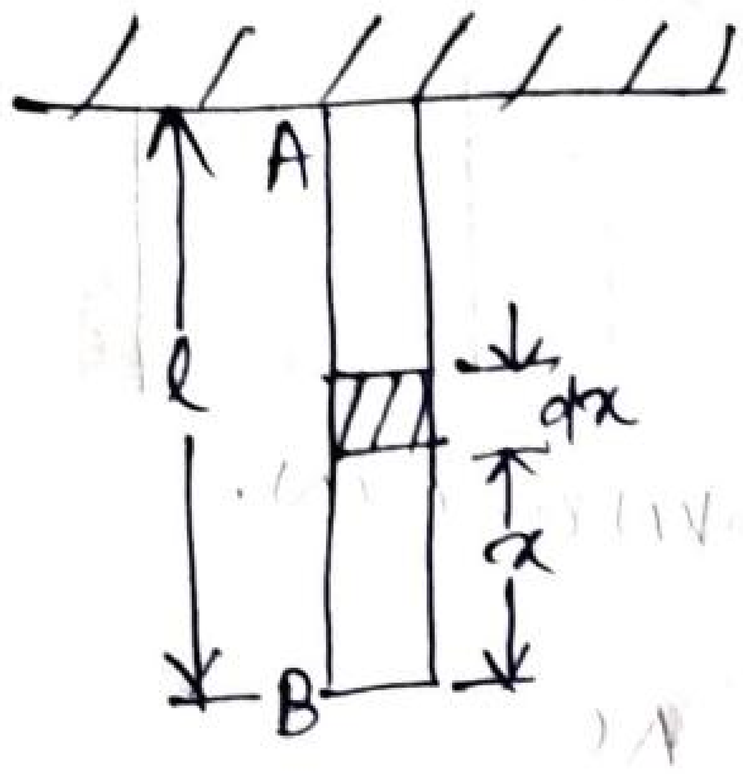
(01) Simple stress & strain

Deformation of a body due to self weight



Consider a bar AB hanging freely under its weight.

Let l = length of AB
A = Cross-sectional area of AB
E = Young's modulus of material of AB
ω = Sp. weight of material of AB

Now take a section dx of the bar AB at a distance 'x' from B.

∴ Weight of AB for a length 'x' ⟶

$$P = \omega A x$$

∴ Elongation of small section $= \frac{Pl}{AE} = \frac{\omega A x \cdot dx}{AE} = \frac{\omega x \cdot dx}{E}$

∴ Total elongation of AB is ——

$$\delta l = \int_0^l \frac{\omega x \, dx}{E} = \frac{\omega}{E}\int_0^l x \cdot dx = \frac{\omega}{E}\left[\frac{x^2}{2}\right]_0^l = \frac{\omega l^2}{2E}$$

or $\boxed{\delta l = \frac{Wl}{2AE}}$ (∵ $W = \omega A l$ = total weight)

Principle of Superposition

Along the length of the body, number of forces are if acting, then some are acting on its outer edges & some are in other sections, when a body is subjected to a number of forces. In such case, the forces are split up & their effects are taken on individual sections. The resulting deformation is then equal to the algebric sum of the deformations of the individual sections. Such a principle is called as Principle of superposition when the resultant deformation is to foundout.

$$\therefore \delta l = \frac{Pl}{AE} = \frac{1}{AE}(P_1 l_1 + P_2 l_2 + \cdots)$$

where, P_1 = force acting on section-1
P_2 = force acting on section-2
l_1 = length of section-1
l_2 = length of section-2

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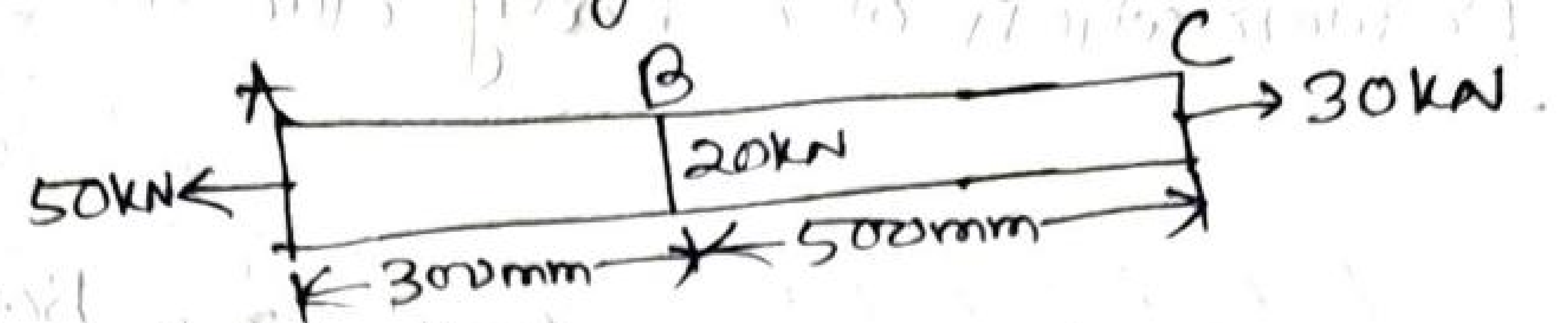
No1 A steel bar of cross-sectional area 200 mm^2 is loaded as shown. Find the change in length of the bar. Take E = 200 GPa.



Soln

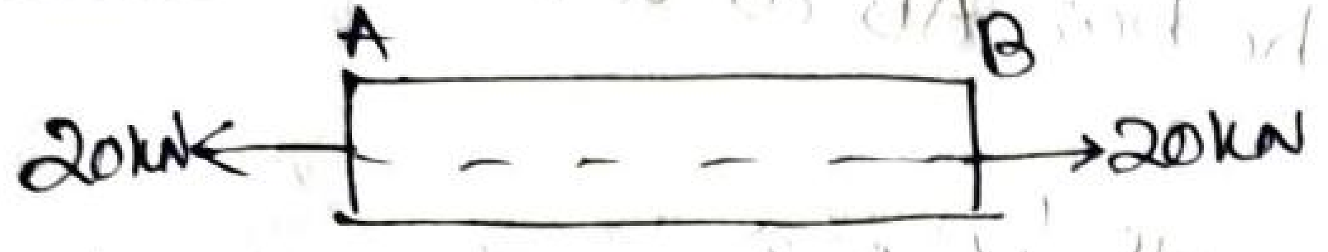
Given

$A = 200 mm^2$

$E = 200 GPa = 200 \times 10^3 N/mm^2$

The force 50 kN may split into 30 kN & 20 kN.

Part AB



Part AC

∴ Change in length of the bar —

$$\delta l = \frac{1}{AE}(P_1 L_1 + P_2 L_2)$$

$$= \frac{1}{200 \times 200 \times 10^3}(20 \times 10^3 \times 300 + 30 \times 10^3 \times 800)$$

or, $\delta l = 0.75 mm$ Ans

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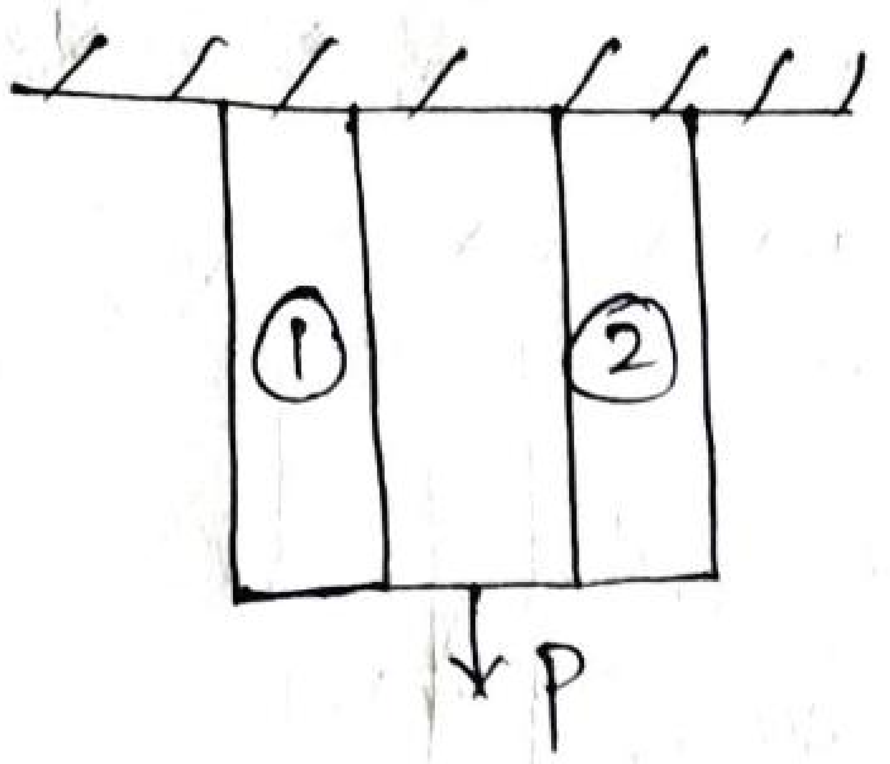
Stresses in the bar of Composite Section



Take a bar made of two different materials as shown in fig.

Let P = total load on the bar
l = length of the bar
A_1 = area of bar-1
E_1 = Modulus of elasticity of bar-1
P_1 = Load on bar-1
& A_2, E_2, P_2 = Corresponding values for bar-2

∴ Total load on the bar —— $P = P_1 + P_2$ —— (1)

For bar-1

∴ $\text{Stress}(\sigma_1) = \frac{P_1}{A_1}$ & $\text{Strain}(\varepsilon_1) = \frac{\sigma_1}{E_1} = \frac{P_1}{A_1 E_1}$

∴ $\text{Elongation}(\delta l) = \varepsilon_1 l = \frac{P_1 l}{A_1 E_1}$ —— (2)

For bar-2

$\delta l = \varepsilon_2 l = \frac{P_2 l}{A_2 E_2}$ —— (3)

As both the elongation are equal, so eqⁿ (2) & eqⁿ (3) are equal.

∴ $\frac{P_1 l}{A_1 E_1} = \frac{P_2 l}{A_2 E_2}$ or, $\frac{P_1}{A_1 E_1} = \frac{P_2}{A_2 E_2}$ —— (4)

From (4) $\longrightarrow P_2 = P_1 \times \frac{A_2 E_2}{A_1 E_1}$

But $P = P_1 + P_2 = P_1 + P_1 \times \frac{A_2 E_2}{A_1 E_1}$

$= P_1 \left(1 + \frac{A_2 E_2}{A_1 E_1}\right)$

or $P_1 = P \times \frac{A_1 E_1}{A_1 E_1 + A_2 E_2}$ —— (5)

Similarly, $P_2 = P \times \frac{A_2 E_2}{A_1 E_1 + A_2 E_2}$ —— (6)

But in eqⁿ (4) $\longrightarrow \frac{P_1}{A_1 E_1} = \frac{P_2}{A_2 E_2}$

or $\frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$ $\left(\because \sigma = \frac{P}{A}\right)$

∴ $\sigma_1 = \frac{E_1}{E_2} \times \sigma_2$

& $\sigma_2 = \frac{E_2}{E_1} \times \sigma_1$

∴ Total load $= P = P_1 + P_2 = \sigma_1 A_1 + \sigma_2 A_2$.

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Principal Stresses & Strains

Principal planes

In a strained material at any point, there are three mutually perpendicular planes which carry direct stresses only & no shear stress. Out of these three direct stresses one will be maximum, the other minimum & the third an intermediate between the two. These particular Planes, which have no shear stress are called as Principal Planes.

Principal stress

Across a principal plane, the value of direct stress is called as Principal stress.

Methods for stresses on an oblique section of a body

There are two methods for calculations.
1) Analytical method, 2) Graphical method

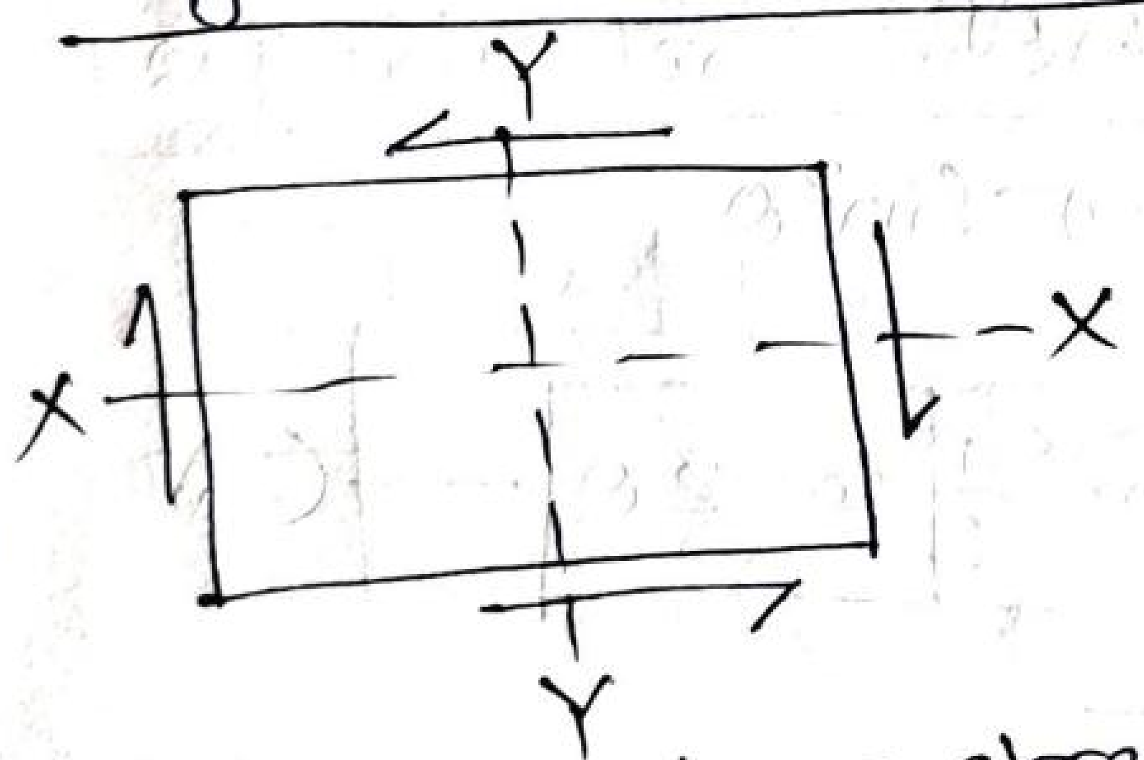
Analytical Method

For stress calculations two conditions are there.

a) when a body subjected to a direct stress in one plane

b) when a body subjected to direct stresses in two mutually perpendicular directions.

Sign conventions for analytical method



→ All the tensile stresses & strains are taken as +ve, while all the compressive stresses & strains are −ve.

→ The shear stress which tends to rotate the element in clockwise direction is taken as +ve, whereas which tends to rotate in anticlockwise direction as negative.

→ shear stress on the vertical faces (~~[illegible]~~) is +ve & shear stress on the horizontal faces is −ve.

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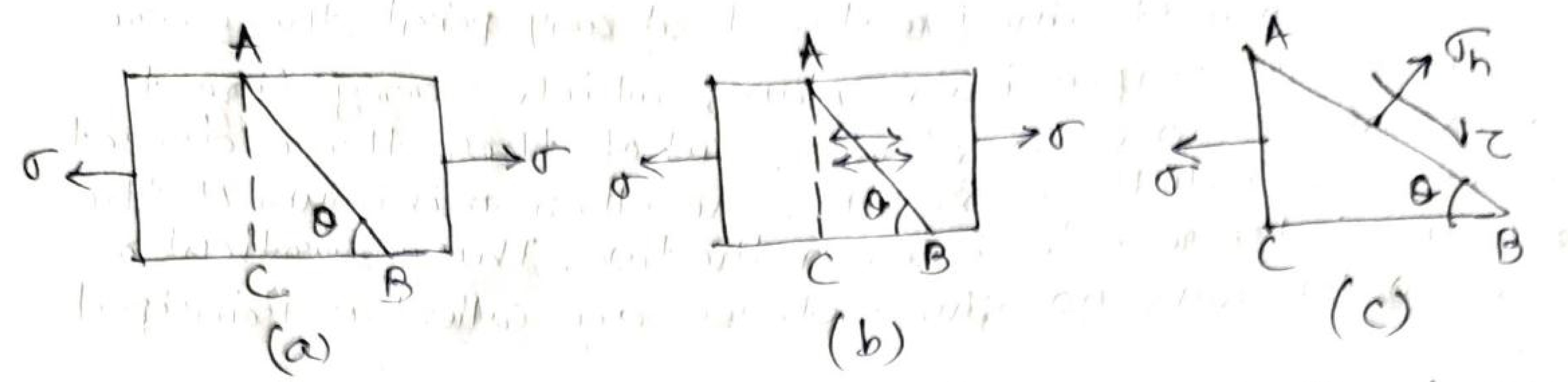
Stresses on an oblique section of a body subjected to a direct stress in one plane



Take a rectangular body of uniform cross-sectional area & unit thickness subjected to a direct tensile stress along X–X axis as shown.
Now take an oblique section AB inclined with X–X.
Let σ = tensile stress across face AC
& θ = angle made by AB with BC in clockwise direction.

Now consider the equillibrium of ABC.
Then the horizontal force acting on AC is —

$$P = \sigma \cdot AC (\leftarrow)$$

Resolving the force perpendicular to AB is —

$$P_n = P\sin\theta = \sigma AC \sin\theta \quad \text{—— (1)}$$

& now resolving the force tangential to AB is —

$$P_t = P\cos\theta = \sigma AC \cos\theta \quad \text{—— (2)}$$

∴ The normal stress across section AB ——

$$\sigma_n = \frac{P_n}{AB} = \frac{\sigma AC \sin\theta}{\frac{AC}{\sin\theta}} = \sigma \sin^2\theta$$

$$\text{or, } \boxed{\sigma_n = \frac{\sigma}{2}(1-\cos 2\theta) = \frac{\sigma}{2} - \frac{\sigma}{2}\cos 2\theta} \quad \text{—— (3)}$$

Then shear stress across AB ——

$$\tau = \frac{P_t}{AB} = \frac{\sigma AC \cos\theta}{\frac{AC}{\sin\theta}} = \sigma \sin\theta \cdot \cos\theta$$

$$\text{or } \boxed{\tau = \frac{\sigma}{2}\sin 2\theta} \quad \text{—— (4)}$$

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Pr-1 A wooden bar is subjected to a tensile stress of 5MPa. What will be the values of normal & shear stress across a section, which makes an angle of 25° with the direction of the tensile stress.

Soln

Given

Tensile stress $(\sigma) = 5\,MPa$

$\theta = 25^\circ$

Normal stress across the section

$$\therefore \sigma_n = \frac{\sigma}{2} - \frac{\sigma}{2}\cos 2\theta = \frac{5}{2} - \frac{5}{2}\cos(2\times 25^\circ)$$

$$= 2.5 - 2.5\cos 50^\circ = 2.5 - (2.5\times 0.64)$$

$$= 2.5 - 1.607 = 0.89\,MPa$$ Ans

Shear stress across the section

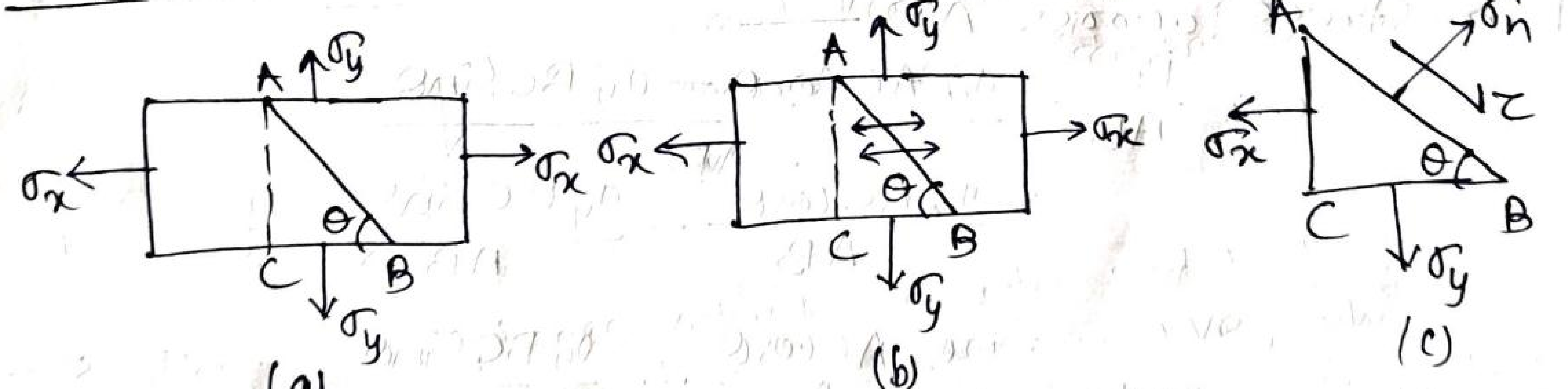
$$\tau = \frac{\sigma}{2}\sin 2\theta = \frac{5}{2}\sin(2\times 25^\circ)$$

$$= 2.5\sin 50^\circ$$

$$= 2.5\times 0.766 = 1.915\,MPa$$

Ans

Direct stresses on an oblique section of a body subjected to two mutually perpendicular directions



Consider a rectangular body of uniform cross-section & unit thickness subjected to direct tensile stresses in two mutually perpendicular directions along X–X & Y–Y. Now take an oblique section AB inclined with X–X.

Let σ_x = Tensile stress along X–X (major tensile stress)

σ_y = Tensile stress along Y–Y (minor tensile stress)

θ = angle which AB makes with X–X.

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Consider the equillibrium of section ABC.
Horizontal force acting on face AC —

$$\text{i.e. } P_x = \sigma_x \cdot AC \,(\leftarrow)$$

& vertical force acting on BC —

$$P_y = \sigma_y \cdot BC \,(\downarrow)$$

Resolving the forces normal to AB —

$$P_n = P_x \sin\theta + P_y \cos\theta = \sigma_x AC \sin\theta + \sigma_y BC \cos\theta \quad \text{—(1)}$$

& now resolving the forces tangential to AB —

$$P_t = P_x \cos\theta - P_y \sin\theta = \sigma_x AC \cos\theta - \sigma_y BC \sin\theta \quad \text{—(2)}$$

∴ Normal stress across section AB —

$$\sigma_n = \frac{P_n}{AB} = \frac{\sigma_x \cdot AC \sin\theta + \sigma_y BC \cos\theta}{AB}$$

$$= \frac{\sigma_x AC \sin\theta}{AB} + \frac{\sigma_y BC \cos\theta}{AB}$$

$$= \frac{\sigma_x AC \sin\theta}{\frac{AC}{\sin\theta}} + \frac{\sigma_y BC \cos\theta}{\frac{BC}{\cos\theta}}$$

$$= \sigma_x \sin^2\theta + \sigma_y \cos^2\theta$$

$$= \frac{\sigma_x}{2} - \frac{\sigma_x}{2}\cos 2\theta + \frac{\sigma_y}{2} + \frac{\sigma_y}{2}\cos 2\theta$$

$$\text{or, } \boxed{\sigma_n = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta} \quad \text{—(3)}$$

Shear stress across AB —

$$\tau = \frac{P_t}{AB} = \frac{\sigma_x AC \cos\theta - \sigma_y BC \sin\theta}{AB}$$

$$= \frac{\sigma_x AC \cos\theta}{AB} - \frac{\sigma_y BC \sin\theta}{AB}$$

$$= \frac{\sigma_x AC \cos\theta}{\frac{AC}{\sin\theta}} - \frac{\sigma_y BC \sin\theta}{\frac{BC}{\cos\theta}}$$

$$= (\sigma_x - \sigma_y)\sin\theta\cos\theta = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta$$

$$\text{or, } \boxed{\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta} \quad \text{—(4)}$$

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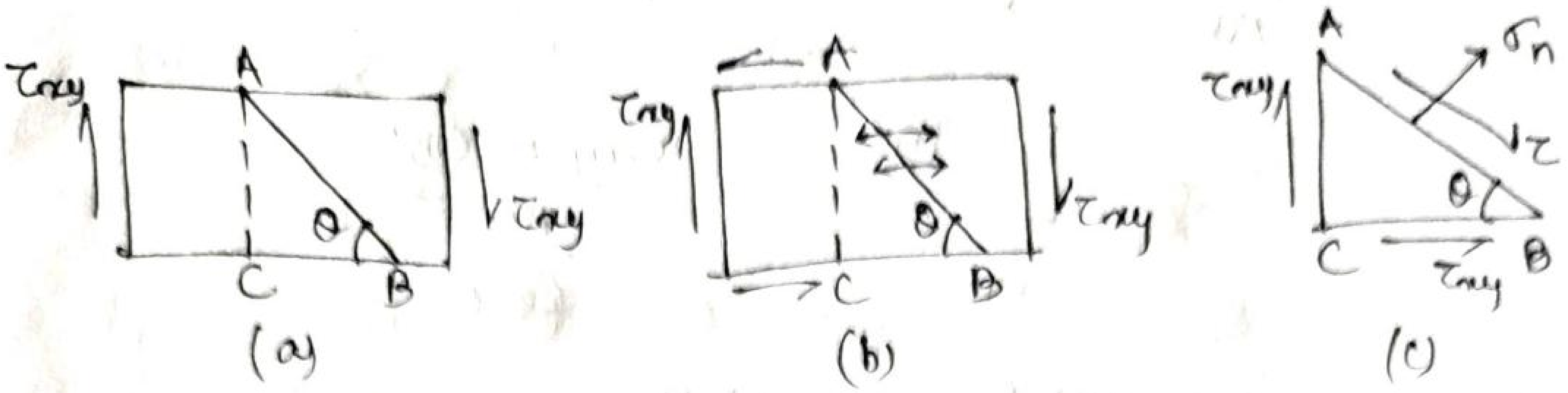
(2) Stresses on an oblique section of a body subjected to a simple shear stress



Take a rectangular body of uniform cross-sectional area & unit thickness subjected to a +ve shear stress along X-X.

Take an oblique section AB inclined with X-X.

∴ Let τ_{xy} = +ve shear stress along X-X

θ = angle made by AB along X-X in anticlockwise direction.

Consider the equillibrium of ABC.

∴ The vertical force acting on AC —

$$P_1 = \tau_{xy} AC (\uparrow)$$

& horizontal force acting on BC —

$$P_2 = \tau_{xy} BC (\longrightarrow)$$

∴ Resolving the forces perpendicular to AB is —

$$P_n = P_1 \cos\theta + P_2 \sin\theta = \tau_{xy} AC \cos\theta + \tau_{xy} BC \sin\theta$$

& resolving the forces tangential to AB is —

$$P_t = P_2 \sin\theta - P_1 \cos\theta = \tau_{xy} BC \sin\theta - \tau_{xy} AC \cos\theta$$

∴ Normal stress across AB —

$$\sigma_n = \frac{P_n}{AB} = \frac{\tau_{xy} AC \cos\theta + \tau_{xy} BC \sin\theta}{AB}$$

$$= \frac{\tau_{xy} AC \cos\theta}{\frac{AC}{\sin\theta}} + \frac{\tau_{xy} BC \sin\theta}{\frac{BC}{\cos\theta}}$$

$$= \tau_{xy} \sin\theta \cos\theta + \tau_{xy} \sin\theta \cos\theta$$

$$= 2\tau_{xy} \sin\theta \cdot \cos\theta = \tau_{xy} \sin 2\theta$$

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($\therefore$ Shear stress across AB ——

$$\tau = \frac{P_t}{AB} = \frac{\tau_{xy} BC \sin\theta - \tau_{xy} AC \cos\theta}{AB}$$

$$= \frac{\tau_{xy} BC \sin\theta}{\frac{BC}{\sin\theta}} - \frac{\tau_{xy} AC \cos\theta}{\frac{AC}{\cos\theta}}$$

$$= \tau_{xy} \sin^2\theta - \tau_{xy} \cos^2\theta$$

$$= \frac{\tau_{xy}}{2} - \frac{\tau_{xy}}{2}\cos 2\theta - \frac{\tau_{xy}}{2} - \frac{\tau_{xy}}{2}\cos 2\theta$$

$$= -\tau_{xy}\cos 2\theta$$ (-ve means the normal stress is opposite to that across AC)

Now for maximum & minimum normal stresses may found by equating the shear stress to zero.

$$\therefore -\tau_{xy}\cos 2\theta = 0$$

This equation is possible, if $\theta = 45^\circ$ or 135°
i.e. $2\theta = 90^\circ$ or 270°.

Pr-2 The stresses at a point on a component are 100 MPa (tensile) & 50 MPa (compressive). Find the value of normal & shear stress on a plane inclined at 25° with tensile stress. Also find the direction of the resultant stress & the value of maximum intensity of shear stress.

Soln Given $\sigma_x = 100$ MPa $\theta = 25^\circ$
$\sigma_y = -50$ MPa

$\therefore$ normal stress across inclined plane is $\longrightarrow$

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta$$

$$= \frac{100 + (-50)}{2} - \frac{100 + 50}{2}\cos(2 \times 25^\circ)$$

$$= 25 - 75\cos 50^\circ = -23.21 \text{ MPa} \quad \underline{\text{Ans}}$$

Shear stress on the inclined plane is $\longrightarrow$

$$\tau = \frac{\sigma_x - \sigma_y}{2}\sin 2\theta = \frac{100 - (-50)}{2}\sin(2 \times 25^\circ)$$

$$= 75 \sin 50^\circ = 57.45 \text{ MPa} \quad \underline{\text{Ans}}$$

Direction of the resultant stress

Let θ = angle which the resultant stress makes with X-X.

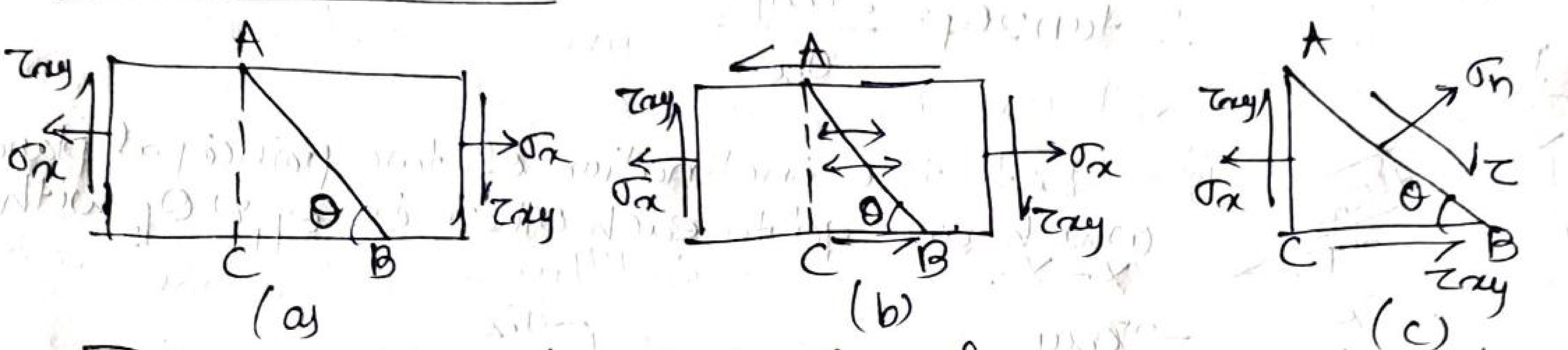
We know that, $\tan\theta = \frac{\tau}{\sigma_n} = \frac{57.45}{-23.21} = -2.4752$

or $\theta = -68^{\circ}$ Ans

Magnitude of the maximum shear stress

$$\tau_{max} = \pm \frac{\sigma_x - \sigma_y}{2} = \pm \frac{100-(-50)}{2} = \pm 75 \text{ MPa}$$ Ans

Stresses on an oblique section of a body subjected to a direct stress in one plane & accompained by a simple shear stress —



Take a rectangular body of uniform cross-sectional area & unit thickness subjected to a tensile stress along X-X accompained by +ve shear stress along X-X.

Let an oblique section AB inclined with X-X.

$\therefore$ σ_x = Tensile stress along X-X

τ_{xy} = Shear stress along X-X (+ve)

θ = angle made by AB with X-X (clockwise direction)

Consider the equillibrium of AB. As per simple shear BC subjected to -ve shear stress

$\therefore$ Horizontal force acting on AC $\rightarrow$ $P_x = \sigma_x AC (\leftarrow)$ — ①

Vertical force acting on AC $\rightarrow$ $P_y = \tau_{xy} AC (\uparrow)$ — ②

& vertical force acting on AC $\rightarrow$ ... Then horizontal force acting on BC $\rightarrow$ $P = \tau_{xy} BC (\rightarrow)$ — ③

Now resolving the forces perpendicular to AB $\rightarrow$

$$P_n = P_x \sin\theta - P_y \cos\theta - P\sin\theta$$
$$= \sigma_x AC\sin\theta - \tau_{xy} AC\cos\theta - \tau_{xy} BC\sin\theta$$

& resolving the forces tangential to AB $\rightarrow$

$$P_t = P_x\cos\theta + P_y\sin\theta - P\cos\theta$$
$$= \sigma_x AC\cos\theta + \tau_{xy} AC\sin\theta - \tau_{xy} BC\cos\theta$$

$$\therefore \sigma_n = \frac{P_n}{AB} = \frac{\sigma_x AC\sin\theta - \tau_{xy} AC\cos\theta - \tau_{xy} BC\sin\theta}{AB}$$

$$= \frac{\sigma_x AC\sin\theta}{\frac{AC}{\sin\theta}} - \frac{\tau_{xy} AC\cos\theta}{\frac{AC}{\sin\theta}} - \frac{\tau_{xy} BC\sin\theta}{\frac{BC}{\cos\theta}}$$

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or, $\sigma_n = \frac{\sigma_x}{2} - \frac{\sigma_x}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$ —— (4)

& $\tau = \frac{P_t}{AB} = \frac{\sigma_x AC\cos\theta + \tau_{xy} AC\sin\theta - \tau_{xy} BC\cos\theta}{AB}$

$= \frac{\sigma_x AC\cos\theta}{\frac{AC}{\sin\theta}} + \frac{\tau_{xy} AC\sin\theta}{\frac{AC}{\sin\theta}} - \frac{\tau_{xy} BC\cos\theta}{\frac{BC}{\cos\theta}}$

or, $\tau = \frac{\sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$ —— (5)

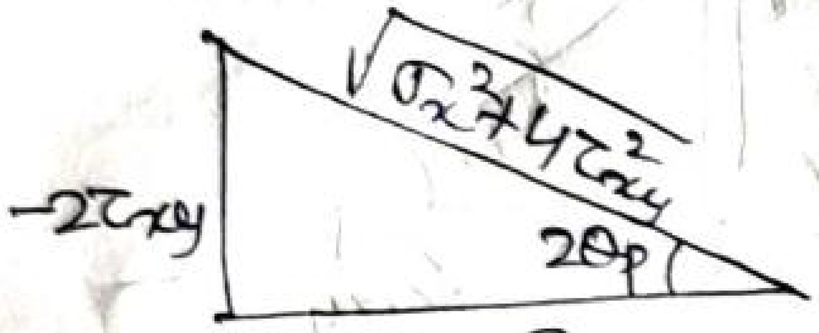
A little consideration will show that σ_x & τ_{xy} are constant & so shear stress varies with the angle θ.

Let θ_p is the value of the angle for which shear stress is zero.

$\therefore \frac{\sigma_x}{2}\sin 2\theta_p - \tau_{xy}\cos 2\theta_p = 0$ or $\frac{\sigma_x}{2}\sin 2\theta_p = \tau_{xy}\cos 2\theta_p$

$\therefore \tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x}$

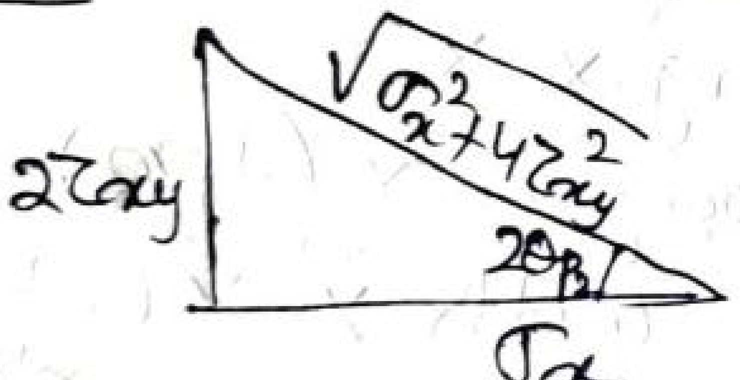
Case-I



Let the inclination of two principal planes which are 90° to each other is θ_{p_1} & θ_{p_2} with X–X.

$\therefore \sin 2\theta_{p_1} = \frac{-2\tau_{xy}}{\sqrt{\sigma_x^2 + 4\tau_{xy}^2}}$ & $\cos 2\theta_{p_1} = \frac{-\sigma_x}{\sqrt{\sigma_x^2 + 4\tau_{xy}^2}}$

Case-II



$\therefore \sin 2\theta_{p_2} = \frac{2\tau_{xy}}{\sqrt{\sigma_x^2 + 4\tau_{xy}^2}}$ & $\cos 2\theta_{p_2} = \frac{\sigma_x}{\sqrt{\sigma_x^2 + 4\tau_{xy}^2}}$

Now, the value of principal stress may found by putting the values of $2\theta_{p_1}$ & $2\theta_{p_2}$ in eqn (4).

$\therefore$ Maximum Principal stress $(\sigma_{p_1}) = \frac{\sigma_x}{2} - \frac{\sigma_x}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$

or, $\sigma_{p_1} = \frac{\sigma_x}{2} - \frac{\sigma_x}{2} \times \frac{-\sigma_x}{\sqrt{\sigma_x^2 + 4\tau_{xy}^2}} - \tau_{xy} \times \frac{-2\tau_{xy}}{\sqrt{\sigma_x^2 + 4\tau_{xy}^2}}$

or, $\boxed{\sigma_{p_1} = \frac{\sigma_x}{2} + \sqrt{\frac{\sigma_x^2}{2} + \tau_{xy}^2}}$

& Minimum Principal stress $(\sigma_{p_2}) = \frac{\sigma_x}{2} - \frac{\sigma_x}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$

or, $\sigma_{p_2} = \frac{\sigma_x}{2} - \frac{\sigma_x}{2} \times \frac{\sigma_x}{\sqrt{\sigma_x^2 + 4\tau_{xy}^2}} - \tau_{xy} \times \frac{2\tau_{xy}}{\sqrt{\sigma_x^2 + 4\tau_{xy}^2}}$

or, $\boxed{\sigma_{p_2} = \frac{\sigma_x}{2} - \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}}$

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Principal stress & Strain

No1 A plane element is subjected to a tensile stress of 100MPa accompanied by a shear stress of 25MPa. Find normal & shear stress on a plane inclined at an angle of 20° with the tensile stress & the maximum shear stress on the plane.

Soln Given

$\sigma_x = 100MPa$, $\tau_{xy} = 25MPa$, $\theta = 20°$

Normal & shear stress

$$\therefore \sigma_n = \frac{\sigma_x}{2} - \frac{\sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$= \frac{100}{2} - \frac{100}{2}\cos 40° - 25\sin 40°$$

$$= 50 - 38.3 - 16.07 = -4.37MPa \quad \underline{Ans}$$

& Shear stress $(\tau) = \frac{\sigma_x}{2}\sin 2\theta - \tau_{xy}\cos 2\theta$

$$= \frac{100}{2}\sin 40° - 25\cos 40°$$

$$= 32.14 - 19.15 = 12.99\ MPa \quad \underline{Ans}$$

Maximum shear stress on the plane

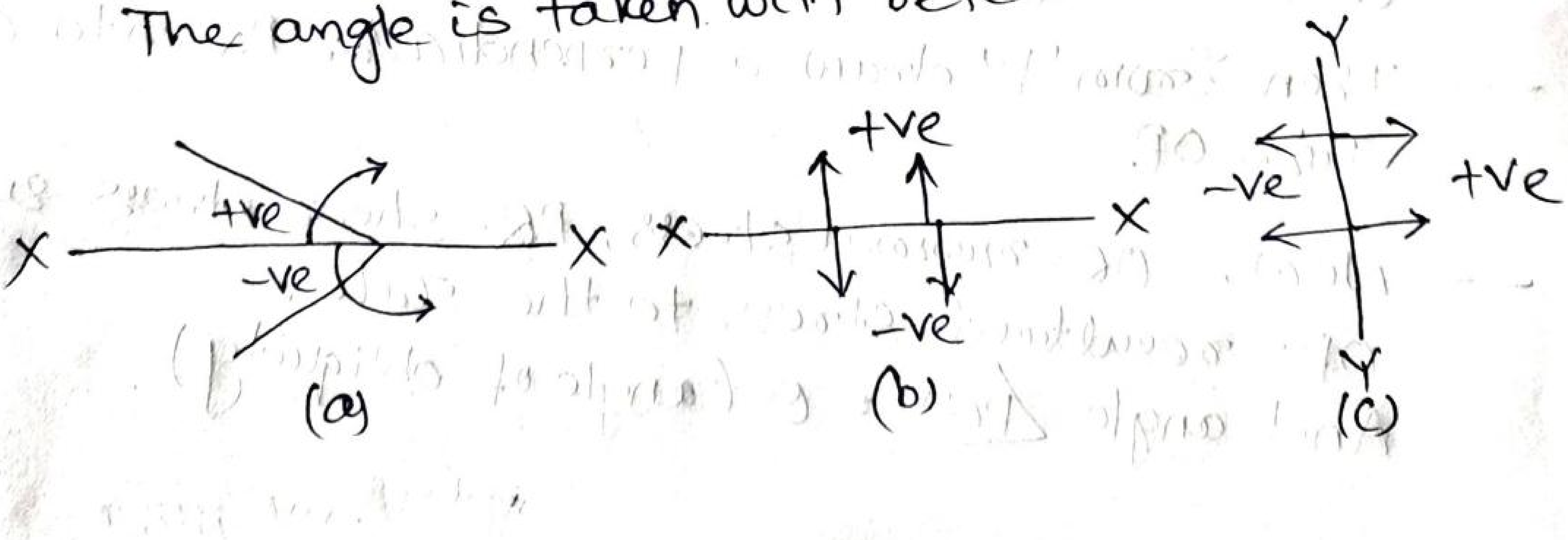
$$\therefore \tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2}$$

$$= \sqrt{\left(\frac{100}{2}\right)^2 + (25)^2} = 55.9\ MPa. \quad \underline{Ans}$$

Graphical Method (Mohr's circle)

Sign Conventions

The angle is taken with reference to X–X.



(a) (b) (c)

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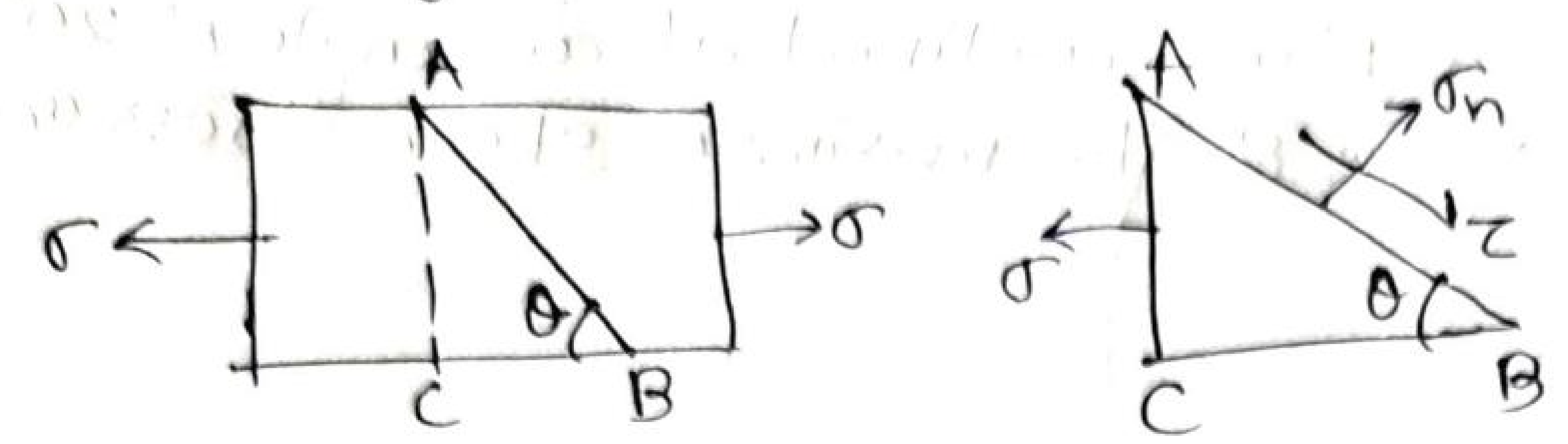
Mohr's circle for stresses on an oblique section of a body subjected to a direct stress in one plane →



Take a rectangular body of uniform cross-sectional area & unit thickness subjected to a direct tensile stress along X–X. Consider an oblique section AB inclined with X–X.

Let σ = tensile stress in X–X

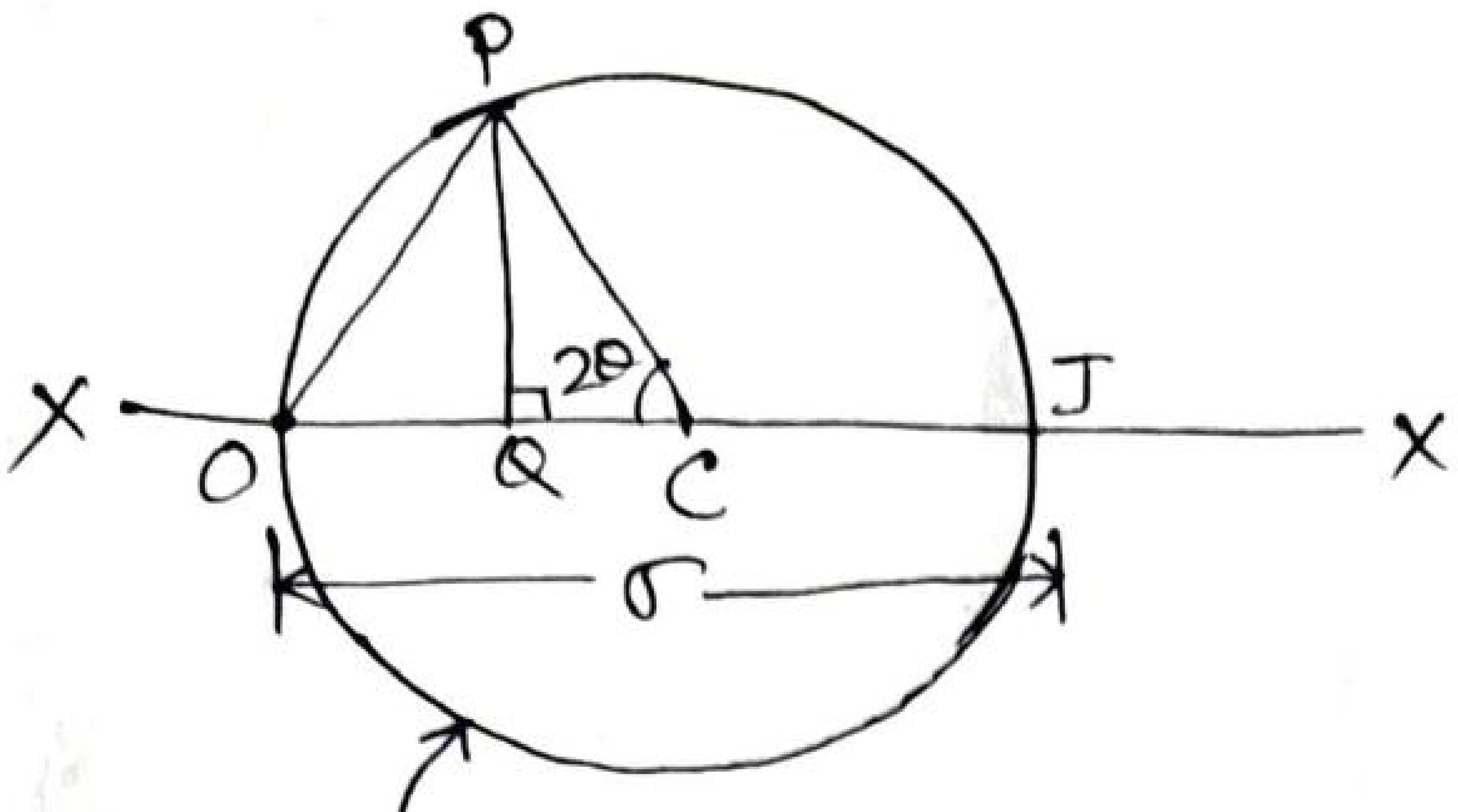
θ = angle made by AB with X–X (clockwise direction)

Consider the equillibrium of ABC.

Steps for Mohr's circle

- Take some suitable point 'O' & through 'O' draw a horizontal line XOX.
- Cut OJ equal to 'σ' to some suitable scale & towards right. Bisect OJ at C. Now 'O' represents stress system on BC, & J represents stress system on AC.
- Now 'C' as centre & radius as OC draw a circle. It is known as Mohr's circle.
- Now through 'C' draw CP by making angle 2θ with CO in the clockwise direction meeting the circle at point 'P'. Point 'P' represents AB of ABC.
- Then from 'P' draw a perpendicular PQ onto OX. Join OP.
- Now, OQ = normal stress, PQ = Shear stress & OP = resultant stress to the scale.

And angle $\angle POJ = \theta$ (angle of obliquity).



No|

A wooden bar is subjected to a tensile stress of 5 MPa. Find the values of normal & shear stresses across a section which makes an angle of 25^0 with the direction of tensile stress. (Using Mohr's circle method).

Ans – Normal stress (σ_n) = 0.89 MPa

Shear Stress (τ) = 1.9 MPa

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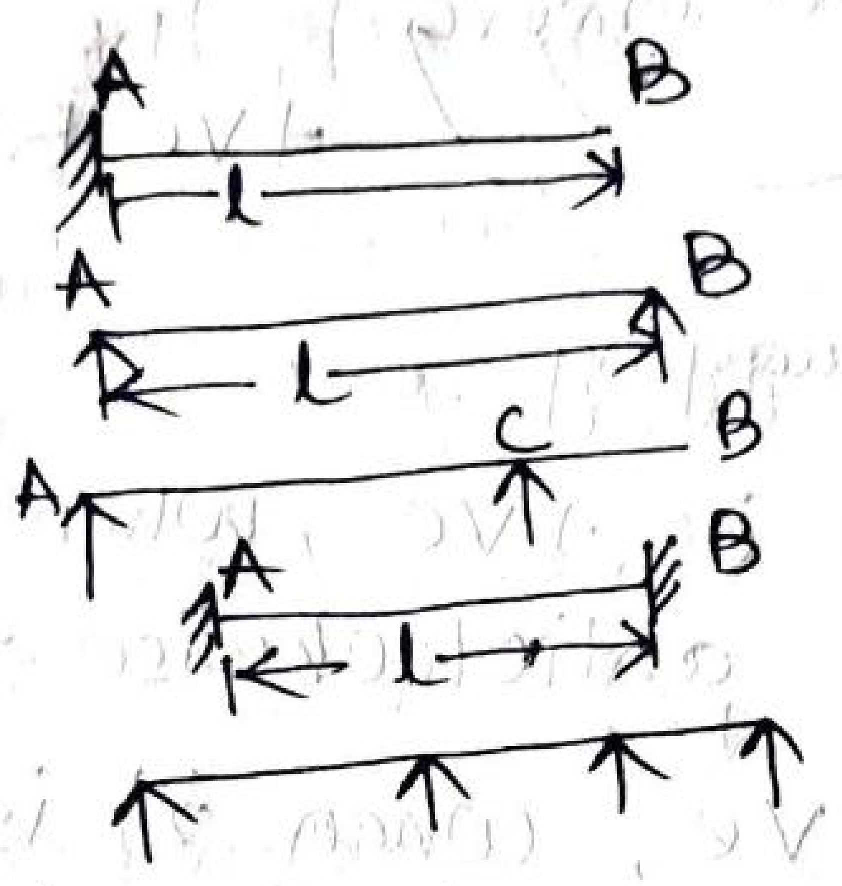
Shear force & Bending moment

Beam

A structural member which is acted upon by some external loads at 90° to its axis is called as "Beam". When a horizontal beam is loaded with vertical load then it bends due to action of the loads. The bending amount is depends upon the types of load & load amount, length of beam, type of beam & elasticity of beam materials. The scientific way of studying the deflection or any other effect is shear force & bending moment.

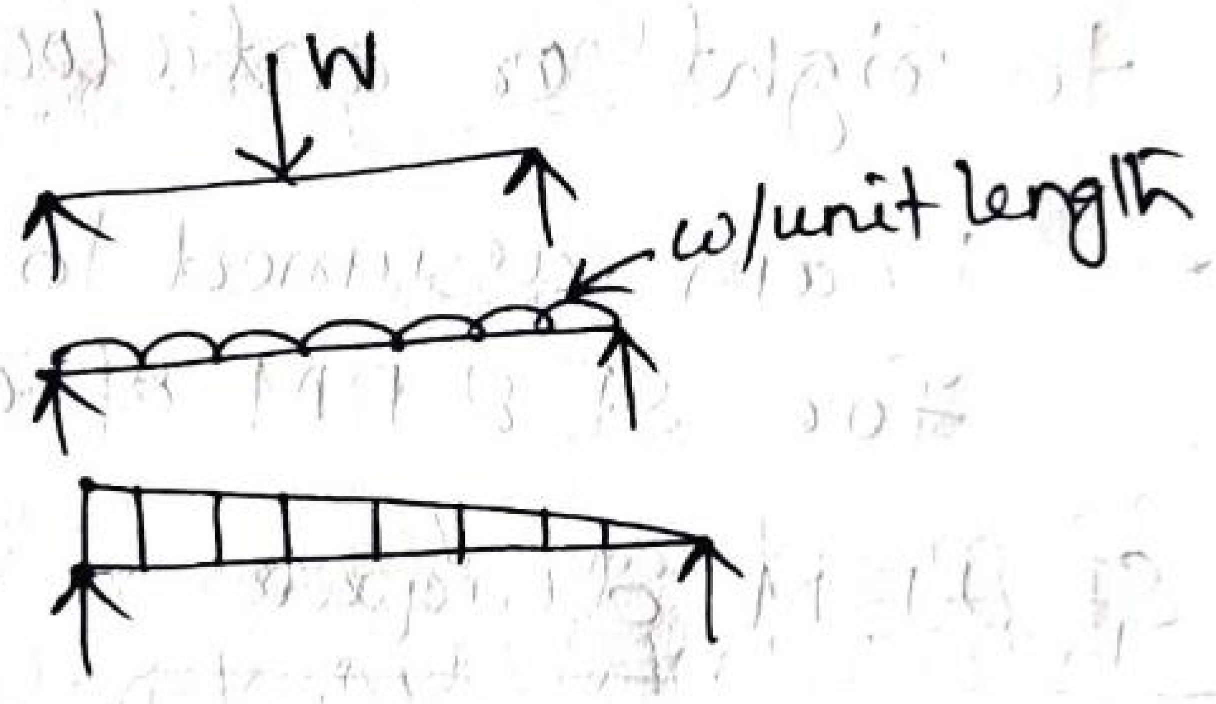
Types of beam

- Cantilever beam
- Simply supported beam
- Overhang beam
- Rigidly fixed beam
- Continuous beam



Types of loading

- Concentrated or Point load
- Uniformly distributed load
- Uniformly varying load



Shear Force (S.F.)

It is defined as the unbalanced vertical force to the right or left of the section of a beam.

Bending moment (B.M.)

It is the algebric sum of the moments of the forces to the right or left of the section.

- While calculating SF or BM at a section of a beam, the end reactions must also be considered along with other external loads.

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Sign Conventions for SF & BM

SF

SF is +ve, when left hand portion tends to slide up or right hand tends to slide down.

SF is −ve, when left hand portion tends to slide down or the right hand tends to slide up.

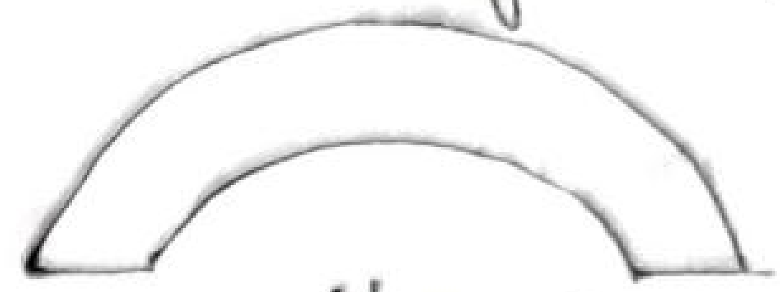
BM

Concavity at top

+ve BM

Sagging

Convexity at top



−ve BM

Hogging

Also BM is +ve, when it is acting clockwise direction to left & anticlockwise to right.

BM is −ve, when it is acting clockwise direction to right or anticlockwise to left.

— Beam assumed to be weightless while calculating for SF & BM of a beam.

SF & BM diagram

SF & BM diagram of a beam shows the manner in which way it behaves when it is loaded.

While drawing SF & BM diagrams, all +ve values are plotted above the base line & −ve below the base line.

Relation between SF & BM

- For point load, the SF line is vertical but the BM remains the same.
- For no loads, the SF line is horizontal & BM line is an inclined straight line.
- For UDL, the SF line is an inclined straight line but the BM line is a parabola.

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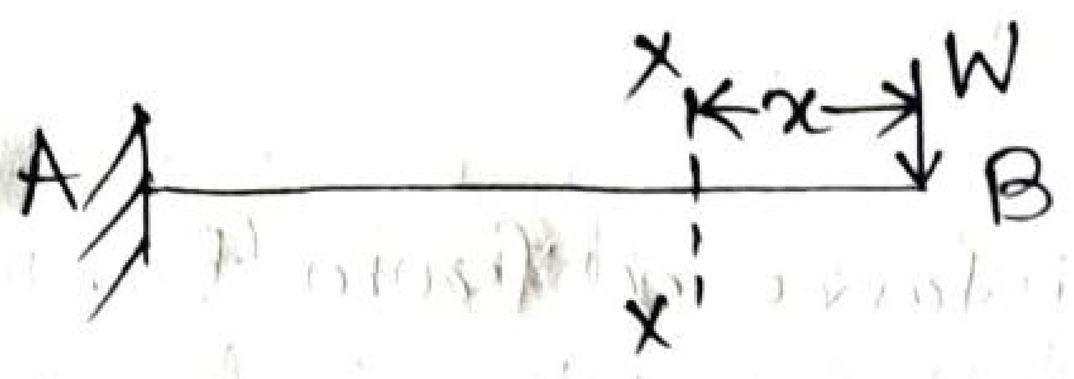
Cantilever with a Point load

Take a cantilever beam AB of length 'l' and carry a point load W at its free end B.



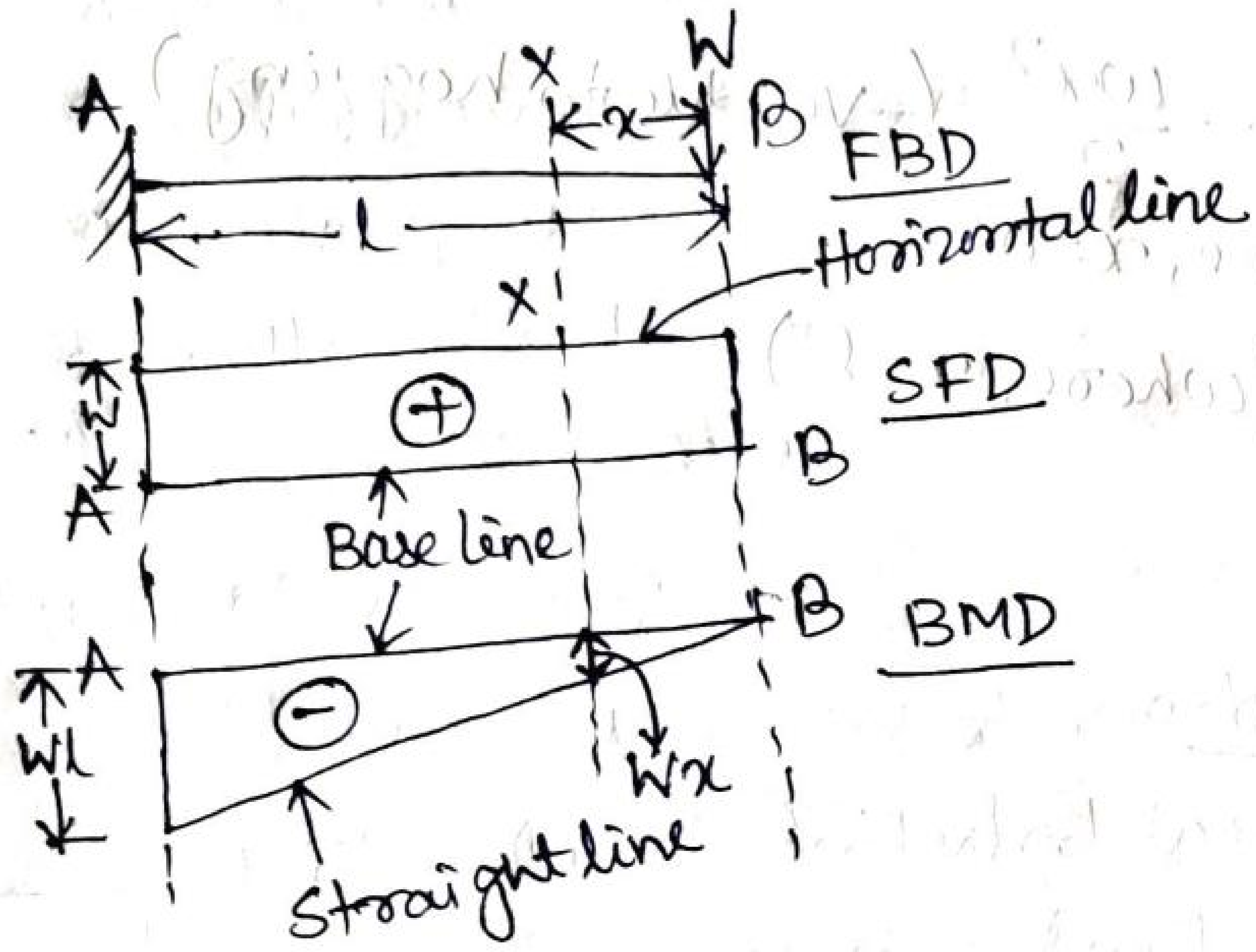
Take a section X–X at a distance 'x' from B. & is equal to the total unbalanced vertical force.

$\therefore\ F_x = +W$ (due to right downward)

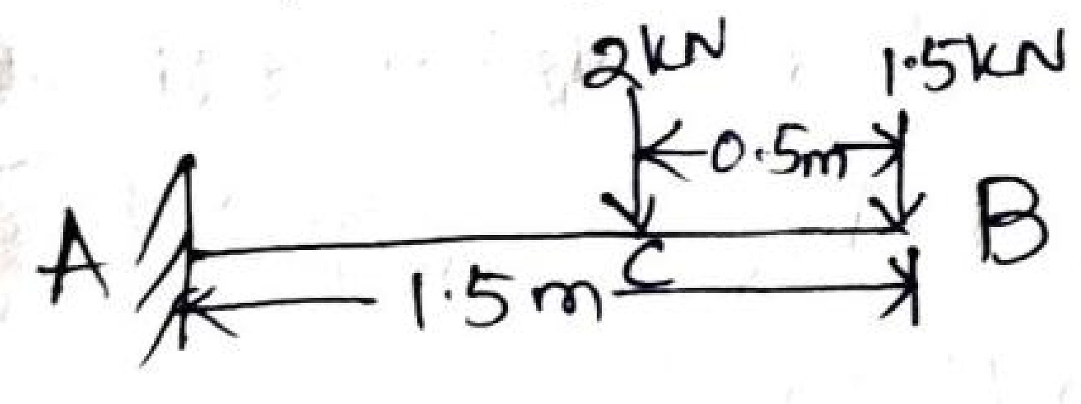
& BM is —
$M_x = -Wx$ (due to hogging)

Then BM at B — $M_B = 0$

& BM at A — $M_A = -Wl$ (where, $x = l$)



No1 Draw SF & BM diagrams for a cantilever beam of length 1·5m carrying point loads of 1·5kN & 2kN as per given diagrams.

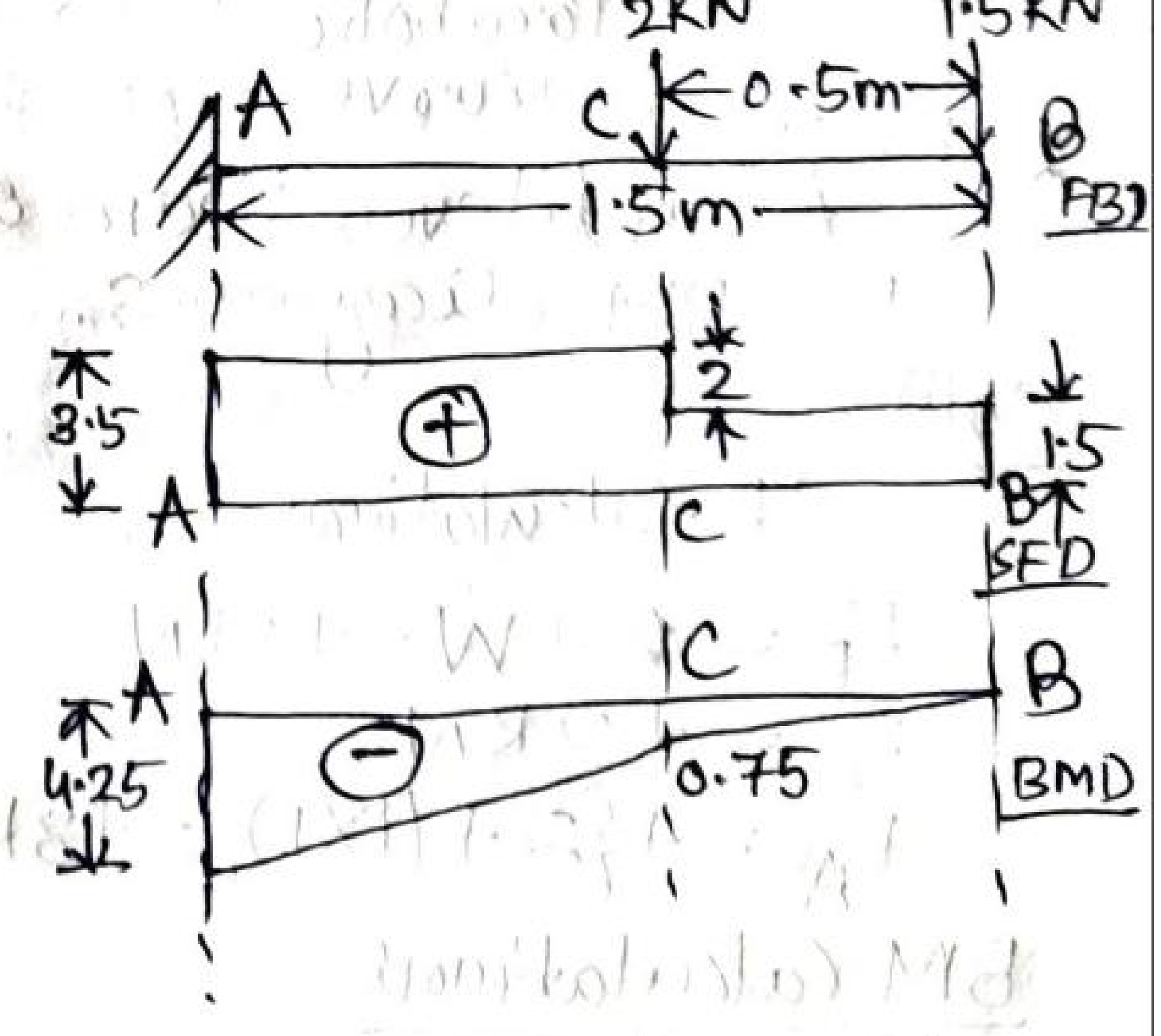


Sol^n SF Calculations

$F_B = +W_1 = +1{\cdot}5\ KN$

$F_C = +(W_1+W_2) = +(1{\cdot}5+2) = +3{\cdot}5\ KN$

& $F_A = +3{\cdot}5\ KN$



BM Calculations

$M_B = 0$

$M_C = -(1{\cdot}5 \times 0{\cdot}5) = -0{\cdot}75\ KNm$

& $M_A = -\{(1{\cdot}5 \times 1{\cdot}5) + (2 \times 1)\} = -4{\cdot}25\ KNm$

Ans

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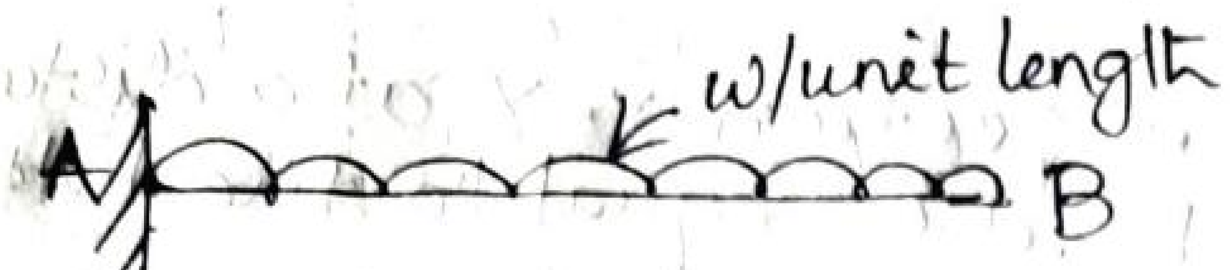
Cantilever with a UDL

Take a Cantilever beam AB of length 'l' carrying a UDL of w/unit length over the whole length of beam AB.



Consider a Section X–X at a distance 'x' from B.

SF Calculation

$F_X = + wx$ (+ve due to right downwards)

$\therefore F_B = 0$ (where, $x=0$)

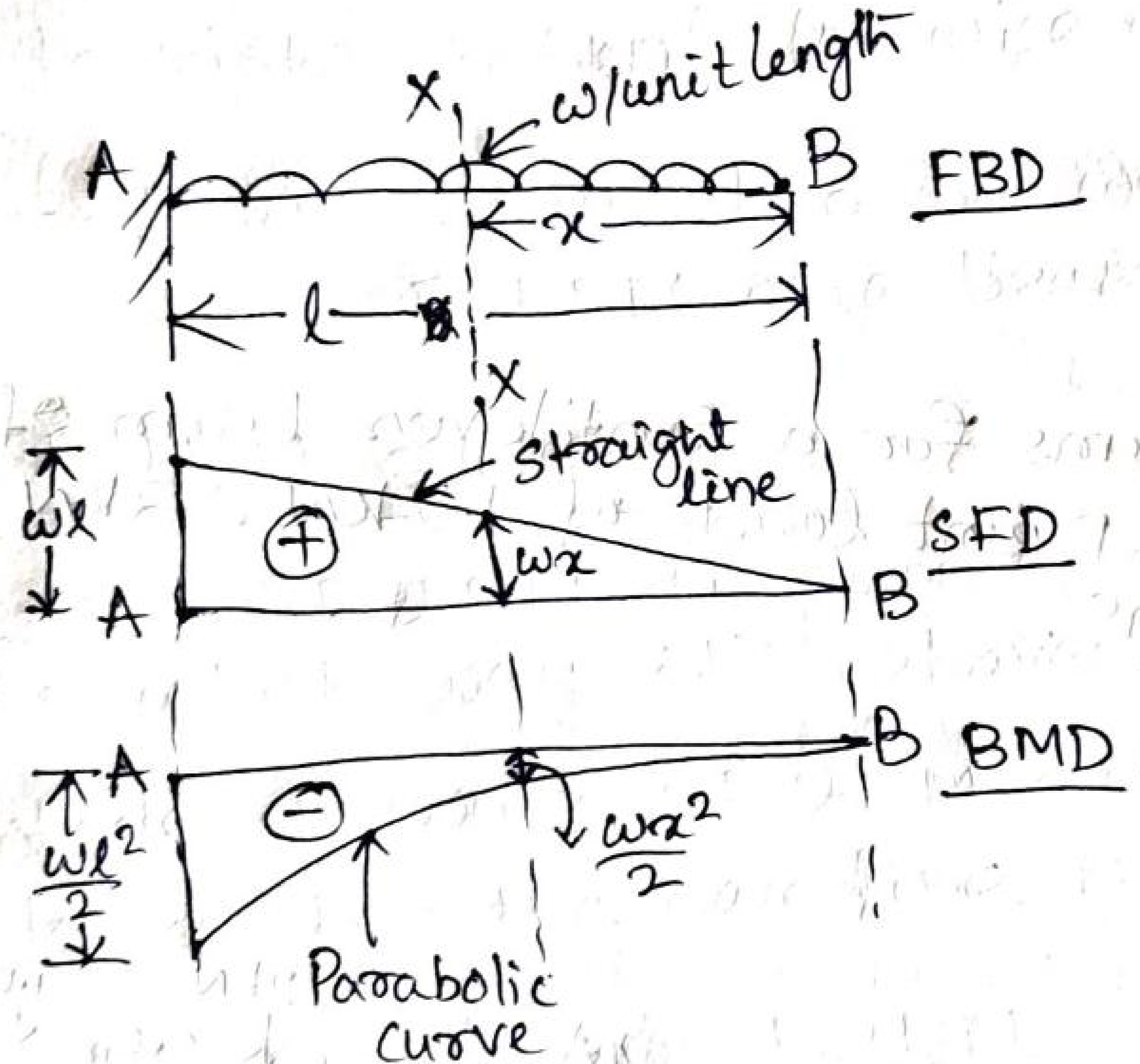
& $F_A = + wl$ (where, $x = l$)

BM Calculation

$M_X = -wx \cdot \frac{x}{2} = -\frac{wx^2}{2}$ (-ve due to hogging)

$M_B = 0$, (where, $x=0$)

& $M_A = -\frac{wl^2}{2}$ (where, $x=l$)



No1 A cantilever beam of 1.5m length is loaded as shown. Draw the SF & BM diagrams.

Soln

SF Calculations

$F_B = +W = +2KN$

$F_C = +2KN$

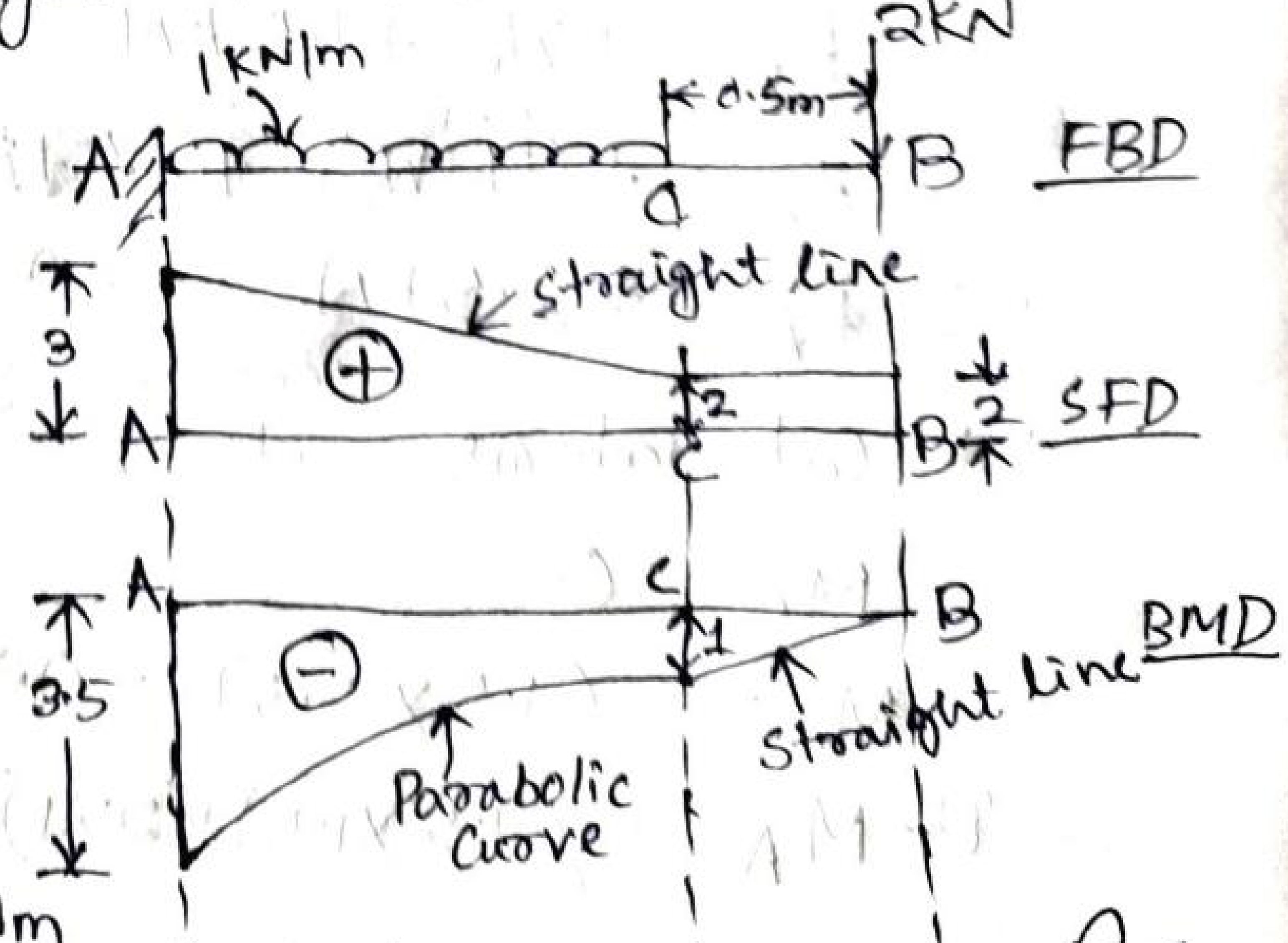
$F_A = +\{2+(1\times1)\} = +3KN$

BM Calculations

$M_B = 0$

$M_C = -(2\times0.5) = -1KNm$

$MA = -[(2\times1.5)+(1\times1)\times\frac{1}{2}] = -3.5 KNm$



Ans

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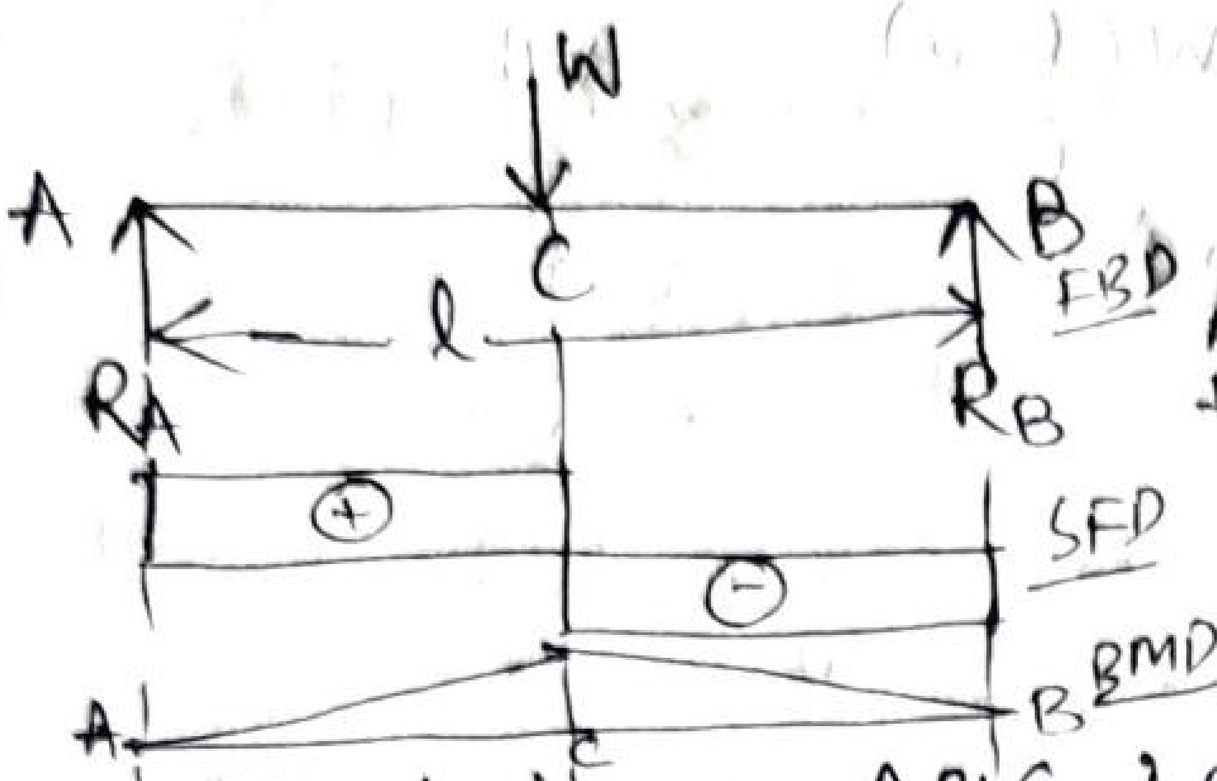
(2)

Shear force & Bending moment

Simply supported beam

Take a simply supported beam AB of length 'l' carrying a point load 'W' at its mid point 'C'.



Let R_A & R_B are reaction forces at A & B respectively.

As the load is acting at mid of the beam, so —

R_A & $R_B = 0.5W$

($\because$ W is acting at mid point 'C')

Then SF between A & C i.e. just before W is constant & equals to $+0.5W$. Similarly SF between C & B i.e just after W is also constant & equals to $-0.5W$ (i.e. $\frac{W}{2} - W = -\frac{W}{2} = -0.5W$)

Then bending moment at A & B is —

$$M_A = M_B = 0.$$

It then increases by a straight line law & is maximum at mid point 'C' where SF changes from +ve to −ve.

$$\therefore M_C = \frac{W}{2} \times \frac{l}{2} = +\frac{Wl}{4} \text{ (+ve due to sagging)}$$

— If the load is not act at mid point, then R_A & R_B are obtained & the diagrams are drawn as usual.

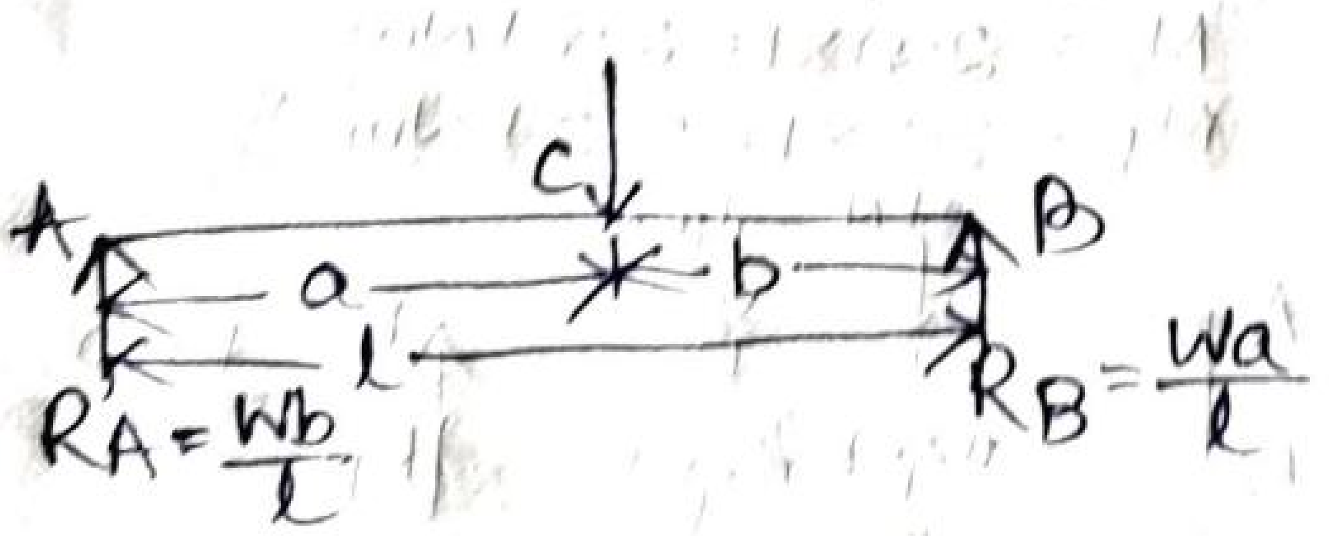
BM at a distance 'x' from B in CA —

$$M_x = +\frac{Wx}{4} - W\left(x - \frac{l}{2}\right)$$

— Also BM at supports for simply supported beam is always zero.

Simply supported beam carrying a concentrated load not at Mid point —

Let AB is the beam of length 'l' carrying at load of 'W' at a distance 'a' from A & a distance 'b' from B.



Taking moments about 'A' —

$R_B \times l = W \times a$

or $R_B = \frac{Wa}{l}$

But $R_A + R_B = W$

$\therefore R_A = W - \frac{Wa}{l} = \frac{Wb}{l}$

$$\therefore R_A = \frac{Wb}{l}, \quad R_B = \frac{Wa}{l}$$

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SF Calculations

SF just to Left of B —

$$+\frac{Wa}{l}$$

It remains constant upto C.

& SF just to the left of 'C' is —

$$\frac{Wa}{l} - W = \frac{-W(l-a)}{l} = -\frac{Wb}{l}$$

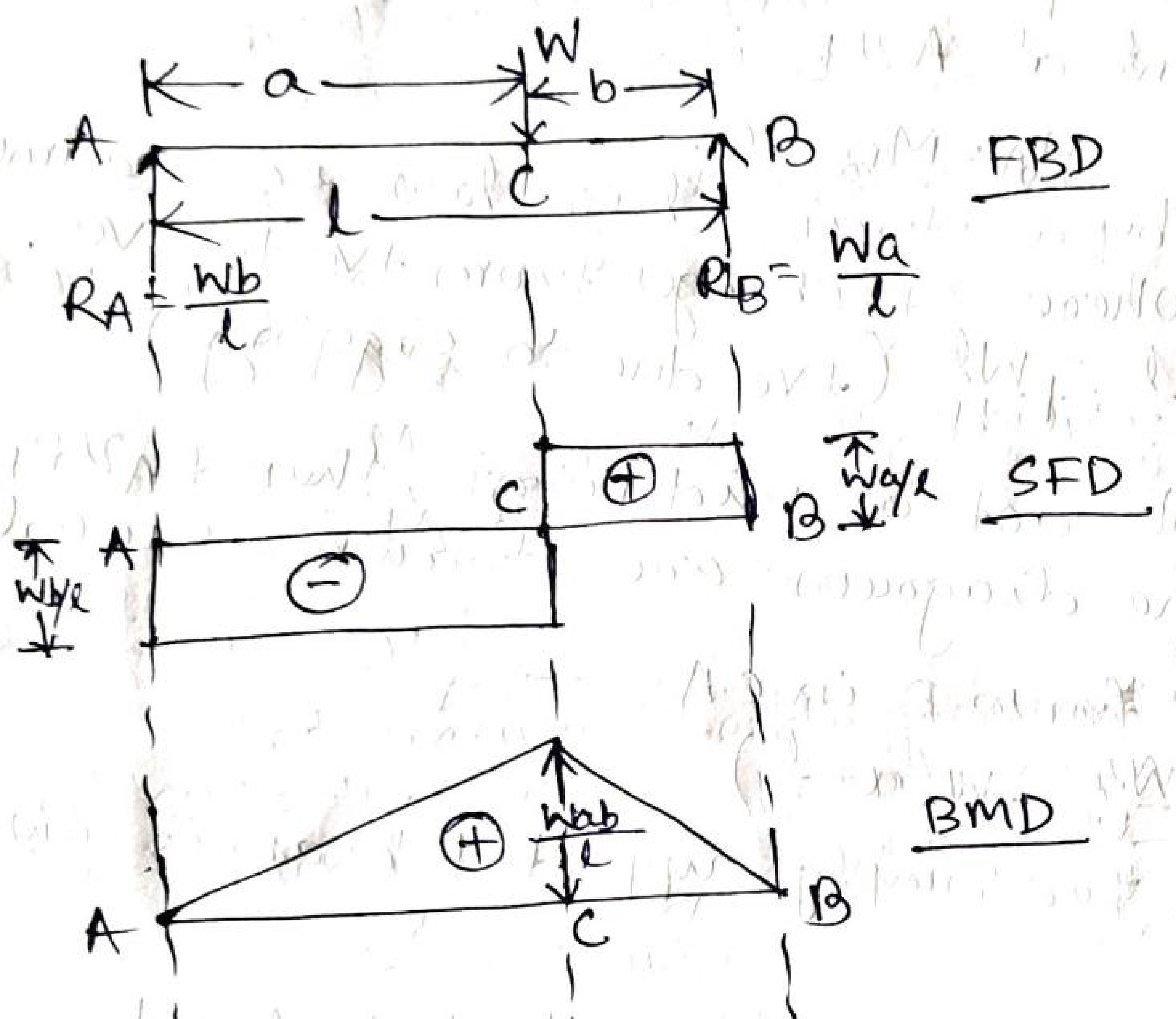
It remains constant upto A.

BM Calculations

$M_A = M_B = 0$

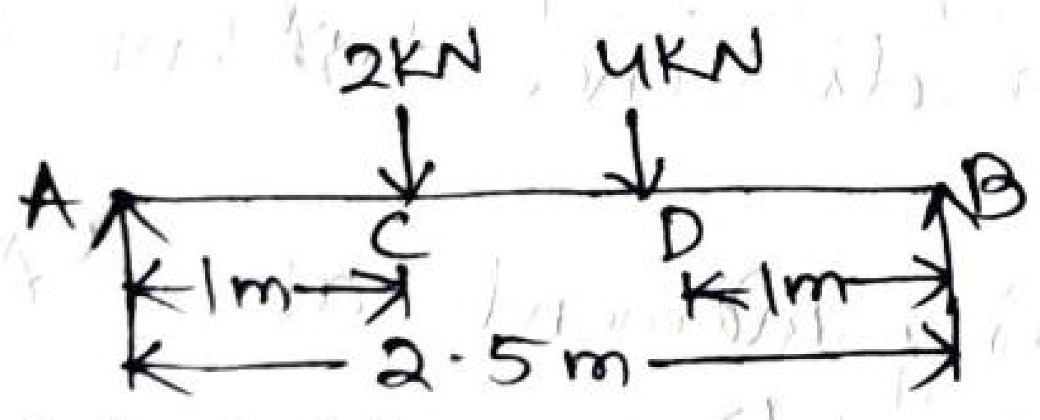
$M_C = \frac{Wa}{l} \times b = \frac{Wab}{l}$

At BM is maximum where SF changes sign.



No1 A simply supported beam of length 2.5m carrying two points load as shown. Draw the SF & BM diagram of the beam.

Soln



SF Calculations

Taking moments about A —

$R_B \times 2.5 = (2\times1)+(4\times1.5) = 8$

or $R_B = 3.2KN$

& $R_A = 2+4-3.2 = 2.8KN$

Then, $F_A = +R_A = 2.8KN$

$F_C = +2.8-2 = 0.8KN$

$F_D = 0.8-4 = -3.2KN$

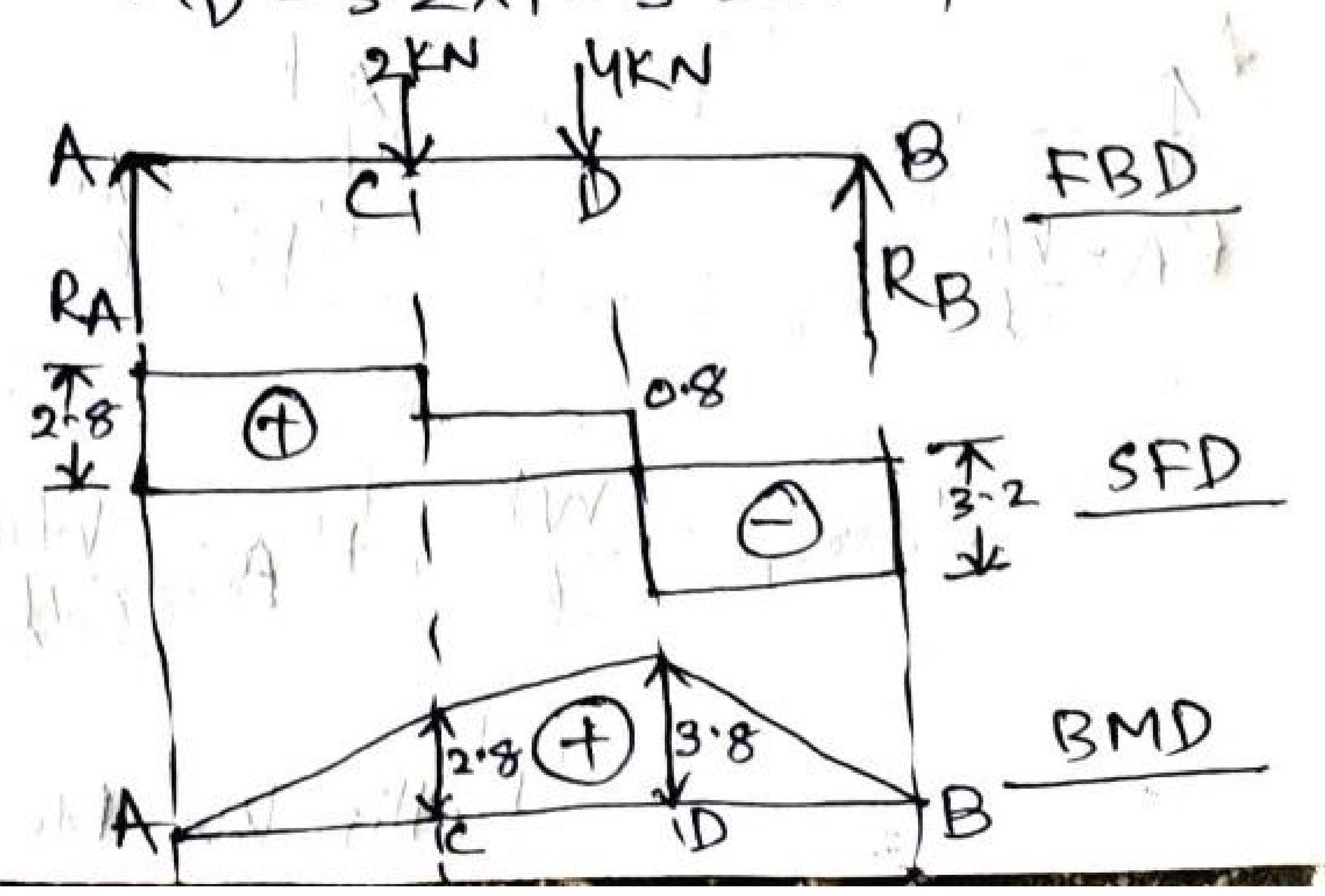
& $F_B = -3.2KN$

BM Calculations

$M_A = M_B = 0$

$M_C = 2.8\times1 = 2.8KNm$

$M_D = 3.2\times1 = 3.2KNm$



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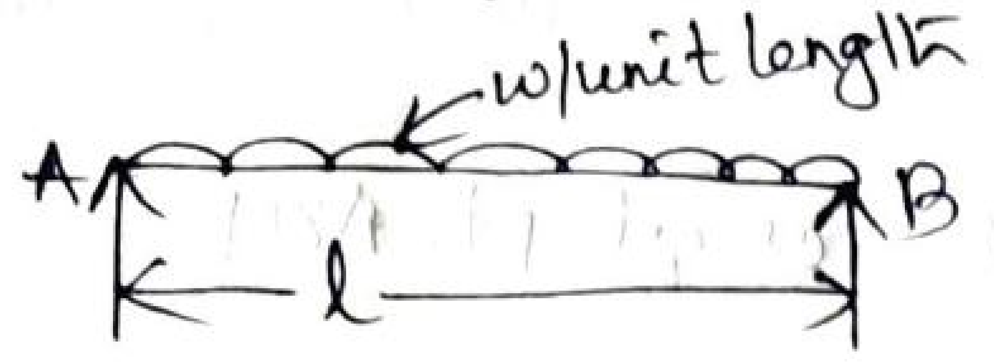
Simply supported beam carrying a UDL

Take a simply supported beam carrying a UDL of w/unit length over the whole span.



$\therefore$ Reactions at A & B are

$$R_A = R_B = \frac{wl}{2}$$

SF at the section x–x at a distance 'x' from B →

$$F_x = +\frac{wl}{2} - wx$$

$$\therefore F_B = +\frac{wl}{2} \quad (\because x = 0)$$

$$F_C = +\frac{wl}{2} - \frac{wl}{2} = 0 \quad \left(\because x = \frac{l}{2}\right)$$

$$\& \; F_A = +\frac{wl}{2} - wl = -\frac{wl}{2} \quad (\because x = l)$$

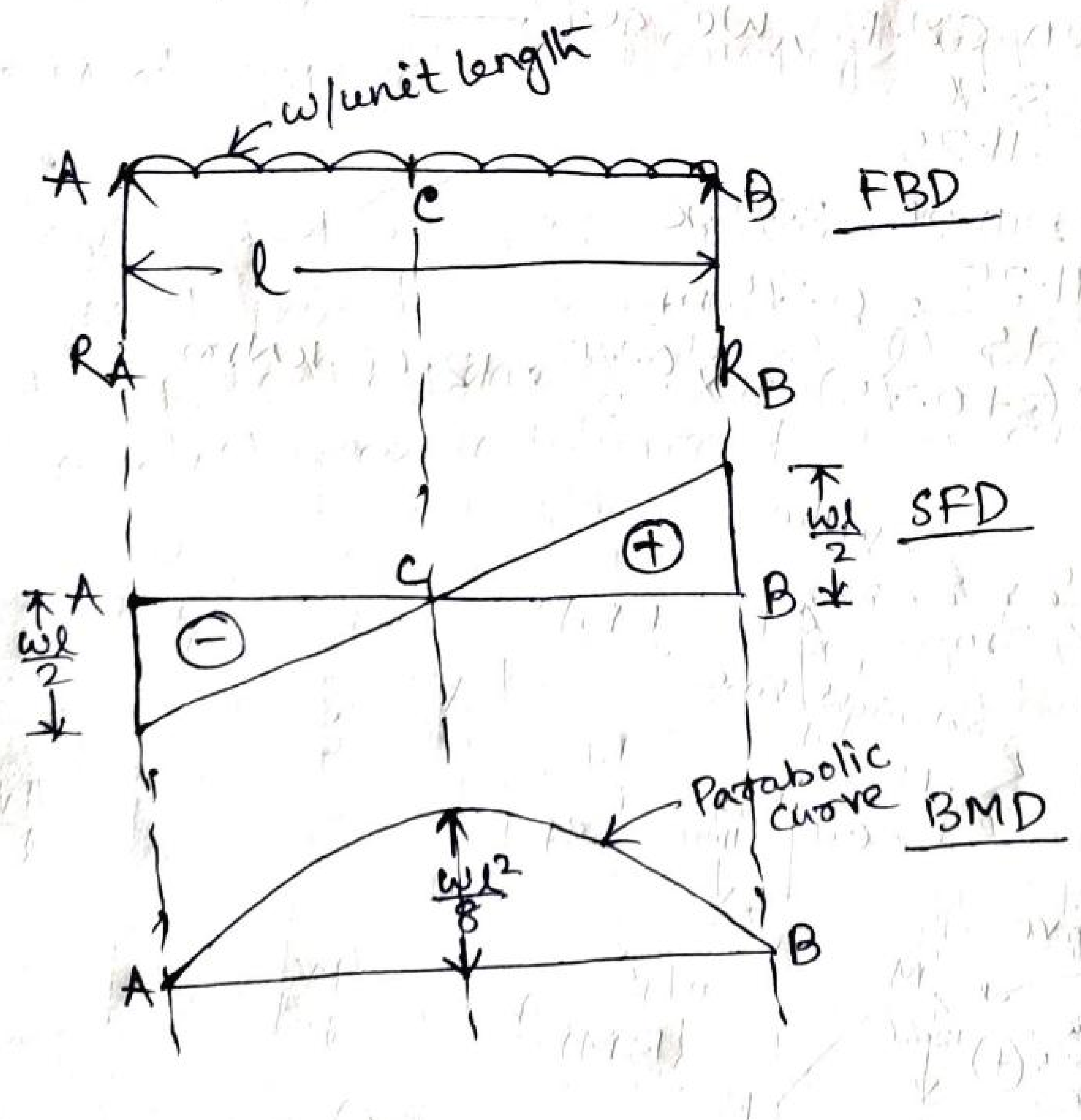
BM at the section x–x at a distance 'x' from B →

$$M_x = +\frac{wl}{2}x - \frac{wx^2}{2}$$

$$\therefore M_B = 0 \quad (\because x = 0)$$

$$M_C = +\frac{wl}{2}\times\frac{l}{2} - \frac{w}{2}\times\left(\frac{l}{2}\right)^2 = +\frac{wl^2}{8} \quad \left(\because x = \frac{l}{2} \text{ \& here BM to be maximum. Here SF is zero}\right)$$

$$\& \; M_A = \frac{wl}{2}\times l - \frac{wl^2}{2} = 0 \quad (\because x = l)$$



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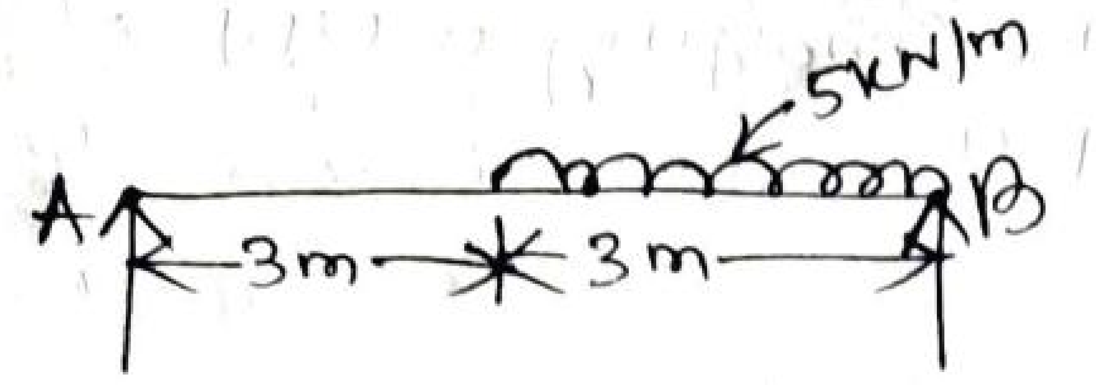
No1 Draw the SF & BM diagram for the given loaded simply supported beam of length 6m.



Soln Let R_A & R_B are the reaction forces at pt. A & B respectively.

Taking moments about 'A' ⟶

$R_B \times 6 = 5\times3\times4.5 = 67.5$

or $R_B = \frac{67.5}{6} = 11.25$ kN

& $R_A = (5\times3) - 11.25 = 3.75$ kN

SF diagram

$F_A = +R_A = +3.75$ kN

$F_C = +3.75$ kN

& $F_B = +3.75 - (5\times3) = -11.25$ kN

BM diagram

$M_A = 0$

$M_C = 3.75\times3 = 11.25$ kNm

& $M_B = 0$

Maximum BM

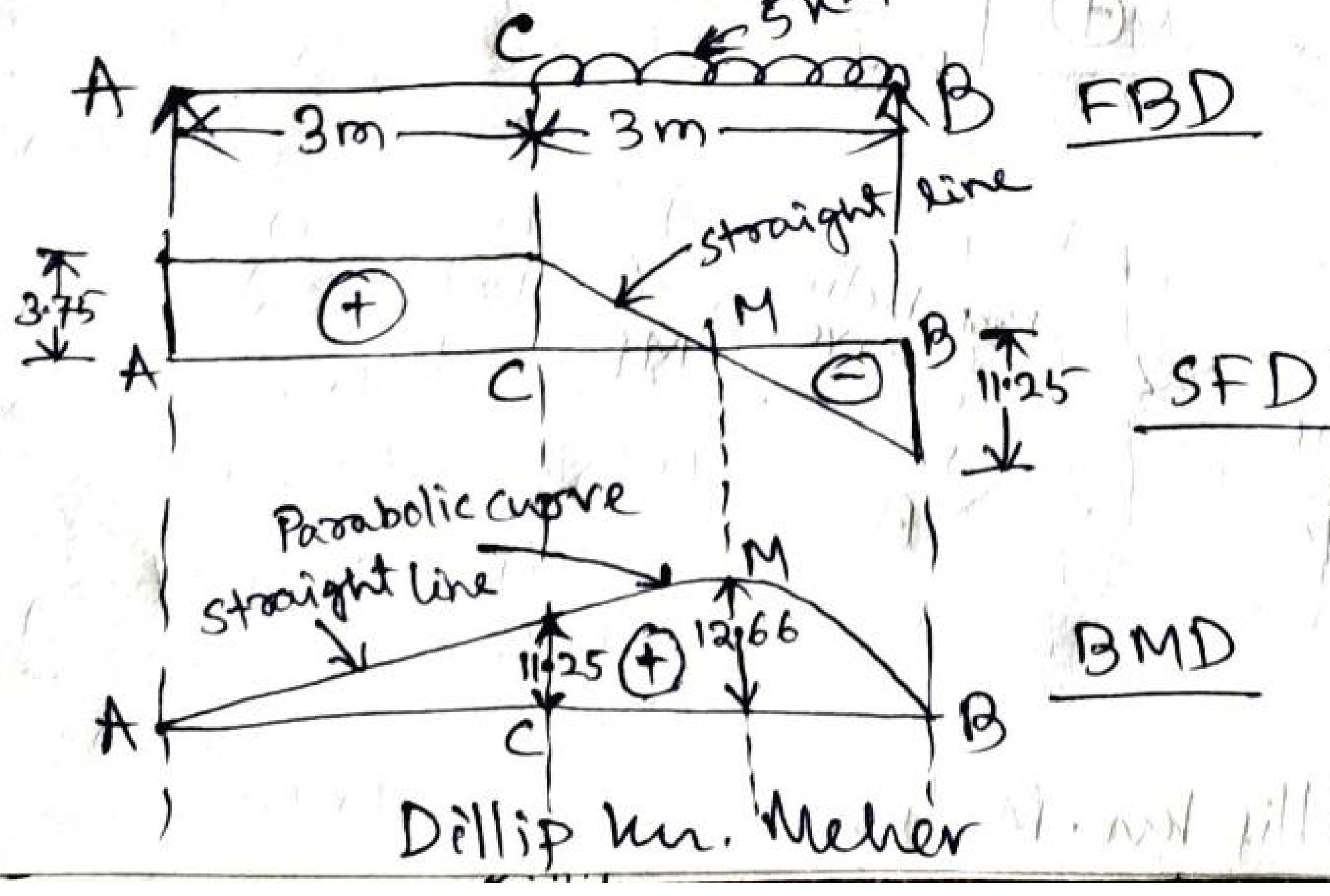
Let maximum bending moment will occurs at M. Let 'x' be the distance between C & M. From the geometry of the figure between C & B, we get —

$$\frac{x}{3.75} = \frac{3-x}{11.25}$$

or, $11.25x = 11.25 - 3.75x$

or, $x = \frac{11.25}{15} = 0.75$ m

$\therefore M_M = 3.75\times(3+0.75) - 5\times\frac{0.75}{2} = 12.66$ kNm



Dillip kr. Meher Ans

Shear force & Bending moment

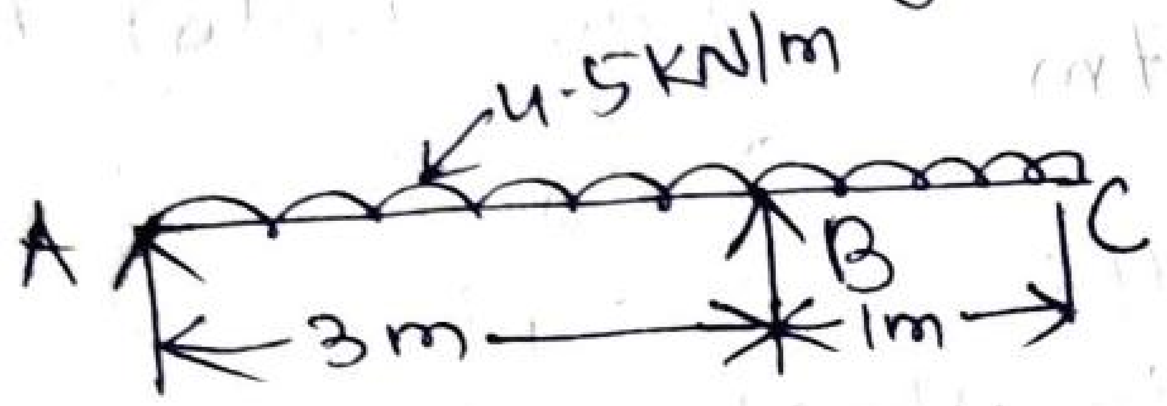
Overhang beam

It is a simply supported beam which overhangs, i.e. extends in the form of a cantilever from its support. An overhang beam may overhang on one side or on both sides of the support.

Point of contraflexture

In an overhang beam there is a point, where the bending moment will changes its sign (i.e. +ve to −ve or −ve to +ve). Such a point is called as point of contraflexture.

No1 An overhang beam ABC is loaded as shown. Draw the SF & BM diagrams & find the point of contraflexture.



Soln Let R_A & R_B are the two reactions at point A & B respectively.

Taking the moment about A ⟶

$$R_B \times 3 = (4.5 \times 4)2 = 36$$

$$\text{or } R_B = 12\,KN$$

$$\text{Then } R_A = (4.5 \times 4) - 12 = 6\,KN$$

SF Calculation

$$F_A = +R_A = +6\,KN$$

$$F_B = +6 - (4.5 \times 3) + 12 = 4.5\,KN$$

$$\& \; F_C = +4.5 - (4.5 \times 1) = 0$$

BM Calculation

$$M_A = 0$$

$$M_B = -\left(4.5 \times 1 \times \tfrac{1}{2}\right) = -2.25\,KNm$$

$$\& \; M_C = 0$$

Maximum BM will occur at 'M' where SF changes sign.

Let x = distance between A & M.

From the geometry of the fig. between A & B, then

$$\frac{x}{6} = \frac{3-x}{7.5}$$

or, $7.5x = 18 - 6x$

or, $x = \frac{18}{13.5} = 1.33m$

$\therefore M_M = (6 \times 1.33) - 4.5 \times 1.33 \times \frac{1.33}{2} = 4 \, kN\text{-}m$

Point of Contraflexture

Let P be the point of Contraflexture at a distance of 'y' from A.

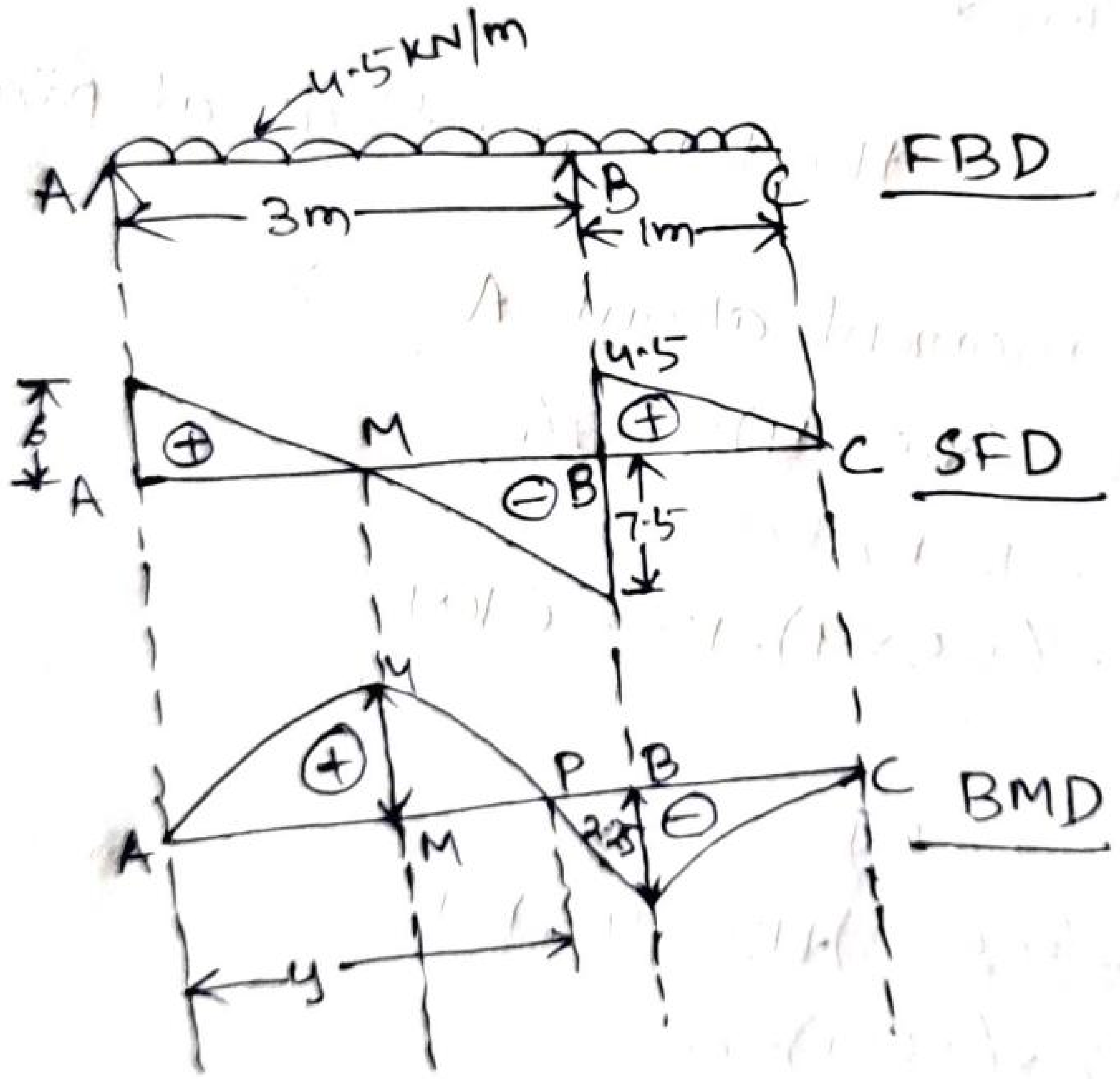
Then BM at point P is ⟶

$$M_P = 6 \times y - 4.5 \times y \times \frac{y}{2} = 0$$

or, $2.25y^2 - 6y = 0$

or, $2.25y = 6$

or, $y = \frac{6}{2.25} = 2.67m$



Ans

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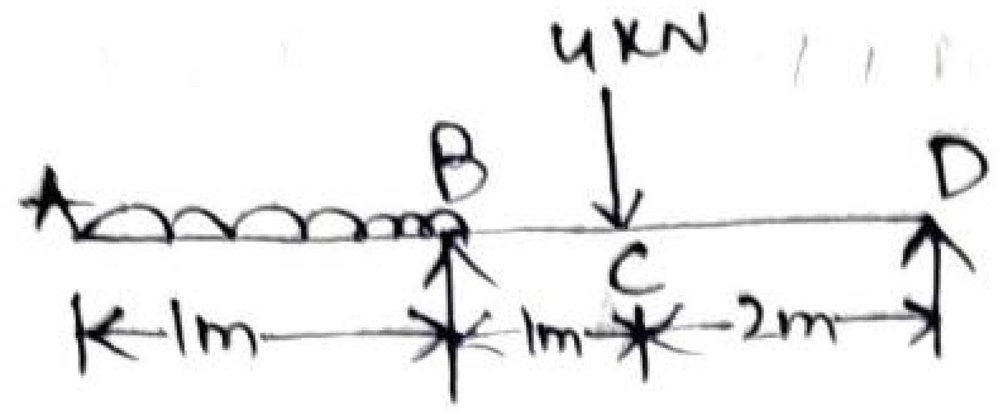
No2 A beam ABCD 4m long is overhang by 1m & carries load as shown. Draw the SF & BM diagram for ABCD & locate the point of contraflexture.



Soln

Let R_D & R_B are the two reaction forces at point D & B respectively.

Taking moments about B ⟶

$$R_D \times 3 = (4\times 1) - (2\times 1)\times \frac{1}{2} = 3$$

$$\text{or } R_D = \frac{3}{3} = 1\,kN$$

Then $R_B = (2\times 1) + 4 - 1 = 5\,kN$

SF Calculation

$$F_A = 0$$
$$F_B = 0 - (2\times 1) + 5 = +3\,kN$$
$$F_C = +3 - 4 = -1\,kN$$
$$\& \quad F_D = 1\,kN$$

BM Calculation

$$M_A = 0$$
$$M_B = -(2\times 1)\,0.5 = -1\,kNm$$
$$M_C = 1\times 2 = +2\,kNm$$
$$\& \quad M_D = 0$$

Maximum BM will occur either at B or C where SF changes sign.

From the geometry of the fig., maximum BM will at B (−ve) & C (+ve).

Point of Contraflexture

Let P = Point of contraflexture at a distance 'y' from B.

$$\therefore \quad \frac{y}{1} = \frac{1-y}{2}$$

$$\text{or } 2y = 1 - y$$

$$\text{or } y = \frac{1}{3} = 0.33\,m$$

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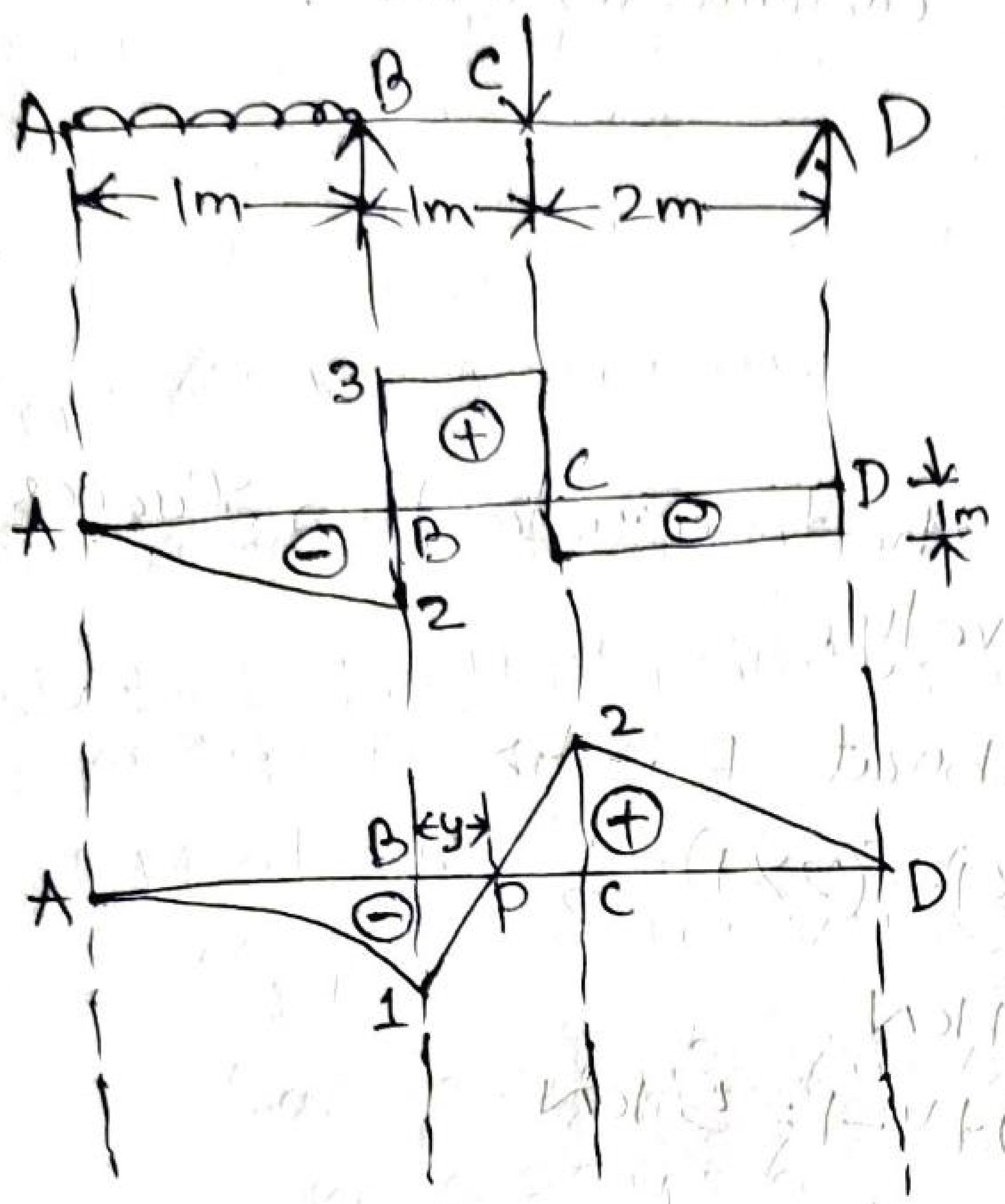

FBD

SFD

BMD

Ans