

PKAIET BARGARH

4RD SEMESTER ELECTRICAL ENGINEERING

TH-3 (ELECTRICAL MEASUREMENT & INSTRUMENTATION)

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Instruments

Instruments or equipments used to measure quantities like Current, Voltage, power etc.

Classification of instruments :-

- (1) Absolute (Primary)
- (2) Secondary

(1) Absolute instruments are used in research and national laboratory. The readings are very accurate. The measured quantities are obtained knowing the constants of the instruments and deflection.

Example:- Tangent Galvanometer.

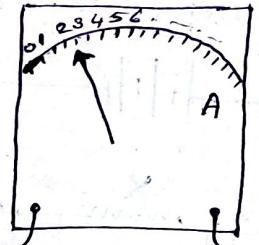
(2) Secondary Instruments are very simple to use but less accurate. Periodically its accuracy is compared using absolute instruments.

Secondary Instruments :-

There are three types of secondary instruments. They are (1) Indicating
(2) Recording (3) Integrating

Indicating instruments :-

These have a pointer and calibrated dial scaled. When the instrument is used, the pointer deflects and indicates the value on the dial. This instrument is very simple and not costly. Example:- Ammeter, Voltmeter, Wattmeter, etc.



scale or dial.

Recording Instruments :- Record instrument has an ink pen and a strip of paper moving at a constant rate. The x-axis of the paper is the time and y-axis is the magnitude of quantity measured. The pen moves up and down making a record on the paper.

Examples:- Recording ammeter, voltmeter, etc.

Integrating instruments :- In the case of integrating instruments, the quantity measured is integrated over time and the quantity is normally shown a gear-train.

Example:- Amper-hour meter & energy meter.

Indicating instruments have the following torque for satisfactory operation of indicating instruments.

- (i) deflecting torque
- (ii) control torque
- (iii) damping torque

Constructional details of indicating instruments :-

Moving system :- The moving parts of the instrument should be fulfilled in order that power required by the instrument for its operation is small. The power expenditure is proportional to the weight of the moving parts and the frictional forces opposing the instrument. The moving system can be made light by using aluminium as far as possible. The frictional forces are ~~obtained~~ reduced by using a spindle mounted between jewel bearings and by carefully balancing the system.

Supporting the moving element :- With operating small forces, the frictional forces ~~kept~~ must be kept to a minimum in order that the instrument reads correctly and is not erratic in action and is reliable. There are several types of supports are used, depending upon the sensitivity required and the operating conditions to be met.

Supports may be of the following types

- (i) suspension.
- (ii) Taut suspension
- (iii) pivot and jewel bearings (double).

Suspension :- It consists of a fine, ribbon shaped metal filament for the upper suspension and coil of fine wire for the lower part. The ribbon is made of a ~~coil~~ spring material like beryllium copper or phosphor bronze. This coiling of lower part of suspension is done in order to give negligible restraint on the moving system. This type of suspension requires careful levelling of the instrument, so that the moving system hangs in correct vertical position. Therefore, this construction is not suited to field use and is employed only in those laboratory applications in which very great sensitivity is required. In order to prevent shocks to the suspension during transit etc., a clamping arrangement is employed for supporting the moving system.

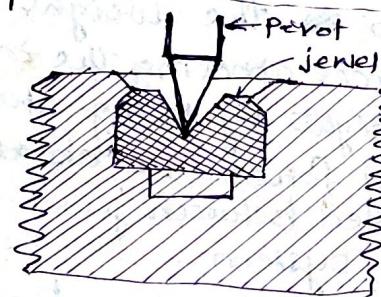
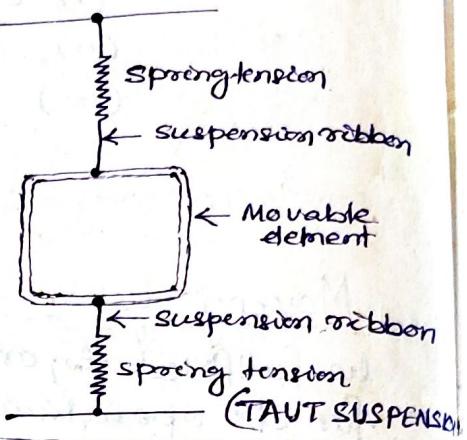
Taut Suspension :- Suspension type of instruments can only be used in vertical positions. The advantage of this suspension is that exact levelling is not required if the moving element is properly balanced.

Suspensions and taut suspensions are customarily used in instruments of Galvanometer class which require a low friction and high sensitivity mechanism.

Pivot and jewel Bearings :-

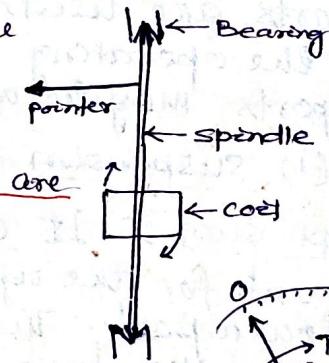
Moving system - spindle made of hardened steel.

The ends of the spindle are conical and polished to form pivots. These ends fit conical holes in jewel located in the fixed part of instruments. Jewels are made of sapphire form the bearings. Combination of steel and sapphire gives lowest friction.

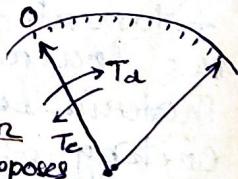


(i) Deflecting torque :- A deflecting torque makes the moving system to deflect and cause the pointer to move over the scale. Without this torque the pointer will always show zero. There are many mechanisms to produce deflecting torque.

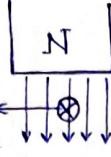
(ii) Control torque :- A control torque makes the pointer to ~~rest~~ come to rest at desired value on steady state. The control torque is opposite to the deflecting torque and their magnitudes are equal. Controlling torque is usually provided by springs.

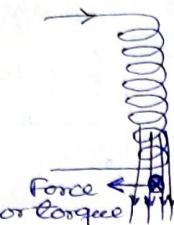


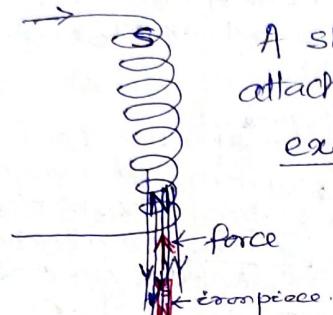
(iii) Damping torque :- Damping torque minimizes or eliminates the oscillations to the pointer about the steady state position. Damping torque opposes the movement of the pointer. When the pointer comes to rest, the damping torque reduces to zero.

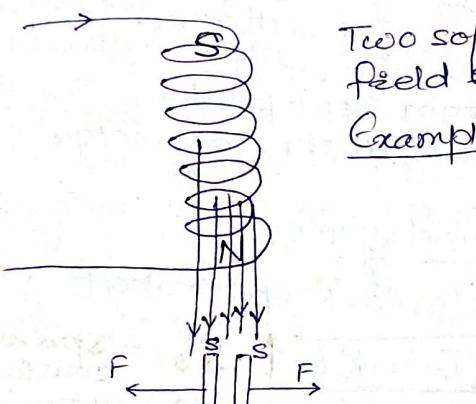


Deflecting torque mechanism:-

- ①  A current carrying conductor kept in a magnetic field experiences a force
Ex:- PMMC Instrument

- ②  When a d.c current flows through a coil produces a magnetic field. A current carrying coil kept in this magnetic field experiences a force
Examples- Electrodynamic instruments

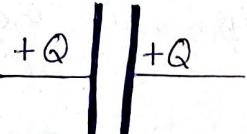
- ③  A soft iron piece kept in a magnetic field is attached inside the magnetic field
example:- (Moving iron attraction type instrument)

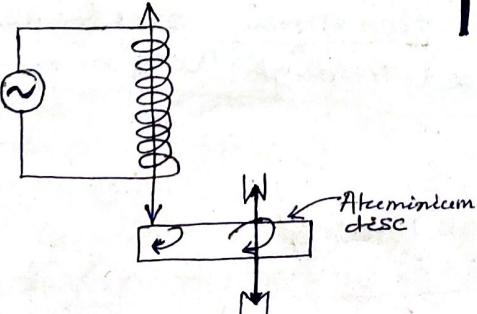
- ④  Two soft iron pieces kept in a magnetic field experiences a force of repulsion.
Example:- (Moving iron repulsion type)

- ⑤  When current flows through a high resistance wire, Copper loss is produced. This increases the temp. of the wire and causes elongation.

Example:- Hot wire instruments

- ⑥ Two metal plates separated by a small distance experiences a force of attraction or repulsion depending upon the nature of the charges on the plates.

 Example:- Electrostatic instruments

- ⑦  Alternating voltage applied to a coil produces an alternating flux along the axis of the coil and aluminium plate kept in this magnetic field gets eddy current induced. This eddy current interacts with the magnetic field and produces a torque on the aluminium plate.

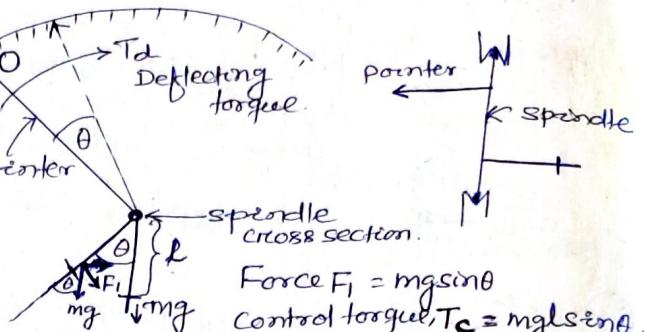
Example:- Induction type of instruments

Control torque :- There are two types of control torque or system which are used in the instruments.

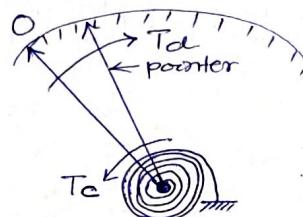
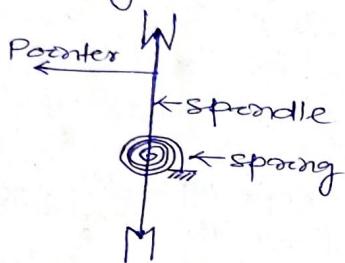
- (1) Gravity control (2) spring control.

Gravity Control :-

In gravity control system, a thin metal rod is attached to the spindle. As the pointer moves, the weight attached to the thin rod also moves upward causing a opposite torque which is called as control torque, $T_c = mg \sin \theta$



Spring Control :-



In spring control system, a thin helical spring made of phosphor bronze wire is attached to the spindle on one end and the other end is attached to a fixed point of the instrument.

As the pointer moves away from zero position, the spring is distorted and this produces an opposing torque or force on the spindle.

In case of spring control system

Control torque, $T_c \propto$ (proportional to) θ

$$\text{or } T_c = k_s \theta \quad k_s = \text{Spring constant}$$

DISTINGUISH BETWEEN GRAVITY AND SPRING CONTROL SYSTEMS

- | | |
|---|--|
| 1. It is very cheap. | 1. It is costly. |
| 2. It is rugged in construction. | 2. It is very delicate. |
| 3. Control constant can be easily adjusted. | 3. Difficult to adjust control constant. |
| 4. There is no fatigue. | 4. Spring is subjected to fatigue. |
| 5. Temp. error is almost zero. | 5. Meter reading varies with temp. |
| 6. The scale is always non-uniform. | 6. The scale may be uniform. |

F In spring control system, two springs may be used and the current can be passed through the springs and coil also.

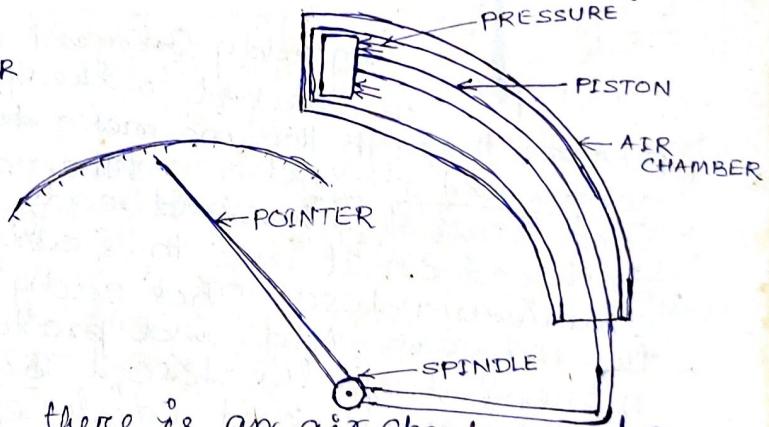
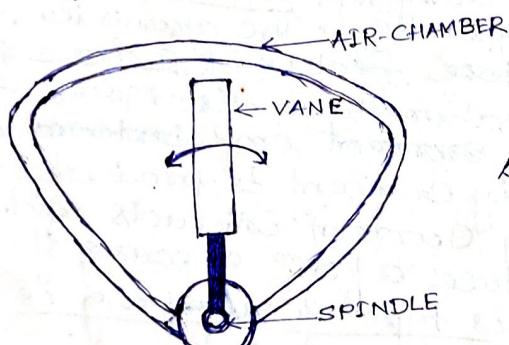
Damping torque :- The damping torque should be of such a magnitude that the pointer quickly comes to its final steady position without overshooting. If the instrument is underdamped, the moving system will oscillate about the final damped position with a decreasing amplitude and will take some time before it comes to rest. When the moving system moves rapidly but smoothly to its final position, the instrument is said to be critically damped or dead beat. If the damping torque is more than what is required for critical damping, the instrument is said to be overdamped. In an overdamped system, the moving system slowly to its final steady position in a lethargic fashion. The readings are very tedious to take in this case.

The damping device should be such that it produces a damping torque only while moving the system is in motion. To be effective the damping torque should be proportional to the velocity of the moving system but independent of the operating current. It must not affect the controlling torque or increase the static friction.

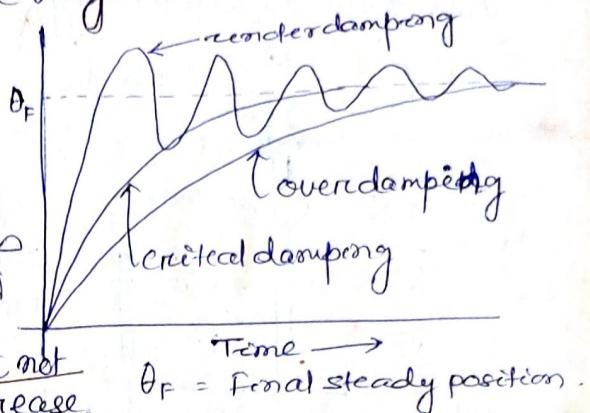
There are four methods of producing damping torque.

- They are (i) air-friction damping
- (ii) eddy current damping
- (iii) fluid friction damping
- (iv) electromagnetic damping (galvano-meter).

Air-friction damping :-

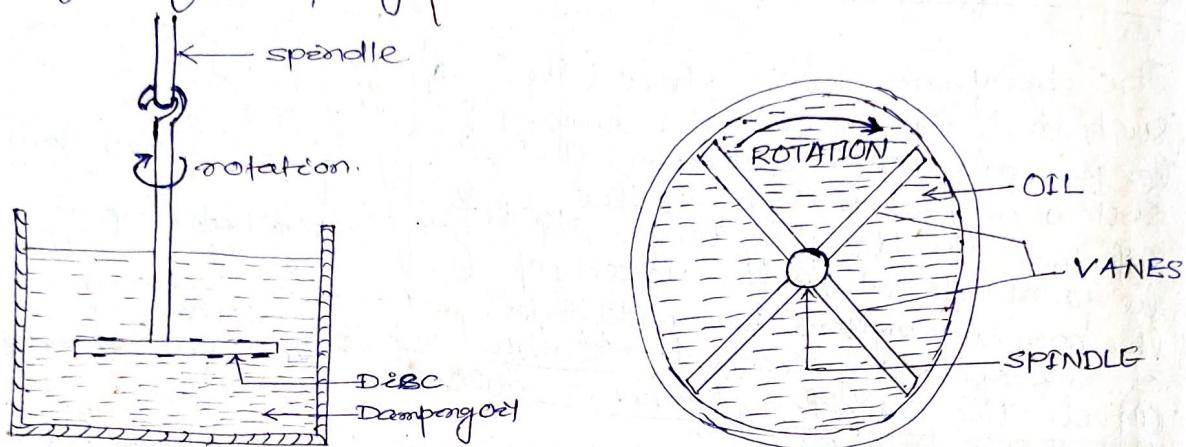


In air-friction damping, there is an air chamber and a piston and the piston is attached to the spindle. The pointer moves to the right, the piston is drawn out of the chamber. The pressure on the close end of the piston chamber decreases and the pressure on the other side is atmospheric. This difference in pressure produces an opposing force in the piston and an opposing torque on the spindle. On steady state the pressure on both sides becomes equal and the torque becomes zero. Air damping can also be obtained using air chamber and van attached to the spindle.

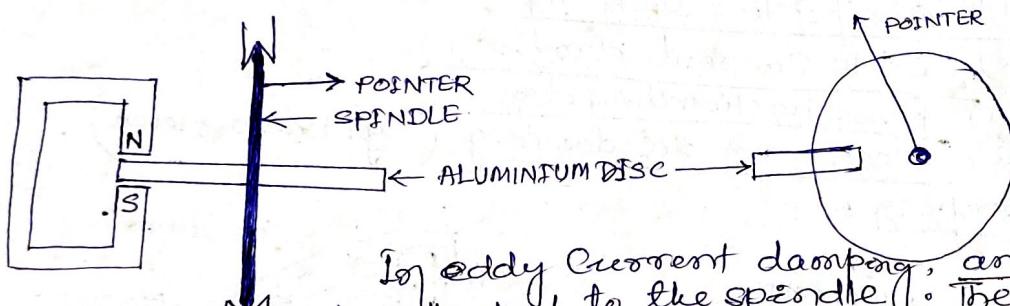


θ_F = Final steady position.

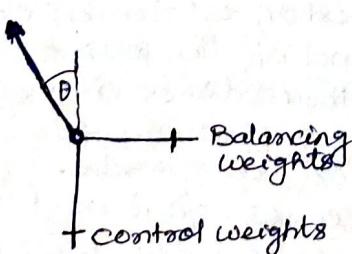
Liquid damping :- This type of damping is similar to air damping. Oil is used in place of air and as the viscosity of oil is greater, the damping force is also correspondingly greater. A disc is attached to the moving system, this disc dips into an oil pot and is completely submerged in oil. When the moving system moves, the disc moves in oil and a frictional drag is produced. This frictional drag always opposes the motion. In the arrangement shown in figure, a number of vanes attached to the spindle. These vanes are submerged in oil and move in a vertical plane. This arrangement gives a greater damping torque. This type of damping is not normally used.



Eddy Current damping :-



In eddy current damping, an aluminium disc is attached to the spindle. The aluminium disc passes through the magnetic field of a permanent magnet. When aluminium disc moves, it cuts lines of flux and eddy current emf is induced in it and thus eddy current is produced in aluminium disc. This eddy current interacts with the magnetic field and produces a force opposing the movement of the disc. This type of damping is very powerful and can be easily adjusted.



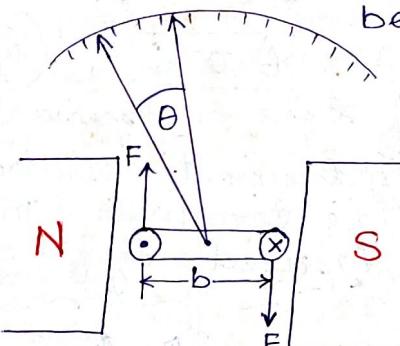
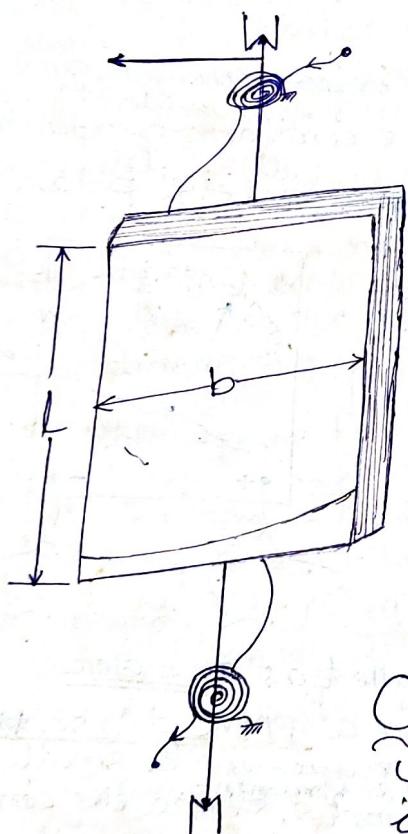
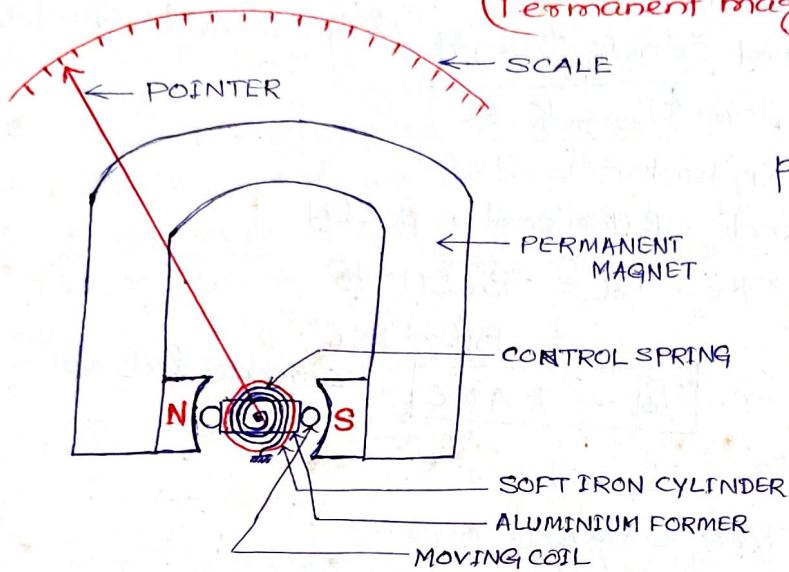
The balanced weight attached to the spindle moves the centre of gravity along the axis of the spindle which causes uniform wear and tear on the spindle and minimise the friction error.

Electromagnetic damping :- The movement of a coil in a magnetic field produces a current in the coil which interacts with the magnetic field to produce a torque. This torque opposes the movement of the coil and slows the response. The magnitude of the current and hence the damping torque is dependent upon the resistance of the circuit to which the instrument is connected. The electromagnetic damping is used in galvanometers.

(I)

PMMC INSTRUMENT

(Permanent magnet moving coil instrument)



PMMC instrument has a permanent magnet. The air gap between them is reduced using soft iron pole pieces and soft iron cylinder. The air gap is very small and the magnetic field is ~~there~~ powerful and flux lines are radial. In such a magnetic field, a rectangular aluminium former with a thin coil is supported in jewel bearing using spindle.

A pointer is attached to the spindle. Spiral hair springs connected to the spindle one on each side of the former. The ends of the coil are connected to the springs and leads are taken out for external connection.

OPERATION :- When current ~~flows~~ flows through the conductor, the force is produced in the conductors and a torque is developed on the moving system. This is called as deflection torque. As the coil

deflects, the springs are distorted and give control torque. When the coil moves, aluminium former cuts lines of flux, eddy emf is produced and eddy current circulates through the former. This current carrying aluminium former interacts with the magnetic field and produces a torque which opposes the

movement of the coil. This torque is the damping torque. Gt.
Called as eddy current damping.

Let B = Flux density in the air gap, Wb/m^2 (Tesla)

L = Length of coil or height of coil, m

b = Breadth or width of coil, m

N = NO. of turns of the coil,

i = Current through the coil, Amps.

T_d = Deflecting torque, N-m

T_c = Controlling torque, N-m.

K_s = Control constant or spring constant, N-m/degree

Controlling torque $T_c = K_s \theta$

Force on each conductor. = Bil

Force on each side of the coil = $BilN$

∴ Deflecting torque, $T_d = BilN \times b$

$$= B(l \times b) N i$$

$$\text{or, } T_d = BANi$$

coerce $l \times b = A$ = area of coil surface

On steady state, $T_c = T_d$

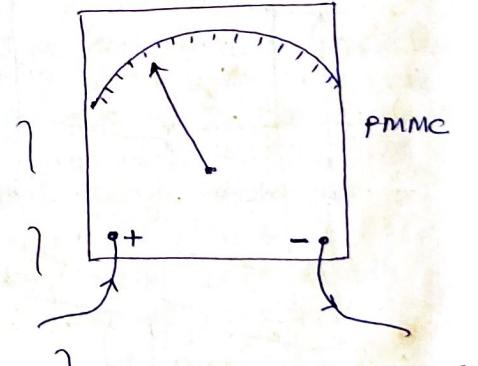
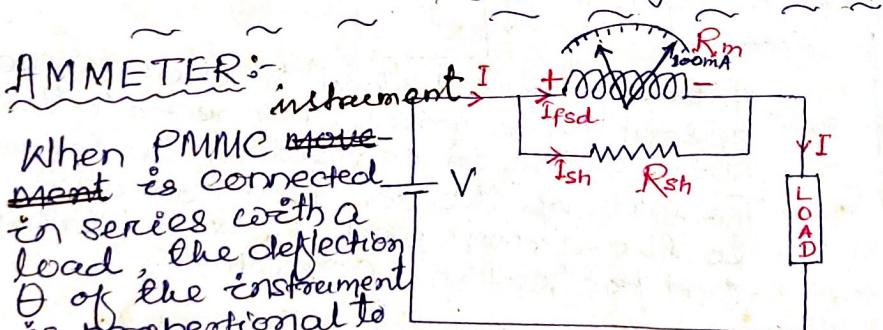
$$\therefore K_s \theta = BANi$$

$$\text{or } \theta = \frac{BANi}{K_s}$$

$$\text{or } \theta \propto i \quad (\text{Where remaining are constant})$$

The scale of this instrument has uniform gradations.

The PMMC instrument will have +ve and -ve polarities marked on the terminals. This instrument is used only for d.c.



The basic ~~method~~ of a d.c ammeter is a PMMC d'Arsonval galvanometer. The working of a basic ~~instrument~~ is small and light and can carry very small currents since the construction of an accurate instrument with a moving coil to carry currents greater than 100mA is impractical owing to the bulk and weight of the coil that would be required.

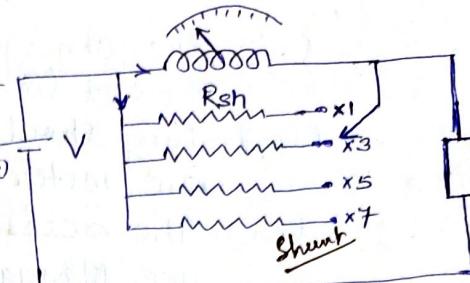
When heavy currents are to be measured the major part of the current is bypassed through a low resistance called shunt.

By connecting a proper shunt resistance across the PMMC instrument, the instrument can be used to measure large current and the scale is marked in terms of the large current.

Different shunts can be selected across PMMC movement and the instrument can be used to measure different ranges of current.

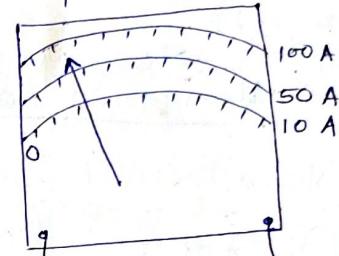
General requirement of shunts :-

1. Temp. coefficient of shunt and instrument should be low and should be as nearly as possibly the same.
2. The resistance of shunts should not vary much with time.
3. They should carry the current without excessive temp. rise.
4. They should have a low thermal electromotive force with copper.
5. Manganin is usually used for shunts in order to minimise the value of temp. error or coefficient of temp.



$$R_{sh} (I - I_{fsd}) = I_{fsd} \times R_m$$

$$\text{or } R_{sh} = \frac{I_{fsd} \times R_m}{I - I_{fsd}}$$



VOLTMETER :- A PMMC d'Arsonval basic meter instrument is converted into a voltmeter by connecting a series resistance with it. This series resistance is known as multiplexer. The combination of the meter instrument and the multiplexer is put across the circuit whose voltage is to be measured. The multiplexer limits the current through the meter so that it does not exceed the value of full scale deflection and thus prevents the instrument from being damaged.

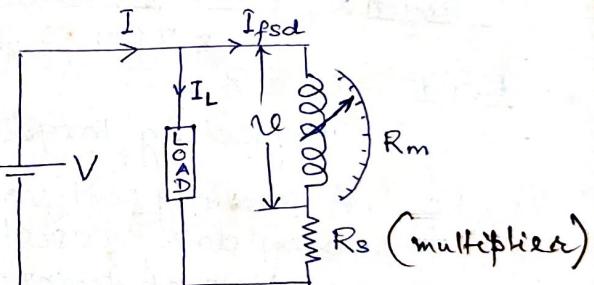
Let I_{fsd} = Full scale deflection current of meter

R_m = Internal resistance of meter movement

V = Voltage across the meter movement for current I_m .

V = Full range of voltage of instrument.

R_s = Series or multiplexer resistance.



From circuit, $V = I_{fsd} \times R_m$ and $V = I_{fsd} \times (R_m + R_s)$ --- (1).

$$\text{From equation (2)} \quad R_s = \frac{V - I_{fsd} R_m}{I_{fsd}} = \frac{V}{I_{fsd}} - R_m$$

$$\text{or. } R_s = \frac{V}{I_m} - R_m$$

$$\text{Multiplying factor of multiplexer, } m = \frac{V}{V} = \frac{I_{fsd}(R_m + R_s)}{I_{fsd} R_m} = \frac{R_m + R_s}{R_m} = 1 + \frac{R_s}{R_m}$$

$$\therefore \text{Resistance of multiplexer, } R_s = (m-1)R_m$$

Hence for the measurement of voltage in terms of the voltage range of the instrument, the series multiplying resistance R_s

By $(m-1)$ times the meter resistance. Thus to extend the voltage range to 10 times the instrument range, $R_s = 9 R_m$

Construction of Multipliers :- The essential requirements of multipliers are (i) their resistance should not change with time.
(ii) the change in their resistance with temp. should be small.
(iii) they should be non-inductively wound for a.c meters.
(iv) the resistance materials used for multipliers are Manganin and constantan.

~~voltmeter~~ As $\theta \propto i$ or $\theta \propto I_{f\text{sd}}$
 $\therefore \theta \propto \frac{V}{R_m + R_s}$ or $\boxed{\theta \propto V}$ voltmeter

Sparking resistance is manganin & constantan.
Manganin have a negligible resistance temp. co-effecient.

Numerical Examples :-

① S-80 :- If the moving coil consists of 100 turns wound on a rectangular former of length 3 cm and width 2 cm & flux density in the air gap is 0.06 wb/m^2 . Calculate the torque acting in the coil when carrying current of 5 mA.

Solution :- Given data :- $N = 100$, $l = 3 \text{ cm} = 3 \times 10^{-2} \text{ m}$,
 $b = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}$, $B = 0.06 \text{ wb/m}^2$, $i = 5 \text{ mA}$,

Reqd. $T_d = ?$

$$\text{Deflecting torque } T_d = PBANi = 0.06 \times 3 \times 10^{-2} \times 2 \times 10^{-2} \times 100 \times 5 \times 10^{-3}$$

② S-80 A moving coil voltmeter has the following data. resistance = 10000Ω , Dimension of coil = $3 \times 3 \text{ cm}$, No. of turns of coil = 100, Flux density in the air gap = 80 mwb/m^2 , Spacing control = $30 \times 10^7 \text{ N-m/degree}$. Find the deflecting torque produced by 200 V.

Soln :- $R = 10,000 \Omega$, $l \times b = 3 \times 3 \text{ cm}^2 = 3 \times 3 \times 10^{-4} \text{ m}^2$,
 $N = 100$, $B = 80 \times 10^{-3} \text{ wb/m}^2$, $R_s = 30 \times 10^7 \text{ N-m/degree}$.

$$V = 200 \text{ V}, \theta = ?$$

$$\text{Hence } R_m + R_s = R = 10,000 \Omega,$$

$$\therefore \text{Current through the voltmeter, } i = \frac{V}{R_m + R_s} = \frac{200}{10,000} = 0.02 \text{ A}$$

$$\begin{aligned} \text{Deflecting torque, } T_d &= PBANi \cdot \text{N-m} \\ &= 80 \times 10^{-3} \times 9 \times 10^{-4} \times 100 \times 0.02 \\ &= 1.44 \times 10^{-4} \text{ N-m.} \end{aligned}$$

$$\text{But control torque, } T_c = T_d = R_s \theta.$$

$$\therefore 1.44 \times 10^{-4} = R_s \theta$$

$$\text{or } \theta (\text{deflection}) = \frac{1.44 \times 10^{-4}}{30 \times 10^7} = 48^\circ \text{ (Ans)}$$

③ W-81 A moving coil voltmeter with a resistance of $10\ \Omega$ gives a full scale deflection with a p.d. of 45 mV . The coil has 100 turns and effective depth of 3 cm and a width of 2.5 cm . The controlling torque exerted by the spring is 0.5 gm-cm for full-scale deflection. Calculate the flux density in the air-gap.

Soln:- $R = 10\ \Omega$, $I_{fsd} = \frac{45 \times 10^{-3}}{10} = 4.5 \times 10^{-3}\text{ A}$.

$$N = 100, l = 3\text{ cm}, b = 2.5\text{ cm}, T_c = 0.5\text{ gm-cm}$$

$$= 0.5 \times 10^{-3} \times 9.81 \times 10^2$$

$$= 4.905 \times 10^{-5}\text{ N-m}$$

We know deflecting torque,

$$T_d = BANi = BAN I_{fsd}$$

But $T_d = T_c$ = controlling torque.

$$\therefore BAN I_{fsd} = 4.905 \times 10^{-5}$$

$$\text{or } B(\text{flux density}) = \frac{4.905 \times 10^{-5}}{AN I_{fsd}} = \frac{4.905 \times 10^{-5}}{3 \times 2.5 \times 10^4 \times 100 \times 4.5 \times 10^{-3}}$$

④ W-82 A millivoltmeter with a resistance of $0.8\ \Omega$ has a range of 24 mV . Explain with all details how it can be converted to

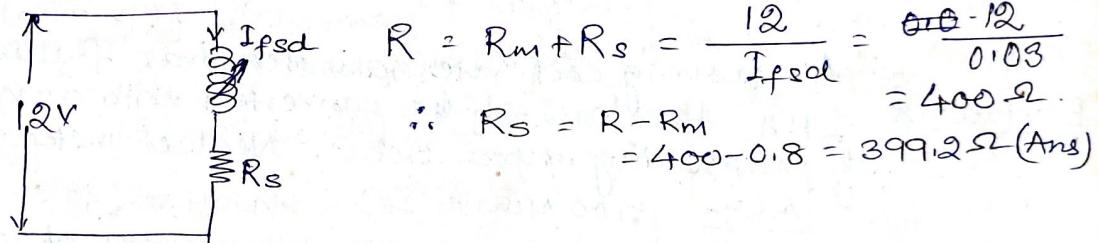
(i) Voltmeter with a range of 12 V .

(ii) An ammeter with a range of 30 A .

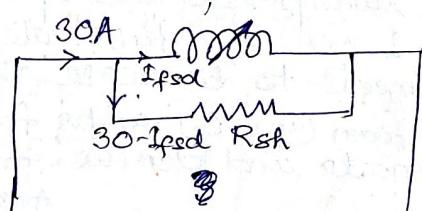
Soln:-

$$R_s = 0.8\ \Omega, I_{fsd} = \frac{24\text{ mV}}{R} = \frac{24 \times 10^{-3}}{0.8} = 0.03\text{ A}$$

(i)



(ii)



$$R_{sh} = \frac{I_{fsd} \times R_m}{30 - I_{fsd}}$$

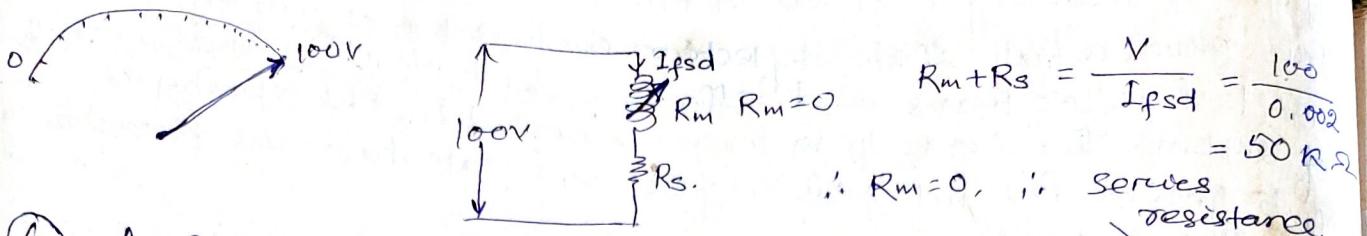
$$= \frac{0.03 \times 0.8}{30 - 0.03} = 0.8008 \times 10^{-3}\ \Omega$$

⑤ S-91 The coil of a moving coil permanent magnet voltmeter is 40 mm long and 30 mm wide, and has 100 turns on it. The control torque exerts a torque of $120 \times 10^6\text{ N-m}$ when the deflection is 100 divisions of full scale. If the flux density of the magnetic field in the air gap is 0.5 wb/m^2 . Estimate the resistance that must be put in series with the coil to give $1\text{ V}/\text{dev}$. The resistance of the voltmeter coil may be neglected.

Soln:- $l = 40\text{ mm}, b = 30\text{ mm}, N = 100, T_c = 120 \times 10^6\text{ N-m}$ for full scale divisions of 100 dev , $B = 0.5\text{ wb/m}^2$, $R_s = ?$ for $1\text{ V}/\text{dev}$.

Deflecting torque, $T_d = BANi = N \cdot M = T_c$ controlling torque.

$$\therefore I_{fsd} = \frac{T_c}{BAN} = \frac{120 \times 10^6}{0.5 \times 40 \times 30 \times 10^6 \times 100} = 0.002\text{ A}$$



⑥ A PMMC has a full scale deflection of 5mA and the ~~Rs = 50KΩ~~ meter resistance is 15Ω . Calculate the shunt resistance which when connected parallel to the instrument will convert the full scale deflection into one ampere.

Soln :-

$$I_{fsd} = 5 \text{ mA}, R_m = 15 \Omega,$$

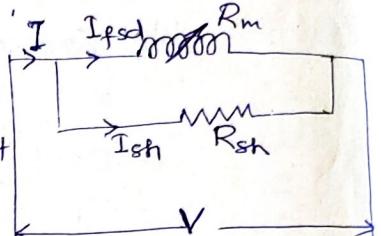
$$V = I_{fsd} \times R_m = 5 \times 10^{-3} \times 15 = 0.075 \text{ Volt}$$

For $I_{fsd} = 1 \text{ amp}$ and $R_m = 15 \Omega$.

$$R_{sh} = \frac{I_{fsd} \times R_m}{I_{sh}} = \frac{0.075}{5 \times 10^{-3}} \quad (1)$$

$$R_{sh} I_{sh} = 0.075 \text{ V.}$$

?



$$R_{sh} = \frac{I_{fsd} \times R_m}{I - I_{fsd}}$$

$$\text{or } R_{sh} = \frac{1 \times 15}{I - 15}$$

$$\text{or } \frac{5 \times 10^{-3} \times 15}{I - 5 \times 10^{-3}} = \frac{15}{I - 15}$$

$$\text{or } 0.075I - 0.075 \times 15 = 15I - 0.075 \times 15$$

$$\text{or } I(15 - 0.075) = 0.075(1 - 15) = -14$$

S-81

Electrodeals

A moving coil microammeter has full scale deflection of 1 mA. How can it be converted into a voltmeter capable of measuring upto 500V. Neglect meter resistance.

Ans:- $500 \text{ M}\Omega$.

KL-87

Electronics

A d.c. meter has a shunt resistance of 1000Ω and ~~full scale deflection~~ requires 1 mA for full scale deflection.

Determine the arrangements to increase the range to
(i) 500V full scale deflection (ii) 0.1 amps full scale deflection
State the assumptions made and derive formulae used.

Ans:- (i) $499 \text{ K}\Omega$
(ii) $10.10 \text{ }\Omega$.

Errors in PMMC type instrument :-

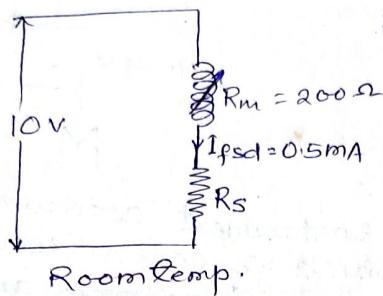
- (i) Weakening of permanent magnets due to ageing at temperature effects.
- (ii) Weakening of springs due to ageing and temp. effects.
- (iii) Change of resistance of the moving coil with temp..

W-88

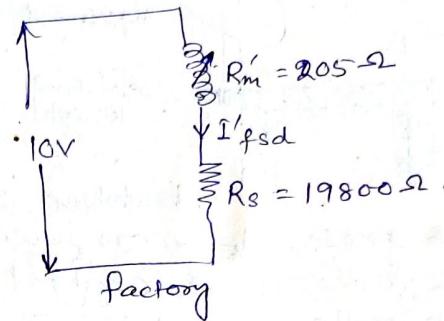
Electronics

The coil resistance of a meter is 200Ω gives full scale deflection of 0.5 mA at room temp. The range of the meter is adjusted to be 10 V full scale deflection by a suitable shunting resistance at room temp. Determine the full scale deflection error in measurement if this meter is used in a factory environment when the coil resistance increases to 205Ω .

Soln:-



$$R_s = \frac{10}{0.5 \times 10^{-3}} - 200 = 19800\Omega$$



$$I'_{fsd} = \frac{10}{19800 + 205}$$

$$= \frac{10}{20005} = 4.9987503 \times 10^{-4} \text{ A.}$$

$$\therefore \% \text{ error} = \frac{I'_{fsd} - I_{fsd}}{I_{fsd}} \times 100 = -0.025\% \text{ (Ans)}$$

W-92

Electronics

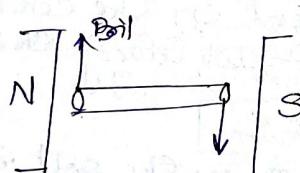
An N -turns rectangular coil of width w_c & length l_c is in a uniform magnetic field B between rectangular pole pieces of permanent magnet of dimensions w_m ($> w_c$) and l_m ($< l_c$). The deflecting torque is proportional to

(A) $w_c l_c$ (B) $w_m l_c$ (C) $w_c l_m$ (D) $w_m l_m$.

Soln:-

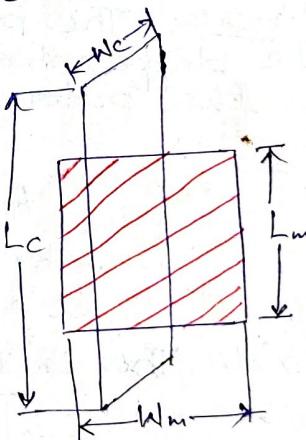
$$T_d \propto B A N i = B l b N i$$

$$\text{or } T_d \propto l b.$$



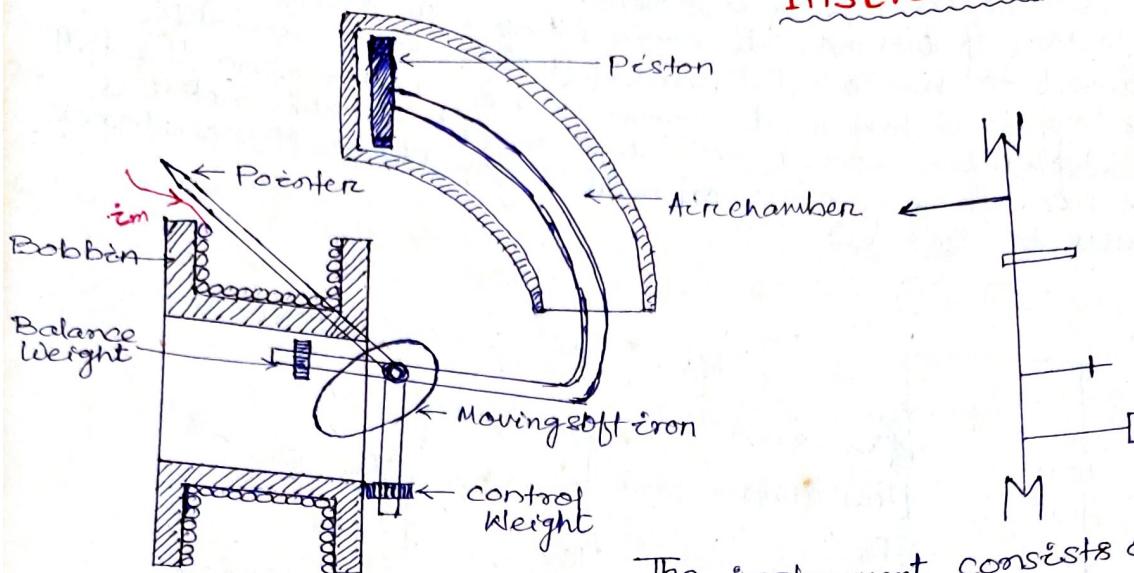
$$L = l_m$$

$$b = w_c$$



II

Moving Iron Attraction Type Instrument



The instrument consists of insulated bobbin over which a coil is wound. A spindle is provided on jewel bearing. A pointer, an oval shaped soft iron, a control weight and piston are attached to the spindle.

Principle of operation :- Current flowing through the coil produces a magnetic field along the axis of the coil. This magnetic field induces pole on the soft iron and the soft iron piece is attracted inside the coil. This attractive force produces a torque (Deflecting torque) on the moving system. This makes the moving system to turn clockwise. As the pointer moves clockwise, the control weight lifted upwards and the gravitational force produces control torque. The piston in the air chamber is drawn out and the differential pressure on the piston produces a force on the piston and a damping torque on the moving system.

Assume the current in the coil be \dot{I}_m
Intensity of magnetic field along the axis of the coil be H .
$$H \propto \dot{I}_m$$

Let The induced pole strength on the soft iron be m'

$$m' \propto H \propto \dot{I}_m$$

Force of attraction $F \propto Hm'$

$$\text{OR } F \propto \dot{I}_m^2$$

\therefore Deflecting torque, $T_d \propto F \propto \dot{I}_m^2$

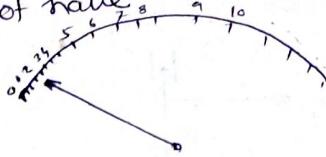
Control torque, $T_c \propto \sin\theta$

On steady state, $T_c = T_d$

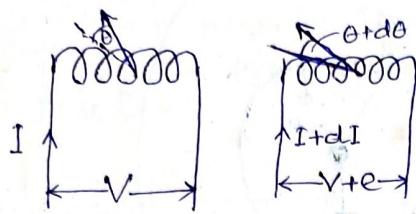
$$\therefore \sin\theta \propto \dot{I}_m^2$$

$$\text{OR } \theta \propto \sin^2 \dot{I}_m^2$$

The relationship between θ and i_m shows that the scale of the instrument is non-linear. This instrument will not have polarity markings. This instrument can be used for both ac and dc measurements.



Expression for the torque in terms of self inductance:



Energy stored in inductance L at I = $\frac{1}{2} L I^2$

$$\begin{aligned}\text{Energy stored in the coil after increase in current} &= \frac{1}{2} (L+dL)(I+dI)^2 \\ &= \frac{1}{2} (L+dL)(I^2 + 2IDdI + dI^2) \\ &= \frac{1}{2} (LI^2 + I^2dL + 2ILDdI + dI^2) \\ &\quad + LdI^2 + dI^2dI \\ &= \frac{1}{2} (LI^2 + 2ILDdI + I^2dL)\end{aligned}$$

Extra energy stored in the inductance

$$\begin{aligned}&= \frac{1}{2} (LI^2 + 2ILDdI + I^2dL) - \frac{1}{2} LI^2 \\ &= ILdI + \frac{1}{2} I^2dL\end{aligned}$$

Energy stored in mechanical system = $(T \frac{d\theta}{dt})dt = Td\theta$
where θ in radian

J. Amp

$$\begin{aligned}\text{Extra energy input} &= e(I+di)dt \\ &= eIdt + eidi \quad \left(\begin{array}{l} \text{Energy} \\ = VIt \end{array} \right)\end{aligned}$$

We know $e = L \frac{dt}{dt}$. But where L & I both are variable

$$\therefore e = \frac{dLI}{dt} = L \frac{dI}{dt} + I \frac{dL}{dt} \quad (\text{Derivative of product})$$

$$\begin{aligned}\therefore eIdt &= Idt \left(L \frac{dI}{dt} + I \frac{dL}{dt} \right) \\ &= LIdI + I^2dL\end{aligned}$$

Now Extra energy stored in inductance
+ Extra energy stored in mechanical system
= Extra input energy

$$\text{or, } LIdI + \frac{1}{2} I^2dL + Td\theta = LIdI + I^2dL$$

$$\text{or, } Td\theta = I^2dL - \frac{1}{2} I^2dL$$

$$\text{or, } T = \frac{1}{2} I^2 \frac{dL}{d\theta} \quad \boxed{\text{Deflecting torque, } T_d = \frac{1}{2} I^2 \frac{dL}{d\theta}}$$

Control torque,

$$T_c = K_s \sin\theta$$

On steady state, $\frac{1}{2} I^2 \frac{dL}{d\theta} = K_s \sin\theta$

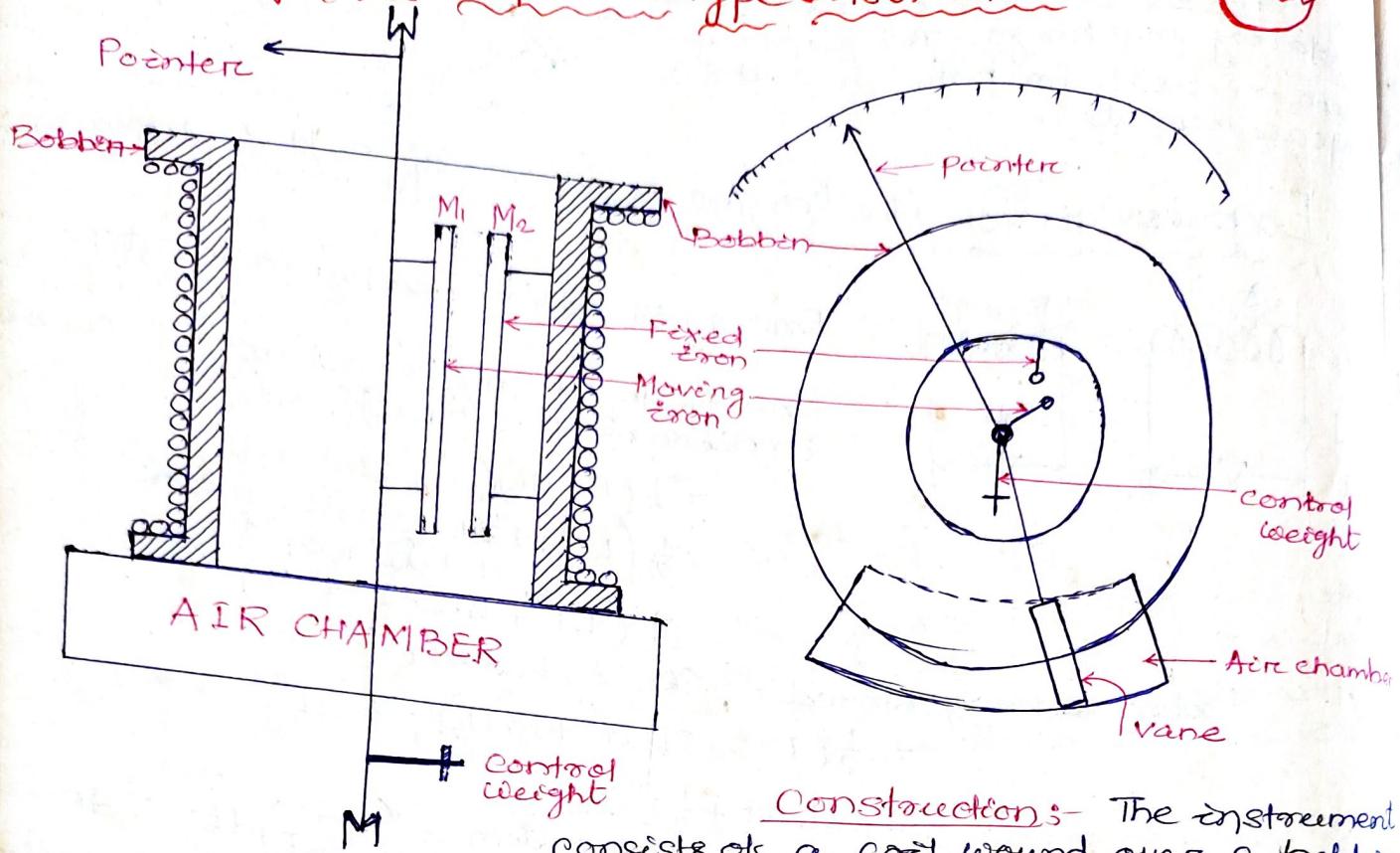
$$\text{or } K_s \sin\theta = \frac{1}{2} I^2 \frac{dL}{d\theta}$$

$$\text{or } \theta = \sin^{-1} \left(\frac{I^2 dL}{2 K_s d\theta} \right) = \sin^{-1} \frac{I^2}{2 K_s}$$

Except I , all things are constant.

Moving Iron Repulsion Type Instrument :-

(III)



Construction: The instrument

consists of a coil wound over a bobbin to which a pointer, an iron rod, a control weight and the vane are attached. There is one more soft iron rod inside the coil which is fixed to the coil.

Principle of Operation :- The current flowing through the coil produces magnetic field along the axis of the coil. This magnetises the two soft iron rods with similar poles and a repulsive force is produced between the rods. This force produces a deflecting torque on the moving system. The control weight produces control torque and the vane and air chamber provides air damping.

Expression for deflecting torque :- The intensity of magnetic field along the axis of the coil is H which is proportional to supply current through coil i.e. $H \propto i_m$

The pole strength induced in moving iron is m_1 , which is proportional to H and i_m i.e. $m_1 \propto H \propto i_m$

The pole strength induced on fixed iron is m_2 which is proportional to both H & i_m

$$\text{i.e. } m_2 \propto H \propto i_m$$

The force of repulsion, $F \propto m_1 m_2$

$$\text{or, } F = \dots^2$$

The deflecting torque, $T_d \propto F \propto i_m^2$

control torque, $T_c \propto \sin\theta$

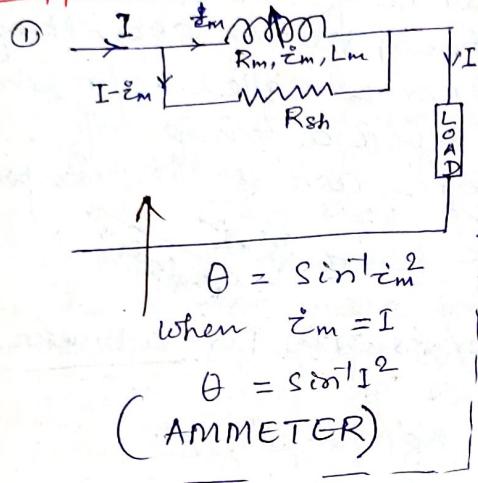
on steady state $T_d = T_c$

$$\therefore \sin\theta \propto i_m^2$$

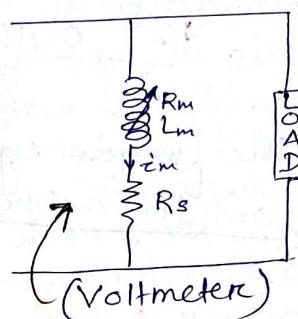
$$\text{or } \theta \propto \sin^{-1} i_m^2$$

This shows that the measuring scale is non-uniform.
This instrument can be used for both a.c and d.c measurements.

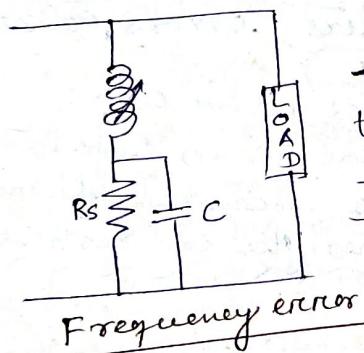
Applications :-



The moving instrument can be used as such for measuring small currents. But for large currents, shunt is used in parallel with the coil.



$$i_m = \frac{V}{R_m + R_s}$$
$$\theta \propto \sin^{-1} i_m^2 \propto \sin^{-1} \frac{V^2}{(R_m + R_s)^2}$$
$$C = \frac{0.41 L_m}{R_s^2}$$



To avoid frequency error, there is capacitor C have to provide parallel with series resistance.

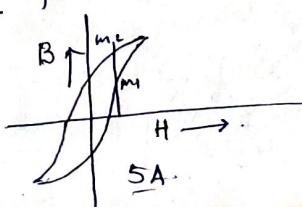
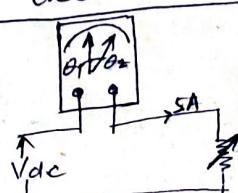
Changes in frequency may cause errors due to changes of reactance of working coil and also due to changes of magnitude of eddy currents setup in the metal parts of instrument.

Errors and remedy of MI instruments :-

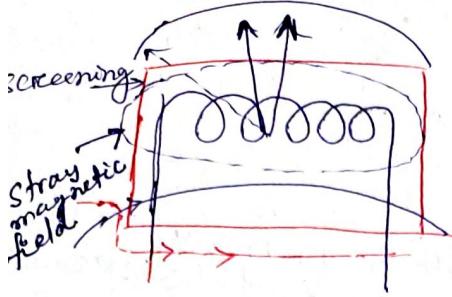
① Hysteresis error :-

The instrument shows a higher value for decreasing value than for increasing value. This error is called hysteresis error. The hysteresis error can be minimized by selecting the material for MI with a smaller hysteresis loop.

measured value of inst. is not proportional to H (magnetic force)



② Stray magnetic field error :-



The error of the instrument depends on the stray magnetic field around the instrument. If the stray magnetic field aids the magnetic field produced by the coil, the deflection shown increases. If it opposes, the deflection decreases.

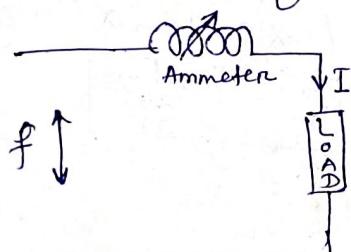
Error can be eliminated by magnetically screening the coil.

③ Frequency error :-

When the frequency of the source varies in case of Voltmeter, the impedance of the voltmeter varies, current varies, deflection shown varies even though the voltage is constant. This error is called as frequency error. It can be compensated over a range of frequencies by connecting a capacitance across the shunt resistance of value

$$C = \frac{0.41 L_m}{R_s^2}$$

$$Z = \sqrt{R^2 + X_L^2} \\ = \sqrt{R^2 + \left(\omega L_m\right)^2}$$



The frequency error is only for Voltmeter not for ammeter.

④ Temperature Error :-

In voltmeters, errors are caused due to self-heating of coil and series resistance; The increase in resistance causes decrease in current for a given voltage which produces a decreased deflection. The series resistance is about 10 times the coil resistance.

Problems

Question The full scale torque of a 5amps MI ammeter is $10 \mu\text{N-m}$. Estimate in $\mu\text{H/radian}$ the rate of change of self-inductance of the instrument at full scale.

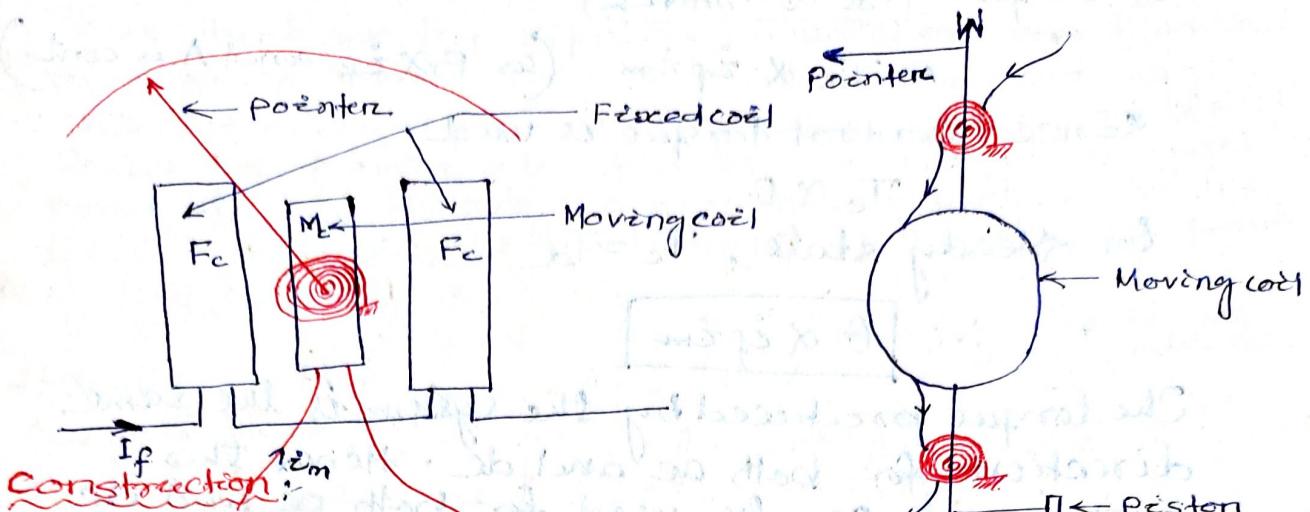
Soln:- $T_d = 10 \times 10^{-6} \text{ N-m}$, $I = 5 \text{ A}$, $\frac{dL}{d\theta} = ?$

We know, $T_d = \frac{1}{2} I^2 \frac{dL}{d\theta}$

or, $\frac{dL}{d\theta} = \frac{2 T_d}{I^2} = \frac{2 \times 10 \times 10^{-6}}{5^2} = 0.8 \mu\text{H/rad}$

~~HSFT
FHST
Frequency
Hysteresis
Stray
Temperature~~

IV ELECTRODYNAMOMETER



The electrodynamometer type instrument has two halves of fixed coil and moving coil. The two halves fixed coil are ~~are~~ identical and are connected in series. The moving coil is connected to a spindle provided in jewel bearings. Special hair springs are connected to the spindle, one on each side of the coil. The ends of coils are connected to the springs and terminals are brought out from these springs. A pointer and a piston for air damping are also connected to the spindle.

Operation: Current through the ~~field~~ coils produce uniform magnetic field along the axis of the coil. The current through the moving coil interacts with the magnetic field and a torque is produced (deflecting torque) on the moving system. As the moving system turns, the springs are distorted and produce control torque on the moving system. The movement of piston in the air chamber produces damping torque.

Expression of deflecting torque :-

Let Intensity of magnetic field along the axis of fixed coil be H which is proportional to the current flows the fixed coils i.e.

$$\text{i.e. } H \propto i_f$$

Again B (magnetic flux density) $\propto H \propto i_f$

Let N be the no. of turns of ~~coil~~ moving coil. A be the area of cross-section of moving coil. i_m be the current in the moving coil.

i. The current carrying moving coil experiences a torque $T_d \propto B A N^2 i_m$

or $T_d \propto i_m$ (as $B A$ if and A is const.)

Since control torque is used

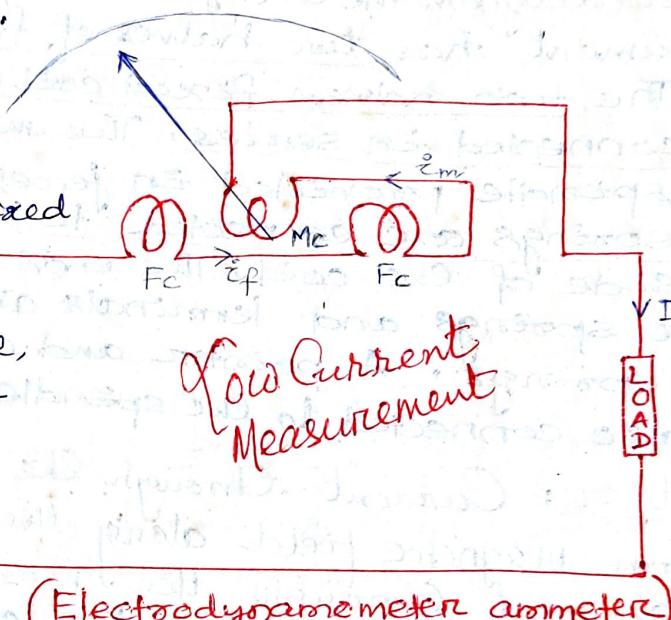
$$T_c \propto \theta$$

On steady state, $T_c = T_d$

i. $\theta \propto i_m$

The torque produced by the system is the same direction for both ac and dc. Hence this instrument can be used for both ac and dc measurements.

Applications :-



In this case, the fixed and moving coils are connected in series and therefore, they carry the same current.

$$\therefore i_f = i_m = I$$

As $\theta \propto \epsilon^2$

$$\therefore \theta \propto I^2$$

In this case, i.e. for this type of connections of fixed coil and moving coil, the instrument is known as ammeter because the deflection of pointer is proportional to the square of the current. This ammeter measures current having a range up to 200 mA.

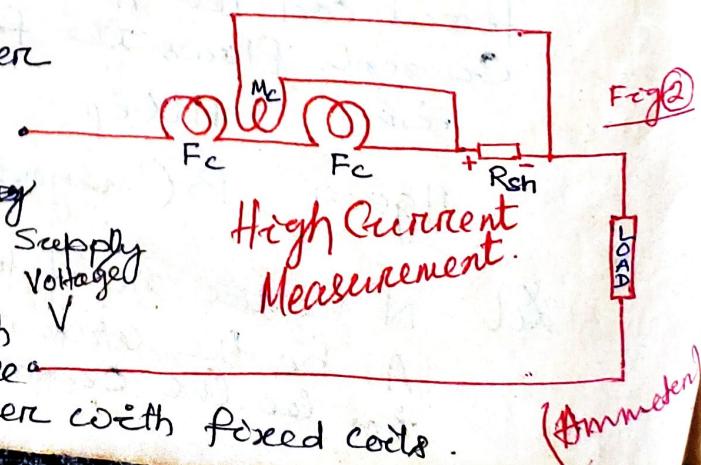
This instrument measures or reads rms value on ac circuit.

② For ammeters of higher capacity, ~~a shunt resistance has to be connected across moving coil~~

~~so that it carries more current~~

~~the moving coil is connected in series with its swamping resistance~~

~~across a shunt together with fixed coils~~



Thus there are two separate parallel branches for fixed and moving coils. In order that the ammeter may indicate correctly at all frequencies, the currents in the fixed and moving coils must be in phase. This requires the constant L/R of both branches to be equal as otherwise the currents in the two branches will not be independent of frequency.

Actually for high current measurements, a shunt resistance is connected in series with fixed coil and the moving coil is connected across the shunt. Fig 1

The purely selecting the R_{sh} , the instrument can be extended for different ranges of currents.

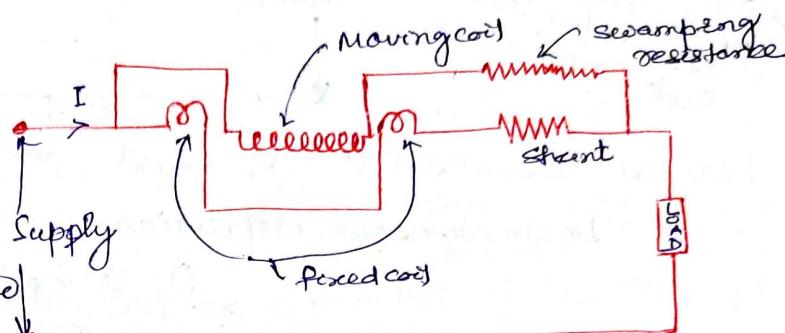
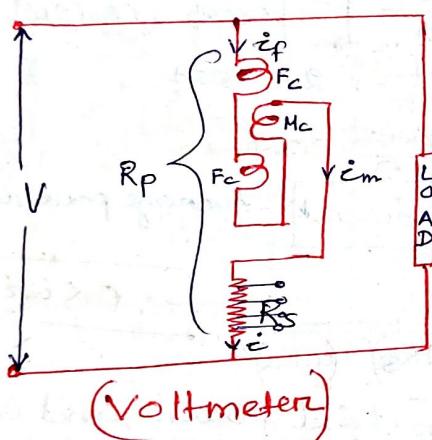


Fig 1 (Ammeter)

(3)



(Voltmeter)

By this type of connection, the instrument measures voltage across the load.

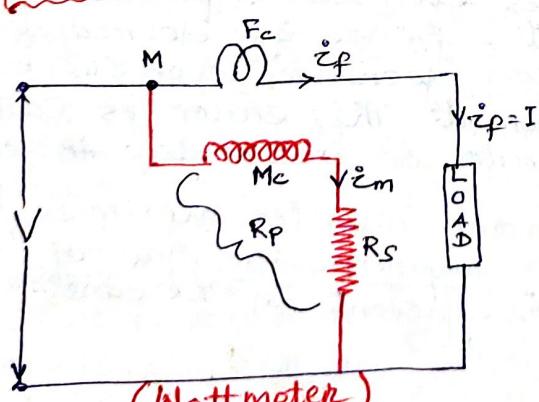
$$\text{Here } i_f = i_m = i, \\ i = \frac{V}{R_p} = \frac{V}{R_m + R_s}$$

Where R_m = meter resistance
 R_s = series resistance

$$\theta \propto i_f i_m \text{ or } \theta \propto i^2 \\ \text{or } \theta \propto \left(\frac{V}{R_p}\right)^2 \\ \text{or } \theta \propto V^2$$

The scale is non-uniform and the meter measures rms Value.

(4)



The instrument measures power. The scale is uniform. This instrument is called Wattmeter.

In case of wattmeter, F_F is called Current coil since it carries load current and M_C is Voltage coil or Voltmeter.

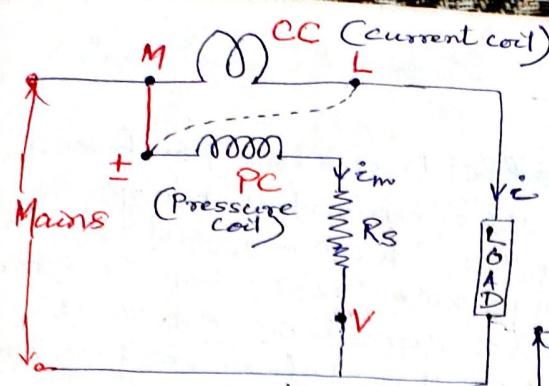
Here $i_f = I$ (current flows through the load)

$$i_m = \frac{V}{R_p}$$

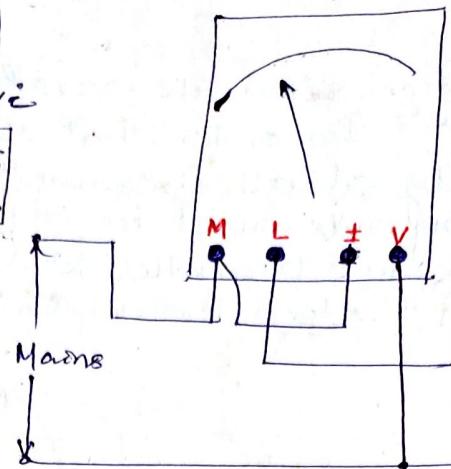
As $\theta \propto i_f i_m$

$$\therefore \theta \propto VI \cdot \frac{1}{R_p}$$

$$\text{or } \theta \propto VI \quad (\because R_p \text{ is constant} \\ \propto \text{Power})$$



M_r stands for service point
for load
 L_s stands for side point



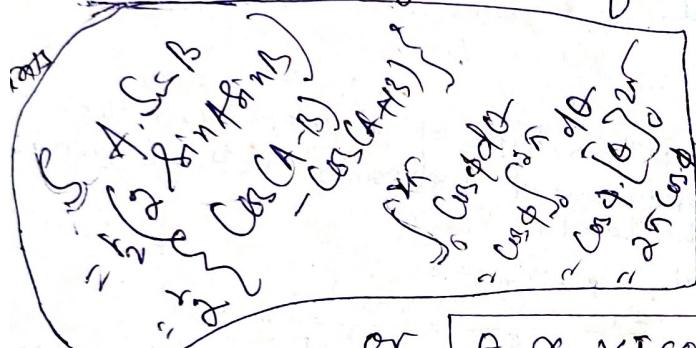
Wattmeter Connection

For AC source $V = V_m \sin \omega t$ and $i_m = I_m \sin(\omega t - \phi)$

Instantaneous deflection,

$\theta_i \propto i_m v_m$ (where i_m and v_m both are variable)

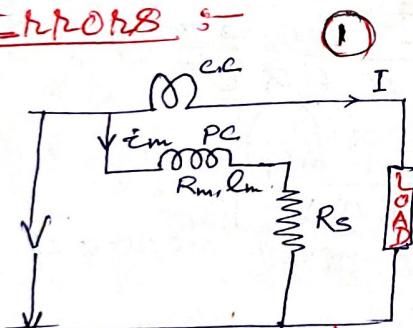
∴ Average value of $\theta = \frac{1}{2\pi} \int_0^{2\pi} \theta_i dt$



$$\begin{aligned} \theta &\propto \frac{1}{2\pi} \int_0^{2\pi} I_m \sin(\omega t - \phi) \cdot V_m \cos \omega t dt \\ &\propto \frac{V_m I_m}{4\pi} \int_0^{2\pi} [\cos \phi - \cos(2\omega t - \phi)] dt \\ &\propto \frac{V_m I_m}{4\pi} \times 2\pi \cos \phi \\ &\propto \frac{V_m I_m}{2} \cos \phi. \end{aligned}$$

or $\theta \propto VI \cos \phi \propto \text{power} \propto \text{Average power.}$

Errors :-



(Wattmeter)

R_p = Swamping resistance

$$i_m = \frac{V}{\sqrt{(R_m + R_s)^2 + (\omega L_m)^2}}$$

$\therefore \theta \propto i_m v_f$

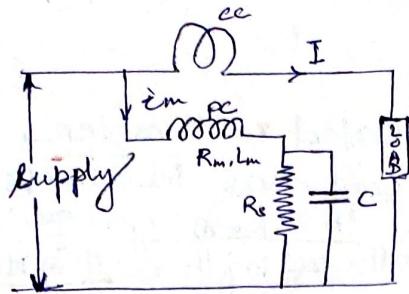
Assume constant power, and constant Voltage, [Temperature $\propto R_s$]

As Temp. increases, the series resistance increases and current decreases even the input voltage is constant. Since i_m decreases, the deflection decreases even the power is constant. This error is called

① temp. error or error due to temp..

This error can be minimised by using manganese wire having lowest temp. co-efficient in the construction of R_s .

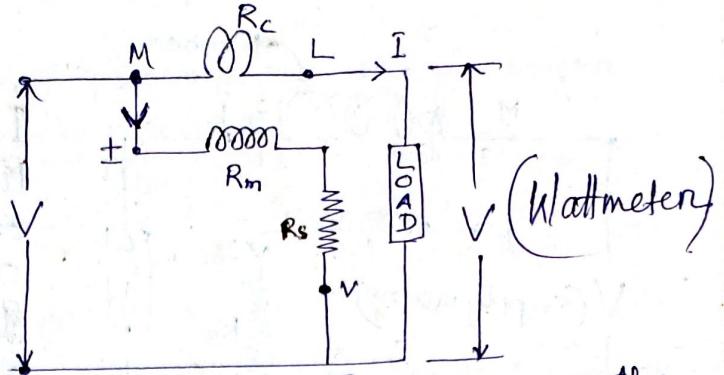
② When the input frequency increases, the reactance of the moving coil increases, the current i_m decreases and θ also decreases. This error is called as frequency error. This error can be minimised by connecting a capacitor of value $\frac{0.413}{f^2 R_s}$ across R_s .



② (Frequency error)

Error due to connections or,

③



Wattmeter reading = (Voltage across the pressure coil) through \times (Current across current coil)

$$W_1 = (V + IR_c)I$$

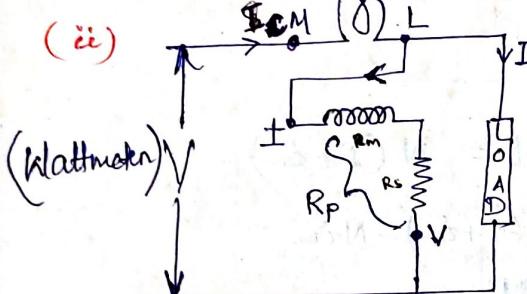
$$= VI + I^2 R_c$$

where VI = true power

$I^2 R_c$ = Error

Here, the pressure coil is connected on the supply side.

For Wattmeter connection, the current through current coil must flow through the load and the supply voltage must be same to the voltage across pressure coil.

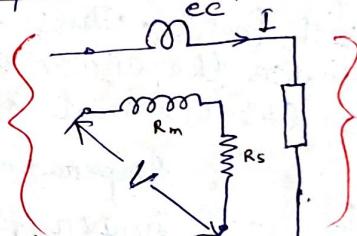


Here, the pressure coil is connected on the load side.

∴ Wattmeter reading

$$W_2 = VI \left(1 + \frac{V}{R_p} \right) = VI + \frac{V^2}{R_p} \} \text{ error.}$$

$$\text{or, } W_2 = VI + \frac{V^2}{R_p} \quad \begin{cases} \text{large error} \\ \text{small power} \end{cases}$$

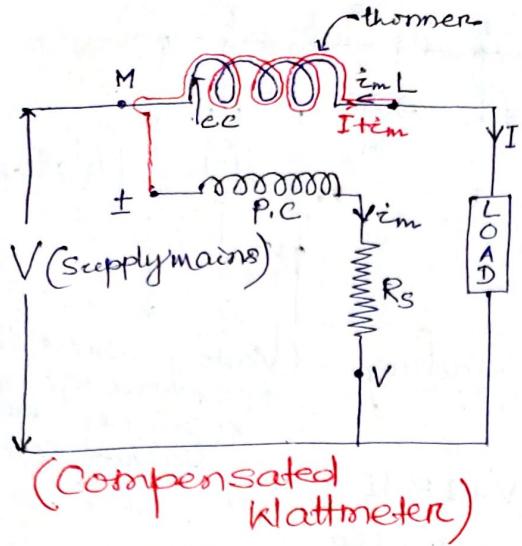


→ For low power measurement, the pressure coil is connected across the supply side.

→ For measuring high power, the pressure coil is connected across the load side.

It is not possible to avoid error in the wattmeter but it can be minimise by selecting the connection of pressure coil.

Compensation :- The power taken by a pressure coil circuit is constant if the voltage is constant and becomes a smaller percentage of total as a larger amount of power is measured. Pressure coil connection to load side may result in large error due to large load current and low power factor since the total power measured is small. Hence in wattmeters which are designed for lower power factor measurements, a compensating coil may be used in the instrument to compensate for the error caused by power loss in the pressure coil side.



In compensated wattmeter, the fixed coil has two coils one with a thinner gauge and the other with a thicker gauge. The coil with a thicker gauge is the normal ~~coil~~ ~~gauge~~ coil. The coil with a thinner gauge is called compensating winding which is connected in series with the pressure coil. The compensating coil is connected ~~in series~~ made

as possible nearly as possible identical and coincident with current coil. It is so connected that it opposes the field of current coil.

The compensating coil carries a current i_m and produces a field corresponding to current of current coil. This field acts in opposition to the current coil field. Thus the resultant field is due to current I only. Hence the error caused by the pressure coil current flowing in the current coil is neutralized.

$$\text{Ampereturns for PC} = N i_m$$

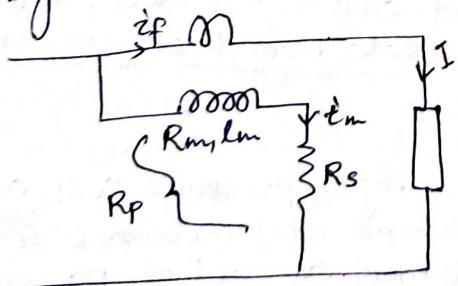
$$\text{Ampereturns for Current coil} = N(I + i_m)$$

$$\therefore \text{Total ampere turns, AT} = N(I + i_m) - N i_m \\ = NI$$

$$\text{where } i_m = \frac{V}{R_p}$$



Error due to power factor of the load :- When the power factor of the load changes say from lagging to leading keeping the power constant, the reading shown by the wattmeter changes. This error is due to power factor.



When inductance is considered the wattmeter shows a higher value for lagging power factor.

Let $\theta_m = 0$

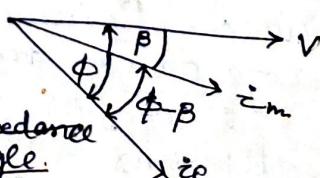
(i)

Hence Deflecting torque
 $T_d \propto i_m \cos \phi$
 $\propto VI \cos \phi$

If $\theta_m \neq 0$

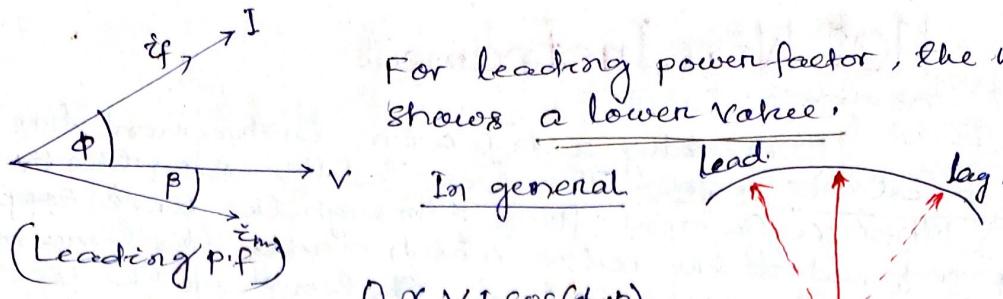
(ii)

Where $\beta = \frac{\text{impedance angle}}{i_m}$



$$T_d \propto i_m \cos(\phi - \beta)$$

or $T_d \propto VI \cos(\phi - \beta)$



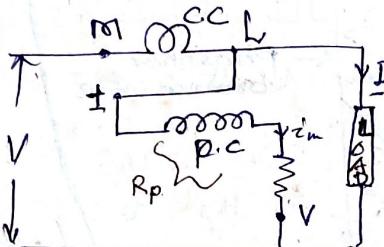
$$\Theta \propto VI \cos(\phi + \beta)$$

To minimize this error, the no. of turns of conductor should be minimum.

Problems

W-80 (1) A dynamometer wattmeter with its voltage coil connected across the load side of the instrument reads 220 watts. If the voltage is 200V and what power is taken by the load. The voltage coil has a resistance of 4000Ω. Neglect the resistance of the current coil. $R_{rc} = 0$.

Soluⁿ:



Wattmeter reading

$$= VI + \frac{V^2}{R_p} \text{ where } R_p = R_m + R_s$$

$$\therefore 220 = \frac{V^2}{4000} VI + \frac{(200)^2}{4000}$$

$$\text{type or } VI = 220 - \frac{40000}{4000}$$

$$= 220 - 10 = 210 \text{ watts (Ans.)}$$

W-86

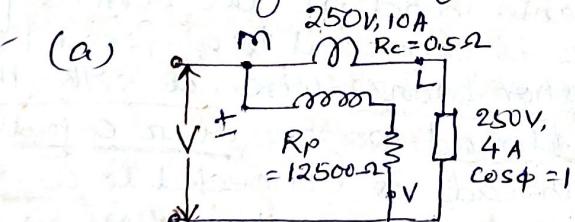
A 250V, 10A dynamometer wattmeter has resistance of current and pressure coil are 0.5Ω & 1250Ω respectively. Find the % error for

(a) the pe is connected on the supply side.

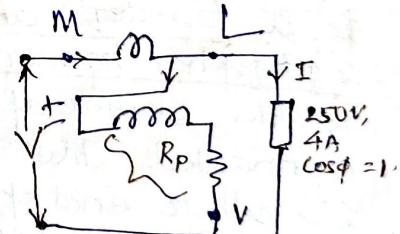
(b) the pe is connected on the load side.

Assume unity powerfactor load at 250V, 4A.

Soluⁿ:



(b)



Wattmeter reading,

$$\begin{aligned} W_1 &= VI + I^2 R_p \\ &= 250 \times 4 + 4^2 \times 0.5 \\ &= 1008 \text{ watts} \end{aligned}$$

$$\therefore \% \text{ error} = \frac{\text{Wattmeter reading - true power}}{\text{true power}}$$

$$\therefore \text{In (a)} \% \text{ error} = \frac{1008 - 1000}{1000} \times 100 = 0.8\% \text{ (Ans.)}$$

$$\text{In (b)} \% \text{ error} = \frac{1005 - 1000}{1000} \times 1000 = 0.5\% \text{ (Ans.)}$$

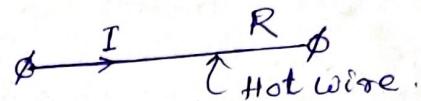
Wattmeter reading,

$$\begin{aligned} W_2 &= VI + \frac{V^2}{R_p} \\ &= 250 \times 4 + \frac{(250)^2}{1250} \\ &= 1005 \text{ watts} \end{aligned}$$

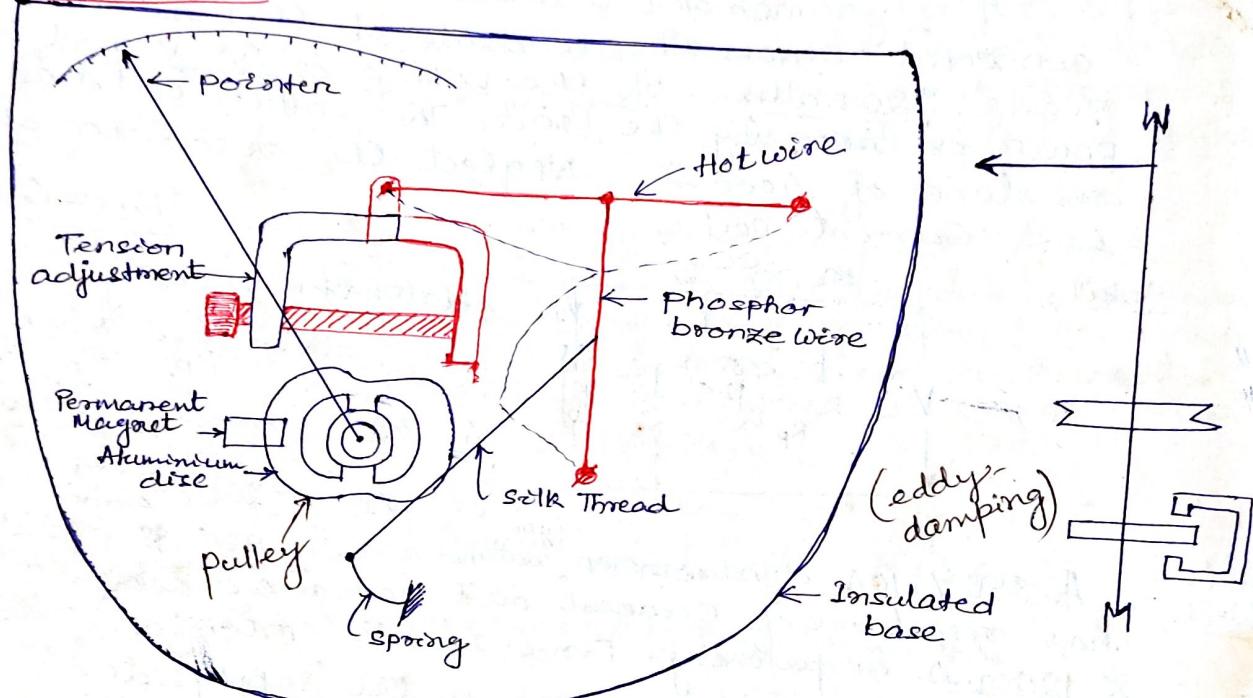
V

Hot Wire Instrument

Basic principle :- Basically a hot wire instrument has a high resistance wire. The wire is fixed on the both the ends. When a current flows through the wire, copper loss is produced in the wire which raises the temperature of wire. Due to increase in temp., the length of the wire increases and sag is formed. The sagging is utilised in moving mechanical system to which a pointer is attached.



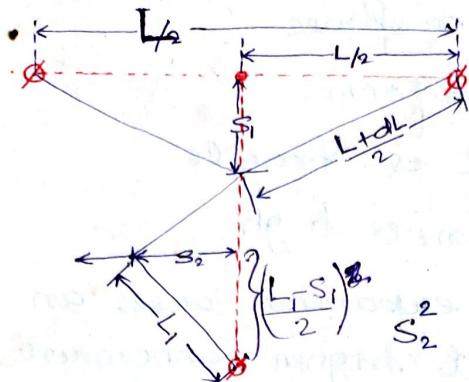
construction :-



This instrument has a high resistance wire of platinum instead of diameter of 0.1 mm. One end of the wire is fixed whereas the other end is connected to a tension adjustment and at the centre of hotwire a phosphor bronze wire is connected to a fixed point. To the centre of phosphor bronze wire, a silk thread is connected. The silk thread passes over a pulley and the other end of silk thread is connected to a spring. The pulley is connected to a spindle which is put on jewel bearing. A pointer and aluminium disc are attached to the spindle. A permanent magnet is provided at the edge of aluminium disc. The whole assembly is mounted on an insulated base.

Operation :- When the current flows through the hot wire, $I^2 R$ is developed in the wire. Due to this power loss $I^2 R$, the temp. of the wire increases. Due to increase in temperature, the length of wire expands and sags. Due to this one end of phosphor bronze wire comes down and the silk thread is pulled over the pulley. The pulley turns and the pointer deflects on the scale.

Expression for the sag :-



This instrument is double sag arrangement which gives the ~~opt~~ amplification.

$$\begin{aligned} S_1^2 &= \left(\frac{L+dl}{2}\right)^2 - \left(\frac{L}{2}\right)^2 \\ &= \cancel{L^2} + dl^2 + 2Ldl - \cancel{L^2} \\ &= \frac{2Ldl}{4} = \frac{Ldl}{2} \Rightarrow S_1 = \sqrt{\frac{Ldl}{2}} \end{aligned}$$

$$S_2^2 = \left(\frac{L}{2}\right)^2 - \left(\frac{L-S_1}{2}\right)^2 = \cancel{L^2} - \cancel{L^2} - S_1^2 + 2LS_1$$

$$= \frac{2LS_1}{4} = \frac{LS_1}{2} \Rightarrow S_2 = \sqrt{\frac{LS_1}{2}}$$

Again, $S_2 = \sqrt{\frac{L}{2}} \sqrt{\frac{Ldl}{2}} = \sqrt{\frac{L}{2} \cdot \frac{L}{2} \cdot Ldl} = \sqrt{\frac{L}{2} \sqrt{\frac{L}{2}}} \cdot \sqrt{Ldl}$

or $S_2 \propto \sqrt{Ldl}$ or $\boxed{S_2 \propto (dl)^{1/4}}$

change in length, dl is proportional to temperature rise
i.e. $dl \propto \text{temp. rise}$

$$\propto I^2 R$$

$$\text{or } dl \propto I^2$$

$$\therefore S_2 \propto (I^2)^{1/4} \text{ or } \boxed{S_2 \propto \sqrt{I}}$$

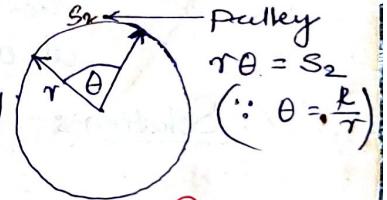
$$r\theta = S_2$$

$$\text{or } \boxed{\theta \propto S_2}$$

$$\boxed{\theta \propto S_2 \propto \sqrt{I}}$$

where

r = radius of the pulley



This instrument is slow in operation and Particularly this type of instrument is helpful to Electronic Engineers.

Applications :- This instrument is used to measure current and the instrument can be used for both ac and dc.

- Advantages :-
- (1) It can be used to measure for both ac and dc current.
 - (2) It measures true r.m.s current.
 - (3) It can be used for a wide range of frequency upto megahertz.
 - (4) It does not have stray magnetic field error.
 - (5) It does not have hysteresis error.

Disadvantages :-

- (1) The overload capacity is almost zero.
- (2) It is slow in response.
- (3) Scale is non-uniform.
- (4) The instrument is fragile.
- (5) Power consumption is high.
- (6) The differential expansion causes an initial reading at higher temperature.

In this instrument, very thin hot wire is used. So skin effect is not so important.

Problems

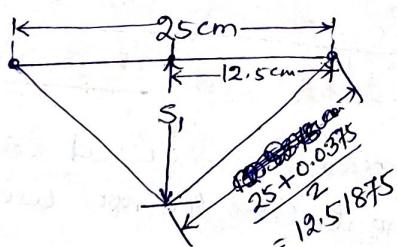
(1)

W-86
Electronics

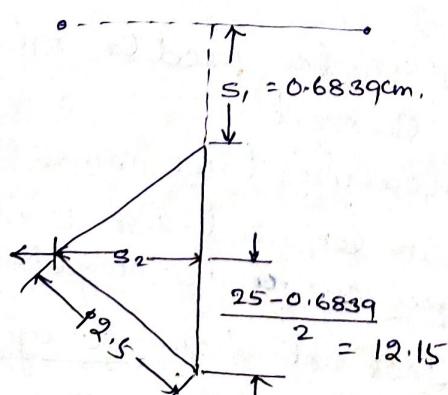
The length of hot wire is 25 cm. It is made of material having co-efficient of expansion is equal to $0.000015 \text{ cm/cm}^{\circ}\text{C}$. The working temperature and ambient temperature are respectively 110°C and 10°C . The length of the auxiliary phosphor bronze wire is also 25 cm. Calculate the magnification of expansion.

Solution :-

$$\begin{aligned} \text{Change in Length, } dL &= 0.000015 \times 25 \times (110 - 10) \\ &= 0.000015 \times 25 \times 100 \\ &= 0.0375 \text{ cm.} \end{aligned}$$



$$\begin{aligned} S_1 &= \sqrt{(12.5)^2 - (12.5)^2} \\ &= 0.6839 \text{ cm.} \end{aligned}$$



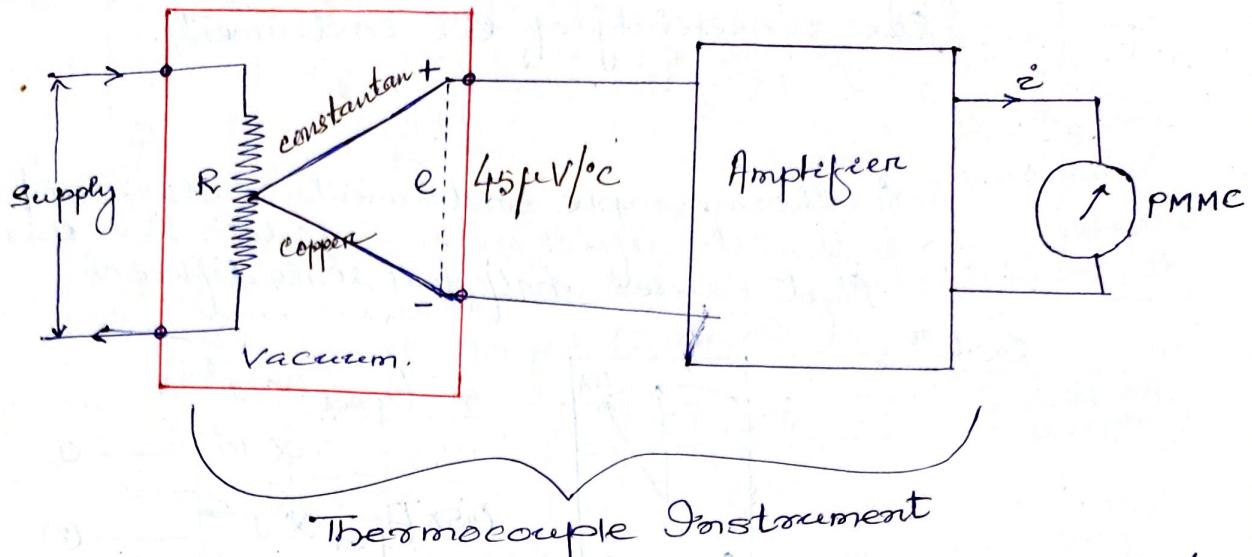
$$S_2 = \sqrt{(12.5)^2 - (12.15)^2} = 2.93$$

Magnification of Expansion

$$\frac{S_2}{dL} = \frac{2.93}{0.0375} = 78.1 \text{ Ans}$$

VI

Thermocouple Instrument



Construction :- This instrument has a thermocouple with two dissimilar metal joint at V junction. Normally Constantan and Copper are used for making the thermocouple. The thermocouple junction is heated by a heating element. The output of this is amplified and measured by a PMMC instrument. The heating element on the thermocouple is normally enclosed in a glass and vacuum is created inside the glass.

Principle :- When current flows through the heating element, heat is produced at the junction of the thermocouple and the thermocouple produces a voltage. This voltage is about $45 \mu\text{V}/^\circ\text{C}$. This voltage is amplified and measured. As current increases, the reading in PMMC is also increases.

- θ (deflection) \propto Current in PMMC
- \propto Thermoelectric emf.
- \propto Temperature rise
- \propto Copper loss in heating element.

$$\text{OR, } \theta \propto I^2 \text{ or, } \boxed{\theta \propto I^2}$$

This instrument measures current.

- Advantages :-
- (1) It measures both ac and dc current.
 - (2) It can be used for wide range of frequency even for megahertz.
 - (3) It does not have stray magnetic field error.
 - (4) It does not have hysteresis loss error.
 - (5) It measures the true rms value.
 - (6) The input waveform may be distorted.

Disadvantages :-

- (1) The conduction and radioactive losses reduce the sensitivity of the instrument.

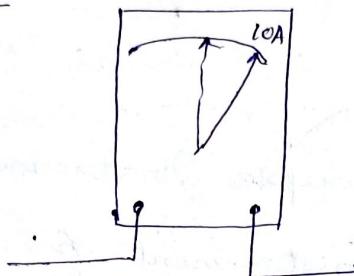
Problem :-

(1)

Wl-84
Electronics

A thermocouple instrument reads 10 amps of full scale deflection. Calculate the current that causes half full scale deflection.

Soln :-



$$\theta_{fsd} \propto I^2$$

$$\propto 10^2 \quad \text{--- (1)}$$

$$0.5 \times \theta_{fsd} \propto I^2 \quad \text{--- (2)}$$

$$(2) \div (1) \Rightarrow \frac{I^2}{10^2} = 0.5 = \frac{1}{2}$$

$$\text{or } I^2 = \frac{10^2}{2} = \frac{100}{2}$$

$$\text{or } I = \sqrt{\frac{100}{2}} = \frac{10}{\sqrt{2}} = 7.07 \text{ A}$$

(Ans).

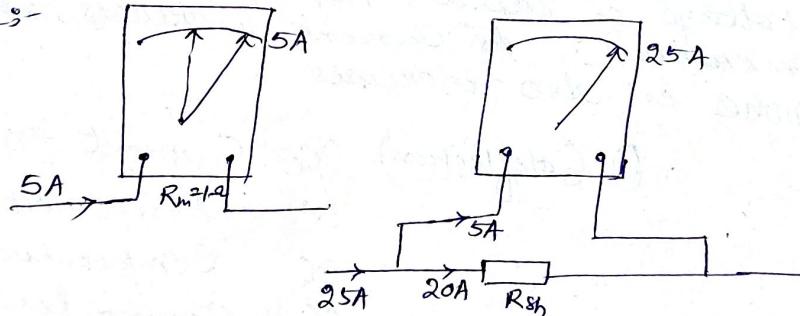
(2)

Wl-85
Electronics

A thermocouple instrument with an internal resistance of 1 ohm reads 5 amps at full scale deflection. Calculate

- The current that causes half full scale deflection.
- The shunt to be used for extending it to read 25 amps at full scale deflection.

Soln :-



$$(i) \quad \theta_{fsd} \propto 5^2 \quad \text{--- (1)} \quad (2) \div (1) \Rightarrow \frac{I^2}{5^2} = 0.5 = \frac{1}{2}$$

$$0.5 \theta_{fsd} \propto I^2 \quad \text{--- (2)}$$

$$\text{or } I = \frac{5}{\sqrt{2}} = 3.535 \text{ A} \quad \text{(Ans)}$$

$$(ii) \quad R_{sh} = \frac{5A \times 1 \Omega}{20A} = \frac{5 \times 1}{20} = \frac{1}{4} = 0.25 \Omega \quad \text{(Ans)}$$

(3)

Wl-87
Electronics

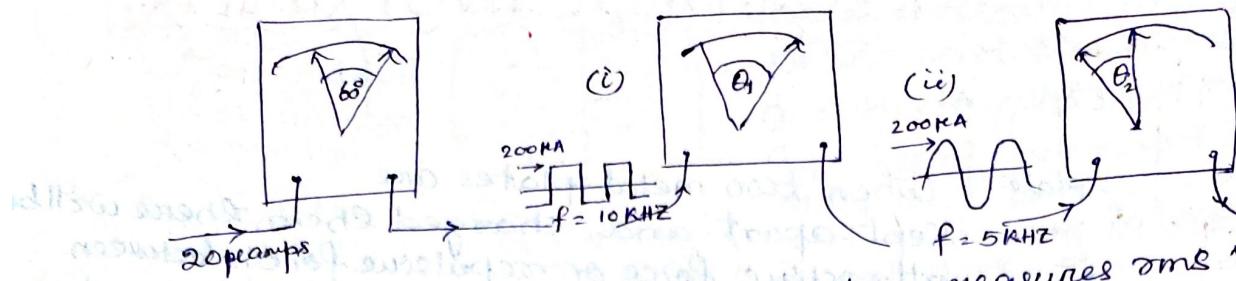
A thermocouple type meter has full scale deflection of 60°. When 200 μA flows through it.

Determine the deflection when

- A square wave current of 10 KHz frequency and 200 mamps. peak flows.

- A sineoidal wave current at 5 KHz frequency and 200 mamps peak flows.

Solution :-



As a thermocouple type meter measures rms value, the rms value of current for square wave current is equal to $200 \mu\text{A}$ (same value).

$$\text{Imp} \quad I_1 \quad T_1 \quad T_1 \quad (I = \sqrt{\frac{I_1^2 T_1 + I_1^2 T_1}{2 T_1}} = I_1) \quad (\text{symmetrical waveform})$$

$$(i) \quad \theta_{sf, d_1} \propto I_1^2 \quad \text{or} \quad 60^\circ \propto (200 \mu\text{A})^2 \quad (1)$$

$$\theta_{sf, d_2} \propto I_2^2 \quad \text{or,} \quad \theta_{sf, d_2} \propto (200 \mu\text{A})^2 \quad (2)$$

$$(2) \div (1) \Rightarrow \frac{\theta_{sf, d_2}}{60^\circ} = \frac{(200 \times 10^6)^2}{(200 \times 10^6)^2} = 1$$

$$\text{or, } \theta_{sf, d_2} = 60^\circ \quad (\text{Ans})$$

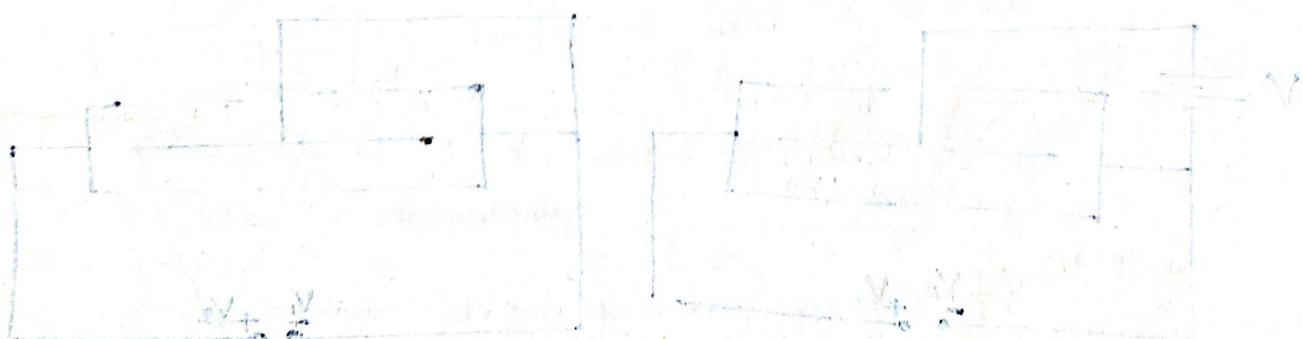
$$(ii) \quad \text{In sinusoidal wave current, } I_{\text{rms}} = \left(\frac{200 \mu\text{A}}{\sqrt{2}}\right) \text{ Ans.}$$

$$\theta_{sf, d_1} \propto I_1^2 \Rightarrow 60^\circ \propto (200 \times 10^6)^2 \quad (3)$$

$$\theta_{sf, d_3} \propto I_3^2 \Rightarrow \theta_{sf, d_3} \propto \left(\frac{200 \times 10^6}{\sqrt{2}}\right)^2 \quad (4)$$

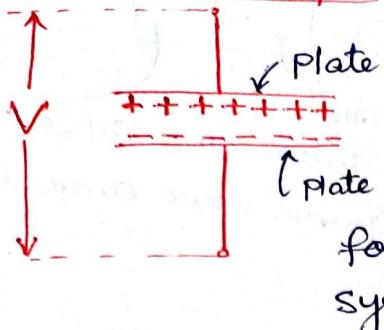
$$(4) \div (3) \Rightarrow \frac{\theta_{sf, d_3}}{60^\circ} = \frac{(200 \times 10^6)^2}{(200 \times 10^6)^2} \times \frac{1}{2}$$

$$\text{or, } \theta_{sf, d_3} = \frac{1}{2} \times 60^\circ = 30^\circ \quad (\text{Ans})$$



(VII) ELECTROSTATIC INSTRUMENT

Basic Principle :-



When two metal plates are kept apart and charged them, there will be attractive force or repulsive force between the plates. These forces can be utilized for producing deflecting torque on the moving system.

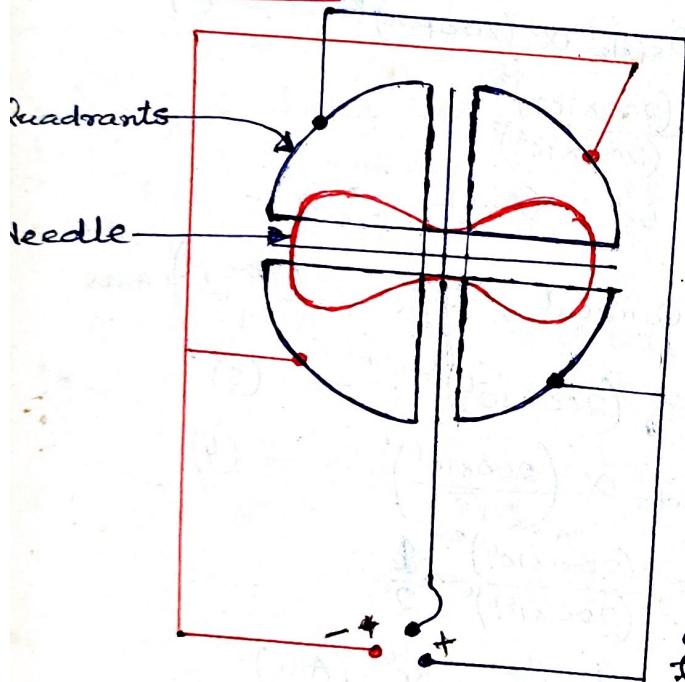
High Voltage Measurement Instrument

There are two types of electrostatic instruments.

- (i) Quadrant type (ii) Attracted disc type

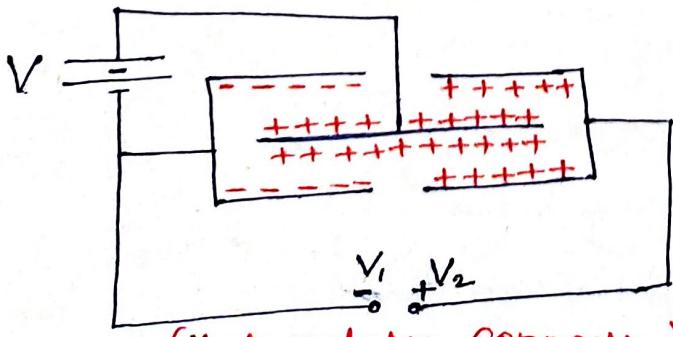
Quadrant type is used up to 20 KV, and attracted disc type is used for more than 20 KV.

Construction :-

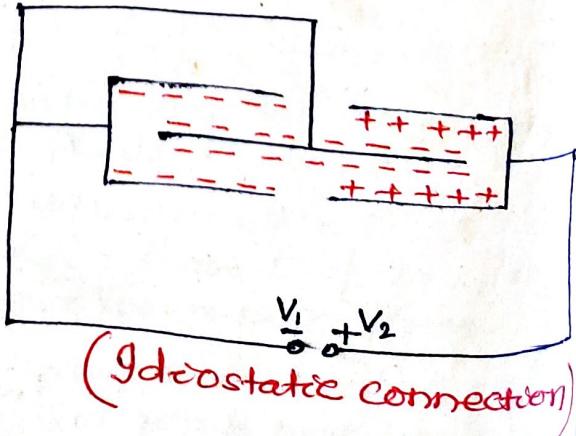


The quadrant type electrostatic has 4 fixed metal double quadrant arranged to form a shallow circular box with short gap between the top and bottom quadrants. In between the quadrants, a double sector needle is suspended from phosphor bronze wire. The diagonally opposite quadrants are connected together and form one terminals of the instrument and similarly other two opposite quadrants are connected together and form the other terminal. Needle forms the third terminal.

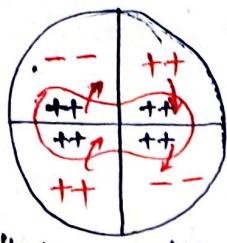
The needle and quadrants may be connected in two different ways. The needle may be directly connected to one set of quadrant called as idioscopic connection and in the other case if it is connected through a high voltage d.c source called as heterostatic connection.



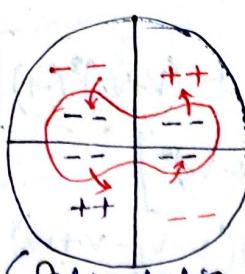
(Heterostatic connection)



(Idioscopic connection)



(Heterostatic connection)

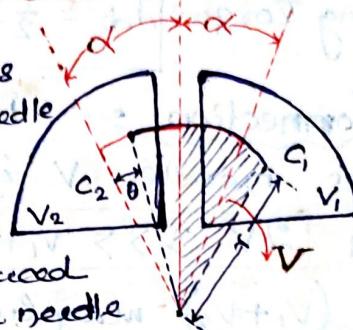


(Homostatic connection)

The deflection charges on the needle and quadrants produce a deflecting torque on the needle. In case of heterostatic connection clockwise torque is produced and in case of homostatic, torque is anticlockwise.

Expression for deflecting torque :- (heterostatic)

Let V_1, V_2 & V be the voltages on the quadrants and the needle respectively. Due to this voltage, charges are induced on the quadrants and needle. A deflection torque is produced on the needle and let the needle turned through an angle θ .



(It is called a/s heterostatic instrument)
(where $V > V_1$ & $> V_2$)

Let, the needle move through angle $d\theta$ in dt seconds. Due to this, there is a change in the energy of mechanical system and the energy in the electrostatic system. Due to conservation of energy, this two changes in energy must be zero. Equal change in mechanical energy equals $T \frac{d\theta}{dt} dt \equiv T d\theta$.

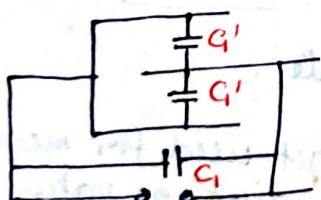
Change in electrostatic energy equals dW where W is the energy stored in the electrostatic system.

$$\therefore T d\theta = dW \text{ or } T = \frac{dW}{d\theta}$$

Deflecting torque is equal to the rate of change of energy stored in the electrostatic system.

Energy stored in capacitance.

$$W = \frac{1}{2} (V - V_1)^2 C_1 + \frac{1}{2} (V - V_2)^2 C_2$$



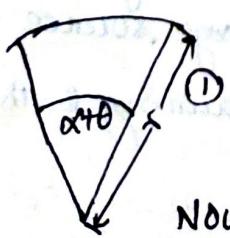
$$C_1 = 2C_1' \quad \text{we know } C_1' = \frac{\epsilon_0 A}{d}$$

In fig. ①, area of the arc,

$$A = \frac{1}{2} r^2 (\alpha + \theta) \quad \left\{ \frac{2\pi}{\alpha} - \frac{\pi r^2}{2\pi} \alpha = \frac{1}{2} \alpha^2 \right\}$$

$$\therefore C_1 = 2 \times \frac{1}{2} \cdot \frac{r^2}{d} (\alpha + \theta) \cdot \epsilon_0 = \frac{r^2 (\alpha + \theta) \epsilon_0}{d}$$

$$\text{similarly, } C_2 = \frac{r^2 (\alpha - \theta) \epsilon_0}{d}$$



$$\text{Now } W (\text{energy stored in capacitance}) = \frac{1}{2} (V - V_1)^2 \cdot \frac{r^2 (\alpha + \theta) \epsilon_0}{d}$$

$$\text{or, } W = \frac{1}{2} \frac{r^2 \epsilon_0}{d} \left\{ (V - V_1)^2 (\alpha + \theta) + (V - V_2)^2 (\alpha - \theta) \right\} + \frac{1}{2} (V - V_2)^2 \cdot \frac{r^2 (\alpha - \theta) \epsilon_0}{d}$$

$$\begin{aligned}
 \text{Now } \frac{dW}{d\theta} &= \frac{1}{2} \frac{r^2 C_0}{d} \left[(V-V_1)^2 (1) + (V-V_2)^2 (-1) \right] \\
 &= \frac{1}{2} \frac{r^2 C_0}{d} \left[(V-V_1)^2 - (V-V_2)^2 \right] \\
 &= \frac{1}{2} \frac{r^2 C_0}{d} (V-V_1+V-V_2)(V-V_1-V+V_2) \\
 &= \frac{1}{2} \frac{r^2 C_0}{d} (2V-V_1-V_2)(V_2-V_1)
 \end{aligned}$$

\therefore Deflecting torque $T_d = \frac{1}{2} \frac{r^2 C_0}{d} (V_2-V_1)(2V-V_1-V_2)$

Heterostatic Connection :- In heterostatic torque, Voltage applied across needle V is much greater than V_1 & V_2 .

$$\text{Again } 2V \gg V_1 + V_2$$

$\therefore (V_1+V_2)$ may be neglected in comparison to V .

$$\begin{aligned}
 \therefore T_d &\approx \frac{1}{2} \frac{r^2 C_0}{d} \cdot 2V \cdot (V_2-V_1) \\
 &\approx \frac{1}{2} \frac{r^2 C_0}{d} 2V \cdot V_2 \quad (\text{Where } V_2-V_1 = V)
 \end{aligned}$$

$$\text{or, } T_d \propto V \quad \text{Control torque, } T_c \propto \theta$$

$$\therefore \theta \propto V \text{ on steady state.}$$

Idiostatic Connection :- In this connection, there is no extra source connected to needle.

$$\begin{aligned}
 \therefore V &= V_1 \\
 \text{so, } T_d &= \frac{1}{2} \frac{r^2 C_0}{d} (V_2-V_1)(V_1-V_2) \\
 &= -\frac{1}{2} \frac{r^2 C_0}{d} (V_2-V_1)^2 \\
 &= -\frac{1}{2} \frac{r^2 C_0}{d} V^2 \quad (\because V_2-V_1 = V)
 \end{aligned}$$

$$\text{or, } T_d \propto V^2 \quad \text{Control torque, } T_c \propto \theta$$

$$\therefore \theta \propto V^2 \text{ on steady state.}$$

Important :- Heterostatic connection is not used for measuring ac voltages because θ measures average value but the average value of sinusoidal voltage source is zero. Idiostatic connection is used for measuring both ac. and dc.

In Heterostatic connection,

$T_d \propto 2V/V$. This type of connection of electrostatic meter is known as Electrometer

In Heterostatic connection

$$T_d \propto N^2$$

In case of heterostatic connection, a large d.c voltage is used for the needle and hence a very small voltage apply to the terminals can be measured.

In this case, it is called as Electrometer.

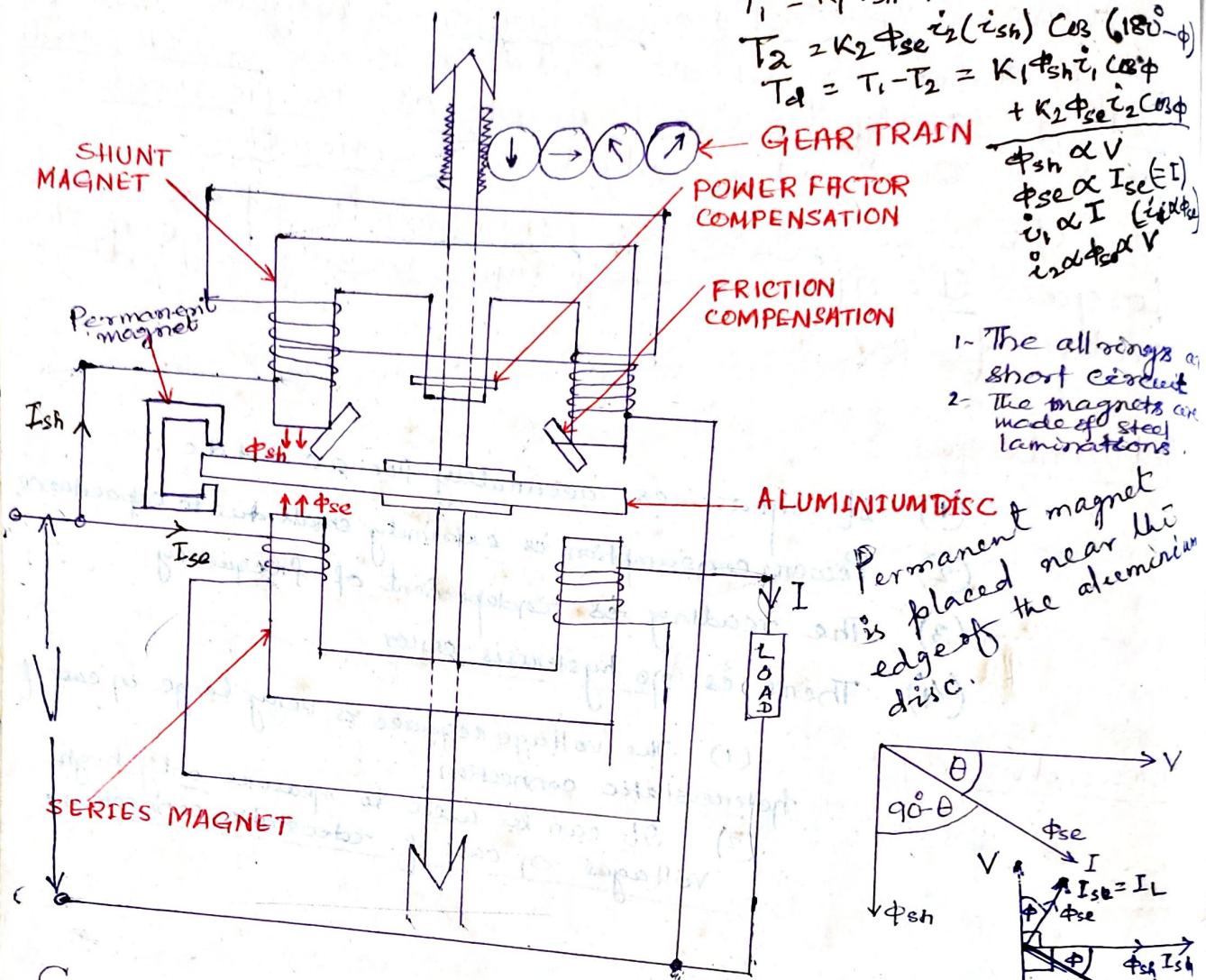
In the case heterostatic connection to get appreciable torque the applied voltage must be very large of the order of KV.

Advantages :-

- (1) It measures accurately for ac and d.c.
- (2) Power consumption is extremely small due to capacitance.
- (3) The reading is independent of frequency.
- (4) There is no hysteresis error.

Disadvantages :- (1) The voltage required is very large in case of heterostatic connection.
(2) It can be used to measure only high voltages in case of heterostatic connection.

(VIII) SINGLE PHASE INDUCTION TYPE ENERGY METER



CONSTRUCTION :- The single phase induction type energymeter has an aluminium disc attached to a spindle. The spindle is provided on jewel bearings. A gear train is attached to the spindle. There are two electromagnets, one magnet is called as series magnet connected in series with the load and another is a shunt magnet connected across the load. In between the two electromagnets, the aluminium disc is provided. At the diagonally opposite ends of the disc, a permanent magnet is provided. Copper shading rings are provided on the centre limb as well as on the outer limbs of the shunt magnet.

OPERATION :- The Load Current flowing through the series magnet produces a flux ϕ_{se} . This induces an eddy current i_{se} in the aluminium disc. Similarly the shunt magnet produces a flux ϕ_{sh} and induces a current i_{sh} in the aluminium disc. Now there are two fluxes ϕ_{se} and ϕ_{sh} and two eddy currents i_{se} and i_{sh} . These two currents and fluxes are interact & produce a deflecting torque on the aluminium disc. The deflection torque, $T_d \propto \phi_{se} \cdot \phi_{sh} \sin(90^\circ - \theta)$

$\phi_{se} \propto I$ and $\phi_{sh} \propto V$ - This is according to Wattmeter.

$$\therefore T_d \propto VI \cos \theta$$

As the aluminium disc rotates, it cuts the flux of the permanent magnet. This induces an eddy current in the aluminium disc which interacts with the flux of the permanent magnet and produces an opposing torque. This opposing torque is a breaking torque T_b .

$$T_b \propto \text{speed } N$$

On steady state, the disc rotates at a constant speed and hence $N \propto \text{power}$ ($\because T_b \propto T_d$).

Multiplying by dt both sides and integrate, we get

$$\int N dt \propto \int \text{power} dt$$

$$(T_b d \frac{\Phi^2 N}{R})$$

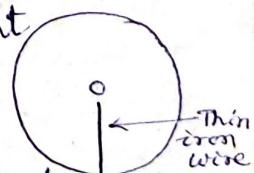
$$\therefore \boxed{\text{No of revolutions} \propto \text{Energy}}$$

The no. of revolutions of the disc is counted by gear train.

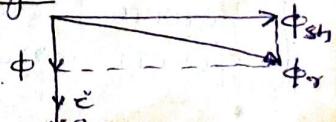
Friction error :- On light loads, the torque produced on the aluminium disc is very small and if it is less than the friction torque of ~~the~~ gear train, the disc will not rotate. This error is called friction error.

To compensate for the friction, the copper shading rings provided on the outer limb are adjusted until the disc just starts rotating. Under this condition the torque produced by the friction compensating device is equal to and opposite to the torque produced by the gear train.

Creeping :- A slow and steady rotation of the disc on no load is called creeping. This error is avoided by embedding a thin iron wire in the disc. When the iron wire passes through the permanent magnet and it is attracted by the magnet and the disc is ~~for~~ stopped rotation.



Power factor error :- The phase ϕ_{sh} does not lag the voltage exactly by 90° . It will be less than 90° . Under this circumstances, the reading shown by the energy meter will be different for leading and lagging power factors even for the same power factor. This error is called error due to power factor. This error is compensated by moving the shading ring on the centre limb of the shunt magnet.



$$T_1 = K_1 i_1 \phi_{sh} \cos \phi, T_2 = K_2 i_2 \phi_{se} \cos(180^\circ - \phi)$$

$$T_d = T_1 - T_2 = K_1 V I \cos \phi + K_2 V I \cos \phi$$

$$\frac{i_1 \alpha \phi_{se} \alpha I_{se} \alpha I_L}{i_2 \alpha \phi_{sh} \alpha I_{sh} \alpha V}$$

Torques are
acted upon
disc in opposite
direction.
(Resistant
torque).

 $\propto V I \cos \phi \propto \text{Power}$

Problem :-

W-91 A 230 Volts single phase domestic energy meter has a constant load of 4 amps current passing through it for 6 hours at unity power factor. If the meter disc makes 2208 revolutions during this period, what is the meter constant in revolutions/kwh. Calculate the power factor of the load if no. of revolutions made by the meter are 1472 when operating at 230V and 5 amps for 4 hours.

Soln :- 1st case 230V, 4A, U.P.F, 6 hours, 2208 revolutions

$$\text{Actual energy} = \frac{VI \cos \phi \times t}{1000}, \text{kwh}$$

$$= \frac{230 \times 4 \times 1 \times 6}{1000} = 5.52 \text{ kwh}$$

2nd case

230V, 5A, 4 hrs, 1472 revolutions,

$$\text{Energy} = \frac{VI \cos \phi \times t}{1000}, \text{kwh}$$

$$= \frac{230 \times 5 \times \cos \phi \times 4}{1000} = 4.6 \cos \phi \text{ kwh}$$

In first case, 2208 revolutions shows 5.52 kwh

$$\therefore 1472 \rightarrow \frac{5.52}{2208} \times 1472 = 3.68 \text{ kwh}$$

$$\therefore 4.6 \cos \phi = 3.68$$

$$\text{or } \cos \phi = \frac{3.68}{4.6} = 0.8$$

or, Power factor. $\cos \phi = 0.8$ (Ans).

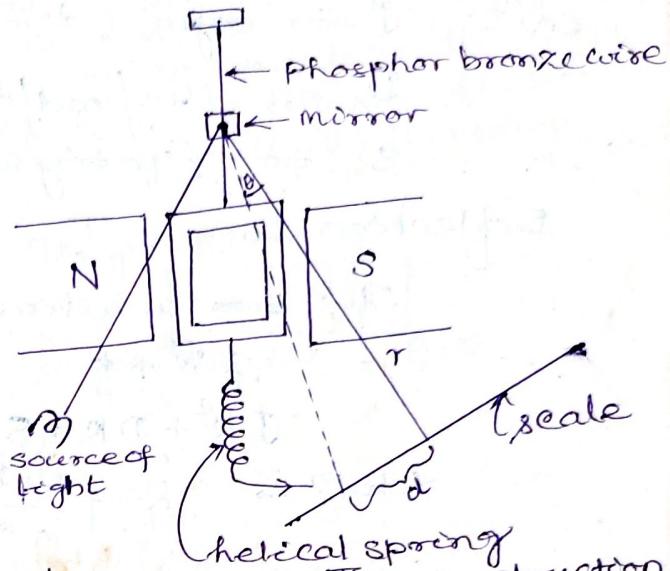
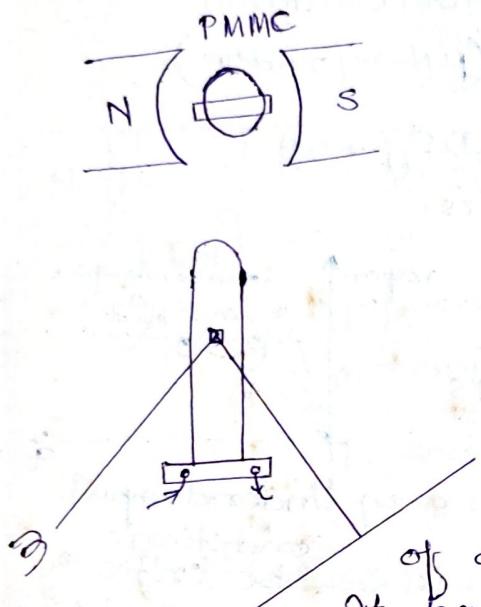
{ This type of problem also is coming ~~for other~~ base on this meter. Type is :- Voltage V, Current I, power factor, time hrs, constant and revolutions are given and error is to be found in the meter.

GALVANOMETER

There are three types of Galvanometers. They are

- (1) D'ARSONVAL GALVANOMETER ✓
- (2) BALLISTIC GALVANOMETER ✓
- (3) VIBRATION GALVANOMETER ✓

D.C. GALVANOMETER



Construction :- The construction

of d.c galvanometer is similar to PMMC. It has a pair of permanent magnet and a soft iron cylinder between magnets. A coil is suspended in the angular space of the magnet by means of a phosphor bronze wire. One end of the coil is connected to the phosphor bronze wire and the other end is connected to a loosely bound helical spring. A small mirror is attached to the phosphor bronze wire and a ray of light is focused on the mirror. The reflected ray falls on a scale which kept about 2 meters away from the mirror.

As in PMMC, the torque produced on the coil is, $T_d = BANi$ and control torque, $T_c = s\theta$ where $s = \frac{\text{control constant}}{\text{spring constant}}$

On steady state

$$s\theta = BANi = Gi \quad \text{where} \quad G = PAIN = \text{constant}$$

$$\text{or, } \theta = \left(\frac{G}{s}\right)i$$

NOW, in d.c. galvanometer, when mirror turns by an angle θ , the reflected ray makes 2θ angle with previous reflected ray.

In figure, if the distance between r is the length of incident reflected light, d is the difference shown in mirror due to deflection of mirror,

$$\therefore 2\theta = \frac{d}{r} \quad \text{or, } d = 2\theta \cdot r \quad (-\theta \text{ is radian})$$

$$\text{or, } D = 2 \cdot \left(\frac{G}{S}\right) i r$$

$$\text{or } D \propto i$$

so, D.C galvanometer measures very small current i.e. mA.

Dynamic Behaviour of Galvanometer :-

Let J in kgm^2 (inertia constant) or (mass moment of inertia)

D in Nm/rad/sec (Friction constant)

S in (Spring constant) ($\mu\text{N-m/radian}$)

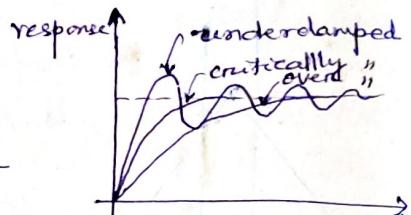
$$\text{Deflection torque, } T_d = J \frac{d^2\theta}{dt^2} + D \frac{d\theta}{dt} + S\theta$$

The ~~one~~ auxiliary equation is

~~If $D^2 < 4JS$~~

$$JK^2 + DK + S = 0$$

$$\text{Solve it is } K = \frac{-D \pm \sqrt{D^2 - 4JS}}{2J}$$



NOW, (i) If $D^2 < 4JS$, the system is in Underdamped condition.

(ii) If $D^2 = 4JS$, " " is said be critically damped condition.

(iii) If $D^2 > 4JS$, " " is said overdamped condition.

(iv) If $D = 0$, then the system is said undamped condition.



Underdamped Condition,

$$T_d = J \frac{d^2\theta}{dt^2} + S\theta \quad (\because D=0)$$

$$\text{or, } \frac{d^2\theta}{dt^2} + \frac{S}{J}\theta = \frac{1}{J}T_d$$

For sinusoidal current

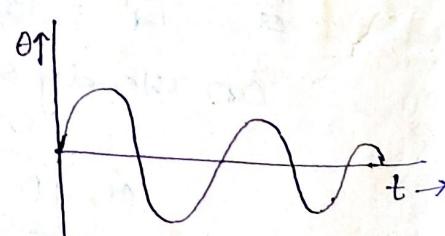
$$\theta = \theta_m \sin \sqrt{\frac{S}{J}} t$$

where Natural frequency of oscillation,

$$\omega_n = \sqrt{\frac{S}{J}}$$

But $\omega_n = \frac{2\pi}{T}$ (where T = time period)

$$\therefore T = 2\pi \sqrt{\frac{J}{S}} \quad \text{--- (1)}$$

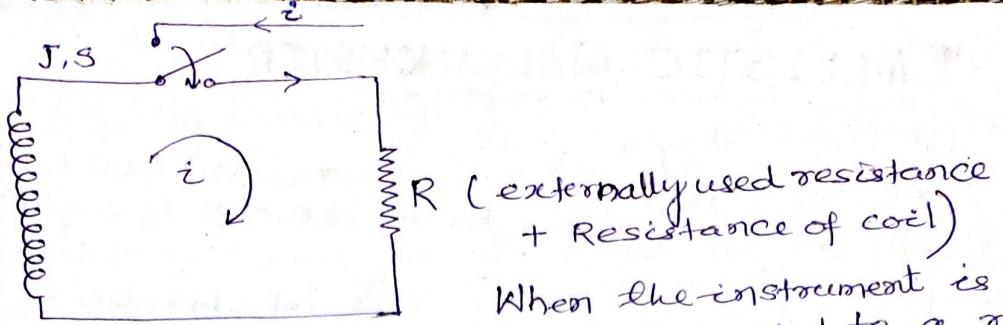


Under critically damped condition,

$$D^2 = 4JS$$

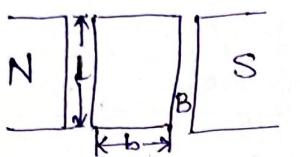
$$\text{or, } D = 2\sqrt{JS}$$

$$\text{Damping co-efficient} = ?$$



R (externally used resistance
+ Resistance of coil)

When the instrument is switched off, it is connected to a resistance. As the coil moves, emf is produced in the coil. A current circulates in the coil and a torque is produced in the coil. The value of R is so adjusted that the moving system has critically damping.



Tangential velocity

$$v = \frac{b}{2} \frac{d\theta}{dt}$$

Emf induced in one conductor = $B l v$

$$\text{or, emf} = Bl \cdot \frac{b}{2} \frac{d\theta}{dt}$$

Emf induced in one turn

$$= 2 Bl \cdot \frac{b}{2} \frac{d\theta}{dt} = BA \frac{d\theta}{dt}$$

Total emf for N turns,

$$e = BN \frac{d\theta}{dt} = Q \frac{d\theta}{dt}$$

$$\text{Now } i = \frac{e}{R} = \frac{Q}{R} \frac{d\theta}{dt}$$

$$\text{Torque, } T_h = BQANi = Qic \text{ or, } T_d = \frac{Q^2}{R} \cdot \frac{d\theta}{dt}$$

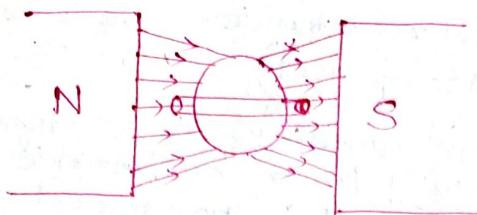
$$\text{But damping torque, } T_d = D \frac{d\theta}{dt}$$

$$\therefore D \text{ (damping coefficient)} = \frac{Q^2}{R}$$

$$\text{or } D = 2\sqrt{JS} = \frac{Q^2}{R}$$

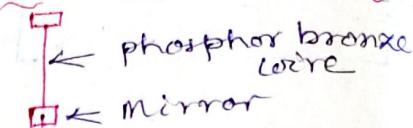
$$\text{or } R = \frac{Q^2}{2\sqrt{JS}} \quad \text{--- (3)}$$

BALLISTIC GALVANOMETER



Powerful magnetic
radial flux
(radial flux)

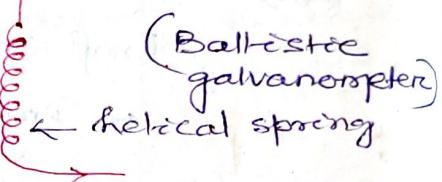
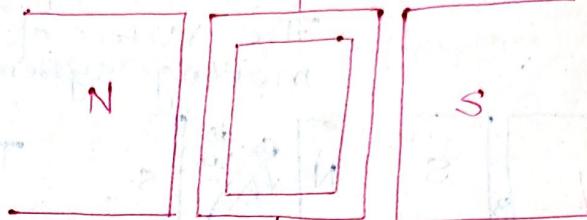
In case of D'Arsonval galvanometer
the weight of the coil is light.



In case of Ballistic
galvanometer,

- J (inertia constant)
is very high
- S (spring constant)
is very small
- D = 0 (damping co-efficient)

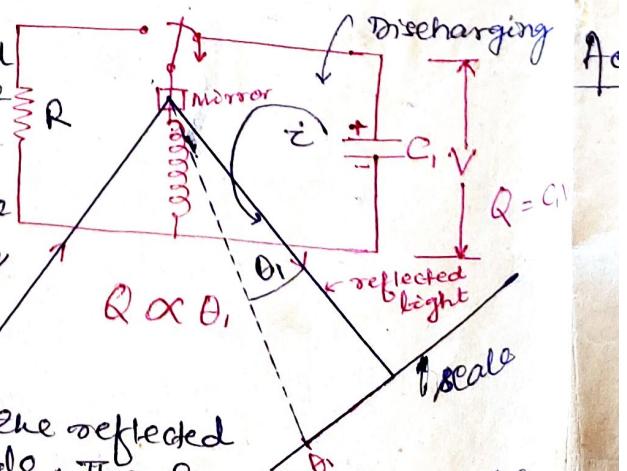
This galvanometer is used for
electricity (charge).



CONSTRUCTION :- The construction of Ballistic galvanometer is similar to D'Arsonval galvanometer. It has pair of permanent magnet and soft iron cylinder between the magnets to reduce the air gap. The flux density in the air gap is uniform and radial. In the air gap a coil is suspended from a phosphor bronze wire. One end of the coil is connected to a very loosely bound helical spring. The weight of the moving system is made heavy and the damping is made very less. The spring constant is also very less. This instrument is used to measure the quantity of electricity.

Operation: Discharging of a capacitor by using Ballistic galvanometer:-

To measure the charge, the Ballistic galvanometer is connected to the capacitance whose charge is to measure. The capacitance discharges the current in the Ballistic galvanometer and a torque is produced. Since the moving system is heavy, during the process of discharge, the coil does not move. After discharge, the coil starts oscillating and the reflected ray of light moves on the scale. The first throw of the reflected light θ_1 is noted. From θ_1 , charge can be calculated. After the measurement of, Ballistic galvanometer is connected to 'R' and the oscillations are stopped.



From the measurement of, Ballistic galvanometer is connected to 'R' and the oscillations are stopped.

Critically damped.

Deering Discharging :-

At the time of discharging, a current flows through the coil. Due to this current, a deflecting torque is produced on the moving system which is stored in the inertia.

Due to heavy weight of coil, the coil does not move i.e. $\theta = 0$.

$$\therefore T_d = Q\dot{c} = J \frac{d^2\theta}{dt^2}$$

or inertial torque,

$$Q\dot{c} = J \frac{d^2\theta}{dt^2}$$

$$\left(\frac{Dd\theta}{dt} = 0 \text{ & } S\theta = 0 \text{ as } \theta = 0 \right)$$

Damping torque Strong spring torque

Integrating both sides with respect to time with limits 0 to τ (i.e. 0 to τ) (where $\tau \rightarrow 0$)

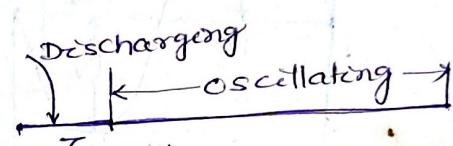
$$\therefore \int_0^\tau Q\dot{c} dt = \int_0^\tau J \frac{d^2\theta}{dt^2} dt$$

$$\text{or, } J \frac{d\theta}{dt} \Big|_0^\tau = Q \int_0^\tau \dot{c} dt \quad (Q = \text{charge})$$

$$\text{or, } J \left[\frac{d\theta}{dt} \Big|_\tau - \frac{d\theta}{dt} \Big|_0 \right] = GQ$$

$$\text{or, } J \frac{d\theta}{dt} \Big|_\tau = GQ$$

$$\text{or, } \boxed{\frac{d\theta}{dt} \Big|_\tau = \frac{GQ}{J}} - ①$$



e.g. Velocity of the coil at the end of the discharging period is $\boxed{\frac{d\theta}{dt} \Big|_\tau = \frac{GQ}{J}}$ where τ is the period of discharge which is very small.

After discharging, current through coil is zero and hence torque is zero. But due to conservation of inertial energy, some oscillations can be formed.

Actually The ballistic galvanometer is rendered over-damped condition.

Now, $J \frac{d^2\theta}{dt^2} + D \frac{d\theta}{dt} + S\theta = 0$ is auxiliary equation.

The solution of this equation is

$$\theta = F e^{-\frac{Dt}{2J}} \sin(\omega dt + \phi)$$

where F & ϕ are unknowns.

Initial values (\dot{c}) at $t=0$, $\theta=0$
in the expression of θ (ii) at $t=0$, $\frac{d\theta}{dt} = \frac{GQ}{J}$

$$(iii) \theta = F \cdot e^0 \cdot \sin(\omega \cdot 0 + \phi) = F \sin \phi$$

By taking $\phi = 0$, $\theta = F e^{\frac{-Dt}{2J}} \sin \omega_d t$

By differentiating θ w.r.t t ,

$$\frac{d\theta}{dt} = F \left[e^{\frac{-Dt}{2J}} \left(-\frac{D}{2J} \right) \sin \omega_d t + e^{\frac{-Dt}{2J}} \omega_d \cos \omega_d t \right]$$

$$\text{At } t=0, \frac{d\theta}{dt} = F \times \omega_d$$

$$\text{or, } \frac{GQ}{J} = F \omega_d$$

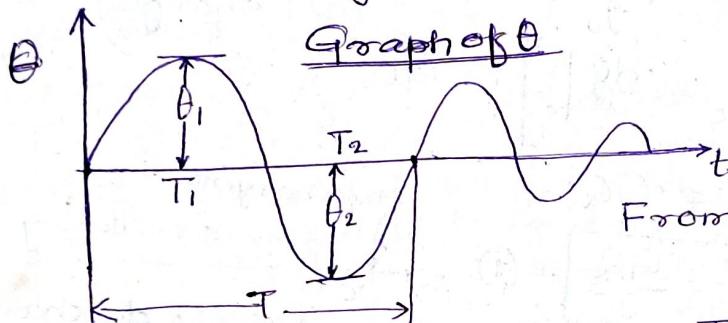
$$\text{or, } F = \frac{GQ}{J \omega_d}$$

$$\text{Now, } \theta = \frac{GQ}{J \omega_d} e^{\frac{-Dt}{2J}} \sin \omega_d t$$

$$\text{Assuming } \omega_d \approx \omega_n = \sqrt{\frac{S}{J}}$$

$$\therefore \theta = \frac{GQ}{J \omega_n} e^{\frac{-Dt}{2J}} \sin \omega_n t$$

$$\text{or, } \theta = \frac{GQ}{J} \times \sqrt{\frac{J}{S}} \cdot e^{\frac{-Dt}{2J}} \sin \sqrt{\frac{S}{J}} t \quad \dots \dots \quad (I)$$



Graph of θ

$$\omega_n = 2\pi f = \frac{2\pi}{T}$$

$$\text{or, } \frac{2\pi}{T} = \sqrt{\frac{S}{J}}$$

$$\text{or, } T = 2\pi \sqrt{\frac{J}{S}}$$

From Graph,

$$T_1 = \frac{T}{4} = \frac{\pi}{2} \sqrt{J/S}$$

$$T_2 = \frac{3T}{4} = \frac{3\pi}{2} \sqrt{J/S}$$

From equatn (I)

$$\theta_1 = \frac{GQ}{J} \sqrt{\frac{J}{S}} \cdot e^{\frac{-D}{2J} \cdot \frac{\pi}{2} \sqrt{J/S}} \cdot \sin \sqrt{\frac{S}{J}} \cdot \frac{\pi}{2} \sqrt{J/S}$$

$$\text{or, } \theta_1 = \frac{GQ}{J} \sqrt{\frac{J}{S}} \cdot e^{-\frac{\pi}{4} \cdot \frac{D}{\sqrt{JS}}} \times 1$$

$$\text{or, } Q = \left(\frac{J}{G} \sqrt{\frac{S}{J}} \cdot e^{\frac{\pi}{4} \cdot \frac{D}{\sqrt{JS}}} \right) \theta_1$$

$$\text{or, } Q \propto \theta_1 \quad \text{constant}$$

Logarithmic decrement :-

$$\theta_1 = \frac{GQ}{J} \sqrt{\frac{J}{S}} \cdot e^{-\frac{\pi}{4} \cdot \frac{D}{\sqrt{JS}}}$$

$$\theta_2 = \frac{GQ}{J} \sqrt{\frac{J}{S}} e^{-\frac{3\pi}{4} \cdot \frac{D}{\sqrt{JS}}}$$

$$\frac{\theta_1}{\theta_2} = \frac{e^{-\frac{\pi}{4} \cdot \frac{D}{\sqrt{JS}}}}{e^{-\frac{3\pi}{4} \cdot \frac{D}{\sqrt{JS}}}} = e^{\frac{\pi}{2} \cdot \frac{D}{\sqrt{JS}}}$$

Taking loge both sides \therefore

$$\log \frac{\theta_1}{\theta_2} = \frac{\pi}{2} \cdot \frac{D}{\sqrt{JS}} = \lambda \text{ (say)}$$

Substituting λ in the expression of θ_1 ,

$$\therefore \theta_1 = \frac{GQ}{J} \sqrt{\frac{J}{S}} \cdot e^{-\frac{\lambda t}{2}}$$

$$\text{or, } \theta_1 = \frac{GQ}{J} \sqrt{\frac{J}{S}} \cdot \frac{1}{e^{\frac{\lambda t}{2}}}$$

$$\text{or, } Q = \frac{J}{G} \sqrt{\frac{S}{J}} e^{\frac{\lambda t}{2}} \cdot \theta_1$$

$$= \frac{J}{G} \sqrt{\frac{S}{J}} \left(1 + \frac{\lambda}{2}\right) \theta_1 \quad \begin{aligned} &(\text{Expansion of } e^{\frac{\lambda t}{2}} \\ &= 1 + \frac{\lambda}{2} + \dots) \end{aligned}$$

$$\text{or, } Q = \frac{S}{G} \sqrt{\frac{J}{S}} \left(1 + \frac{\lambda}{2}\right) \theta_1$$

$$\text{or, } Q = \frac{S}{G} \cdot \frac{T}{2\pi} \left(1 + \frac{\lambda}{2}\right) \theta_1 \quad \left(\because \omega_n = \frac{2\pi}{T} = \sqrt{\frac{S}{J}}\right)$$

By using a battery in place of capacitor, the current through the coil is constant and deflection of reflected ray is also constant.

i. On steady state,

Deflecting torque = control torque

(c) or, $G I_{dc} = S \theta_{dc}$ where $G = BAN$.

$$\text{or, } \frac{S}{G} = \frac{I_{dc}}{\theta_{dc}}$$

$$\therefore Q = \frac{I_{dc}}{\theta_{dc}} \left(\frac{I}{2\pi} \right) \left(1 + \frac{\lambda}{2} \right) \theta_1$$

Problems

① The coil of moving coil galvanometer has 300 turns and suspended in a uniform magnetic field of 0.1 wb/m^2 by a spiral of which the torsion constant is $0.2 \mu\text{N-m/radian}$. The coil is $2 \times 2.5 \text{ cm}$ with a moment of inertia 1.5 gm-cm^2 and the galvanometer resistance is 200Ω . Calculate the value of resistance across the terminals to give critical damping. Assume damping to be entirely electromagnetic.

Soln :- $N = 300, B = 0.1 \text{ wb/m}^2, S = 0.2 \mu\text{N-m/radian}$

$$A = 2 \times 2.5 \times 10^{-4} \text{ m}^2, J = 1.5 \times 10^{-7} \text{ kgm}^2, R_g = 200 \Omega$$

$$\text{We know } R = \frac{Q^2}{2\sqrt{JS}} = \frac{(BAN)^2}{2\sqrt{JS}} = \frac{(0.1 \times 2 \times 2.5 \times 10^{-4} \times 300)^2}{2\sqrt{1.5 \times 10^{-7} \times 0.2 \times 10^6}}$$

$$= 649.51 \Omega \quad (\text{Resistance for critical damping})$$

$$R = R_g (\text{galvanometer}) + R_e (\text{external})$$

$$\text{or, } R_e = R - R_g = 649.51 - 200 = 449.51 \Omega \text{ (Ans.)}$$

② In an undamped galvanometer with a period of 10 secs. A current of 0.1 mA produced a steady deflection of 150 divisions. Find the first swing of 100 divisions. (a) When the instrument is

undamped.

(b) When the instrument is damped so that the decrement is 1.1 (θ_1/θ_2)

Solution :- $T = 10 \text{ sec}$, $I_{dc} = 0.1 \text{ mA}$, $\theta_{dc} = 150 \text{ dev.}$

$$\theta_1 = 100 \text{ dev.}, Q = ?$$

(a) When the system is undamped, $\theta_1 = \theta_2$

$$\text{or, } \frac{\theta_1}{\theta_2} = 1$$

$$\therefore \log_e \frac{\theta_1}{\theta_2} = 0$$

$$\text{or, } \lambda = 0.$$

$$Q = \frac{T}{2\pi} \cdot \frac{I_{dc}}{\theta_{dc}} \left(1 + \frac{\lambda}{2}\right) \theta_1$$

$$= \frac{10}{2\pi} \times \frac{0.1 \times 10^{-3}}{150} \times \left(1 + \frac{0}{2}\right) \times 100$$

$$= \frac{10}{2\pi} \times \frac{0.1 \times 10^{-3}}{150} \times 100 = 106 \mu \text{coulombs (Ans)}$$

(b) When $\frac{\theta_1}{\theta_2} = 1.1$, $\log_e \frac{\theta_1}{\theta_2} = \log_e 1.1 = 0.0953$

$$Q = \frac{T}{2\pi} \cdot \frac{I_{dc}}{\theta_{dc}} \left(1 + \frac{\lambda}{2}\right) \theta_1$$

$$\text{or, } Q = \frac{10}{2\pi} \times \frac{0.1 \times 10^{-3}}{150} \times \left(1 + \frac{0.0953}{2}\right) \times 100$$

$$= 101 \mu \text{coulombs (Ans)}$$

③ A moving coil ballistic galvanometer gives a first swing of 60° for a discharge of $1000 \mu \text{coulombs}$. Find the quantity of electricity to produce

(a) a swing of 90° in the instrument

(b) a spot deflection of 10 mm on a scale 1 m away

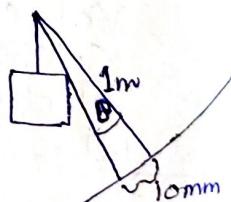
Soln :-

$$(a) Q_1 = 1000 \mu \text{C}, \theta_1 = 60^\circ$$

$$Q = ? \quad , \quad \theta = 90^\circ.$$

$$\frac{Q}{Q_1} = \frac{\theta_1}{\theta_2} \quad \text{or} \quad Q = \frac{1000}{60} \times 90$$

~~But Q_1 is constant~~



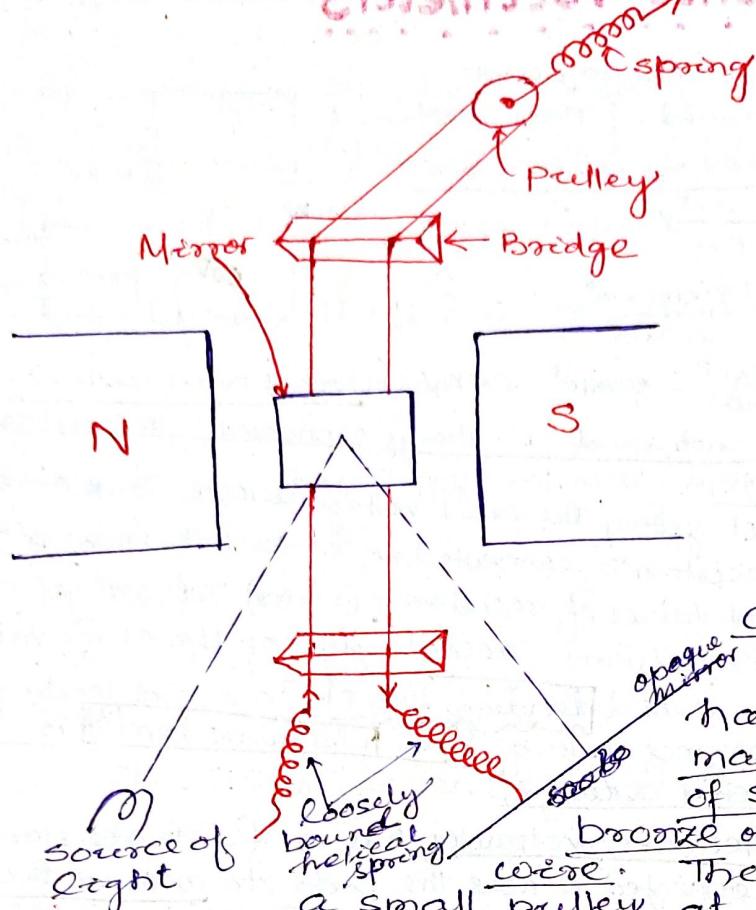
$$= \frac{90}{60} \times 1000 \times 10^6 = 1500 \mu \text{C}$$

$$(b) \theta = \frac{10}{1000} = \frac{10 \text{ mm}}{1000 \text{ mm}} = \frac{10 \text{ mm}}{1000 \text{ mm}} = \frac{10^{-2}}{10^{-3}} = 10^2 \text{ radian}$$

$$10^2 \text{ radian} = \frac{10^2 \times 180}{\pi} = 0.573^\circ$$

$$Q = \frac{0.573^\circ}{60} \times 1000 \times 10^6 = 9.55 \times 10^3 \mu \text{coulombs.}$$

VIBRATION GALVANOMETER



TUNE

DUDDELL VIBRATION GALVANOMETER OR SPOT VIBRATION GALVANOMETER

CONSTRUCTION :-

This instrument has a pair of permanent magnet. It consists of single loop of fine bronze or platinum silver wire. The wire passes over a small pulley at the top and is being pulled right by a spring attached to the pulley. The tension of this spring is adjusting during tuning. The loop of wire is stretched over to every bridge pieces. The distance of bridge pieces are adjustable. This galvanometer is used as a detector in a.c bridges.

OPERATION :- When alternating current flows through the loop of wire, the mirror vibrates and reflected ray of light forms a band of light on the opaque mirror. As the current reduces, the length of the band of light also decreases. When no current flows, the band of light reduces to a spot of light.

Before the vibration galvanometer is used, it is to be tuned for the frequency of the source of the a.c bridge.

The tuning is done as follows:-

The vibration galvanometer is connected to the a.c source. The tension on the spring and the distance between the bridge pieces are adjusted until the vibration is maximum. Under this condition, the bridge is said to be tuned.

— * —

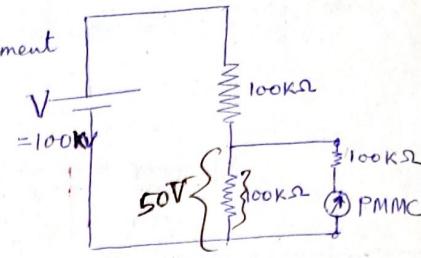
Electronic Voltmeters

In side figure, the voltage across $100\text{ k}\Omega$ which is measured by PMMC instrument
 $= 50\text{ V}$ (if the series resistance of $100\text{ k}\Omega$
 is absent)

Again, Voltage across $100\text{ k}\Omega$

$$= \frac{100}{200\text{ k}} \times \frac{100 \times 100 \times 10^6}{200 \times 10^3}$$

$$= \frac{100 \times 100 \times 10^6}{400 \times 10^3} = 25 \times 10^3 = 25\text{ KV}$$



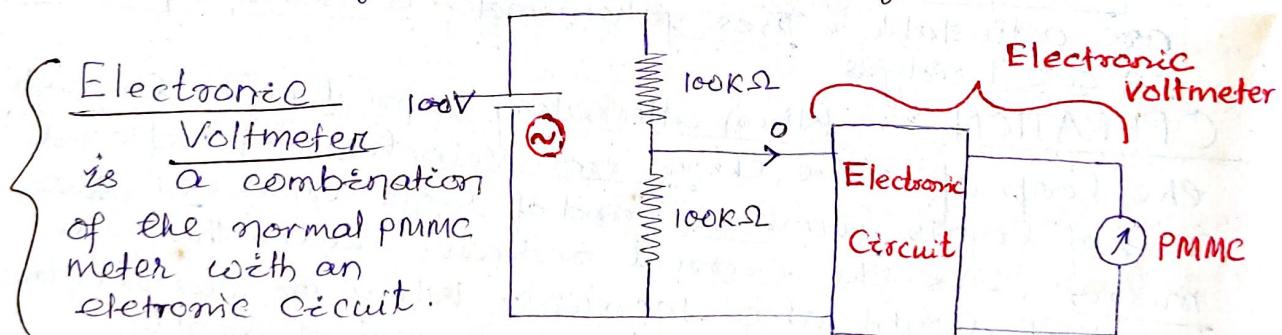
But, the PMMC instrument is always connected with a series resistance to measure voltage.

Here, we are not getting the exact voltage across $100\text{ k}\Omega$ resistance due to series resistance connected in series with PMMC instrument. By putting different values of resistance (series), we will get decrease value across $100\text{ k}\Omega$ resistance accordingly, not the exact value.

That's why it is called loading effect. To avoid loading effect and measure correct voltage, an electronic circuit is used with PMMC inst. connected across it.

Actually, the electronic voltmeter has a finite resistance.

When it is connected across the posts to measure the voltage, the voltmeter draws current from the measuring circuit and measures the voltage less than true value. This effect of the voltmeter is called as loading effect. Loading effect can be minimised by having a voltmeter of high resistance.



The electronic circuit acts as amplifier or rectifier.

The electronic voltmeter has the following advantages.

- (i) It has a very high input impedance and hence loading effect is minimised.
- (ii) The power consumption from the measuring circuit is very small.
- (iii) It can be used to measure very small voltages.
- (iv) It can be used for wide ranges of frequency.
- (v) It can be used to measure average value, rms value and peak value.

Classification of Electronic Voltmeters :-

It is classified into two categories.

- (1) Vacuum type voltmeter (**VTVM**)
- (2) Solid state type voltmeter

Vacuum type voltmeter is classified into two categories.

- (1) Diode VTVM (2) Triode VTVM
(DVTVM)

Solid state type voltmeter is also classified into three types.

- (1) Diode
- (2) Transistor
- (3) FET

DVTVM is classified into two.

- (1) Average reading DVTVM
- (2) Peak reading DVTVM

Peak reading DVTVM is again classified into four categories.

- (1) Series type
- (2) Shunt type
- (3) Compensated shunt type
- (4) Slide back type

Triode VTVM is classified into two categories

- (1) Rms reading Triode VTVM
- (2) Balanced triode VTVM

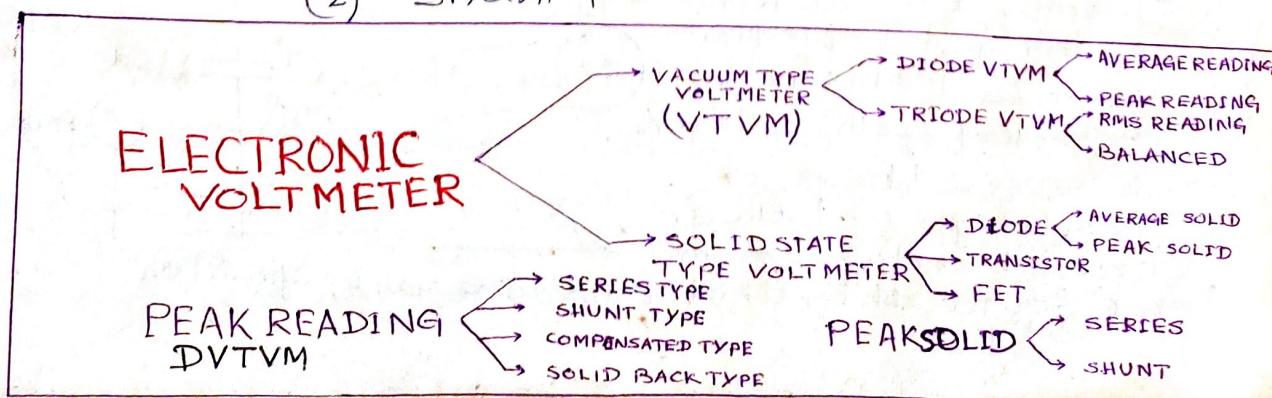
Solid state diode is classified into two categories.

- (1) Average solid state diode
- (2) Peak solid state diode

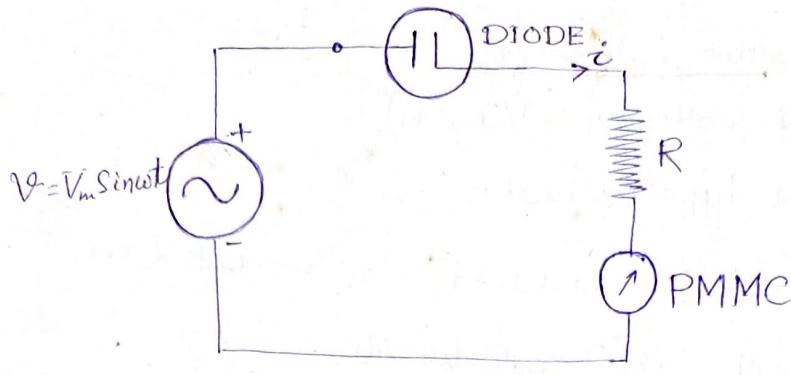
Peak solid state diode is again classified into two

categories. (1) Series peak solid state diode

- (2) Shunt peak solid state diode



① AVERAGE READING DVTVM :-



$$\dot{I} = \frac{V_m}{R} \sin \omega t, 0 \leq \omega t \leq \pi$$

$$= 0, \pi \leq \omega t \leq 2\pi$$

In PMMC, the deflection, $\theta \propto I_{av}$.

$$I_{av} = \frac{1}{2\pi} \int_0^{2\pi} \left(\frac{V_m}{R} \sin \omega t \right) dt$$

$$= \frac{V_m}{\pi R}$$

The deflection θ is proportional to the average value of rectified voltage. Here diode acts as rectifier.

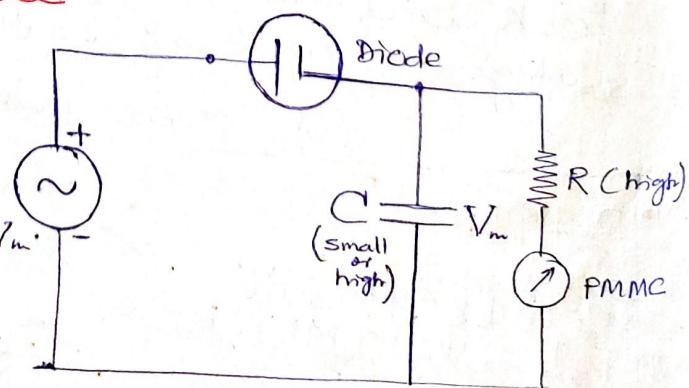
This instrument responds only the + half cycle. That's why it is also called as rectifier instrument.

- Advantages :-
- (1) A very simple circuit
 - (2) The input impedance is very high
 - (3) It measures the average value.

- Disadvantages :-
- (1) It draws d.c current from the measuring circuit and this may affect the operation of the circuit.
 - (2) In case of vacuum tube, voltage less than 10 volt can not be measured.

② PEAK READING DVTVM :-

At the time of forward biasing, the capacitor is charged to its maximum value and it acts as a battery with voltage V_m . In reversed bias, it discharges and the d.c current is drawn from an AC source to which AC source may be affected on.



Voltage across $C = V_m$

The deflection θ shown in PMMC gives peak value.

θ is proportional to I_{av}

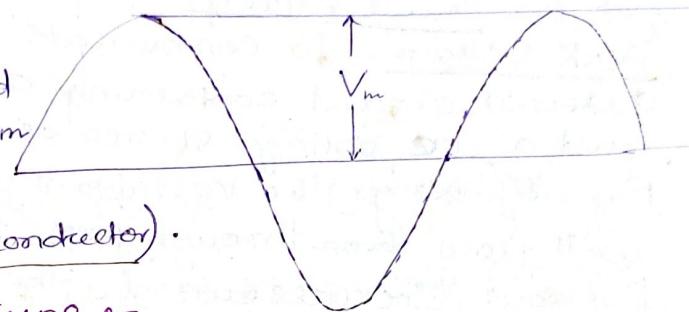
$$\text{or } \theta \propto I_{av}$$

$$\text{or, } \theta \propto I_C$$

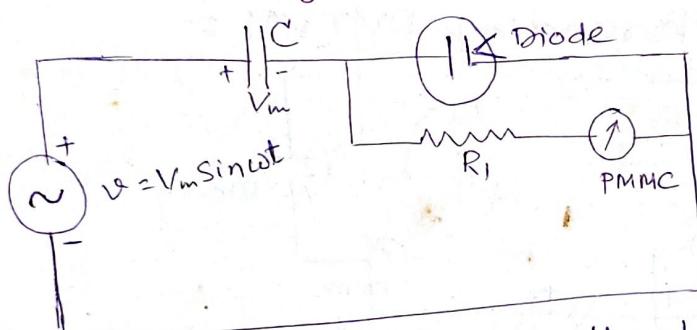
$$\text{or, } \theta \propto V_m$$

Here the value of R is very high by which current flowed through PMMC is minimum. The value of Capacitance may be low or high.

This type of instrument can be changed into Solid type by replacing vacuum diode to solid diode (PN or NP type ~~trans~~ semiconductor).



(i) Peak reading Shunt type :-



In 1st half cycle with source polarity + and -ve, the diode is short circuited and there is no current flow through PMMC because Diode acts as a battery.

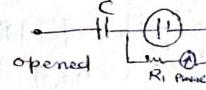
During +ve half cycle, the diode conducts and the capacitance charges to full voltage or the peak value V_m . The charging current flows through the diode. During -ve half cycle, the capacitance discharges and current in the source is in the opposite direction. The discharge current is very small and the value of C is very large. Current is very small and the value of C is very small and hence change in voltage across C is very small and it can be treated as a constant voltage source.

$$\text{Voltage across PMMC} = V_m + V_m \sin \omega t$$

$$V_{av} = \frac{1}{2\pi} \int_0^{2\pi} (V_m + V_m \sin \omega t) d\omega t$$

$$\therefore \theta (\text{deflection}) \propto V_{av} \propto V_m$$

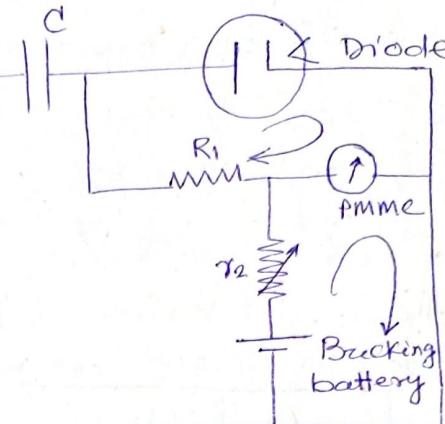
Disadvantages :- When the input terminals are opened and the filament of diode is heated. Some electrons flow through the instrument and the instrument will give a small deflection.



Advantage :- The current drawn from the ac source is ac

(ii) Peak reading Compensated Shunt type DVTVM :-

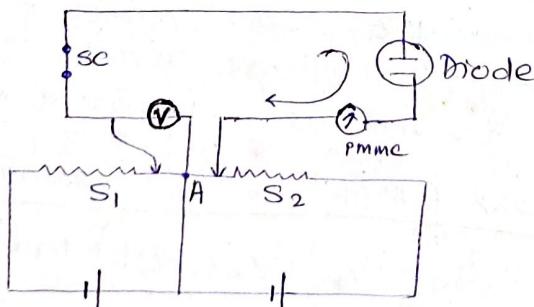
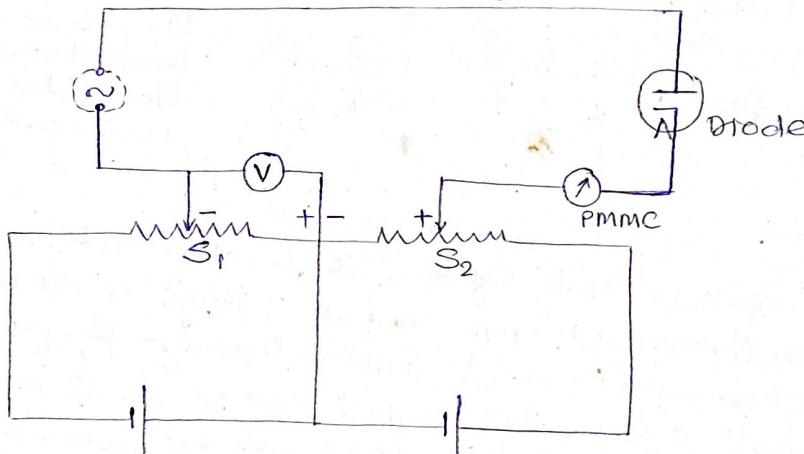
Cohen input terminals are opened and the filament of diode is heated by which some electrons will flow ~~towards~~ towards **OPENED** anode and so current flows through PMMC and PMMC gives shows deflection. Due to this current flows, we will not get the exact value of.



peak voltage. To compensate this current, there is an external circuit containing a ~~resistance~~ variable resistance and a d.c. voltage source is connected across PMMC. By adjusting the variable resistance R_2 , there is current which is equal and opposite to previous clockwise current. Therefore, there is no current flow through and there is no deflection. (Zero adjustment)

(iii) Slide back Peak reading DVTVM :-

Short Circuit



The input terminals are short circuited. The sliding point of S_1 is kept at A and the jockey point of S_2 is varied till the PMMC reads zero. This is the Zero adjustment.

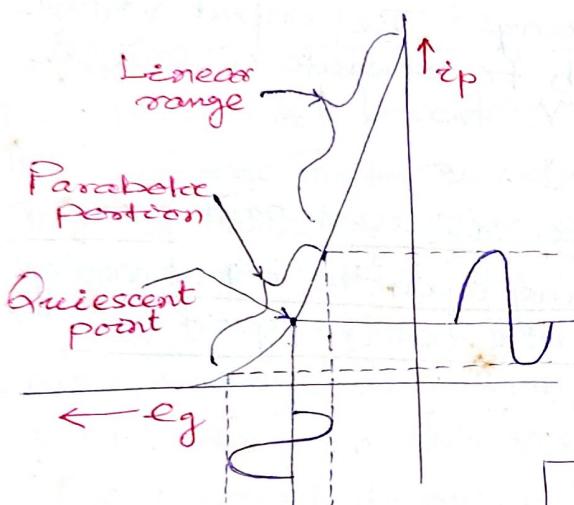
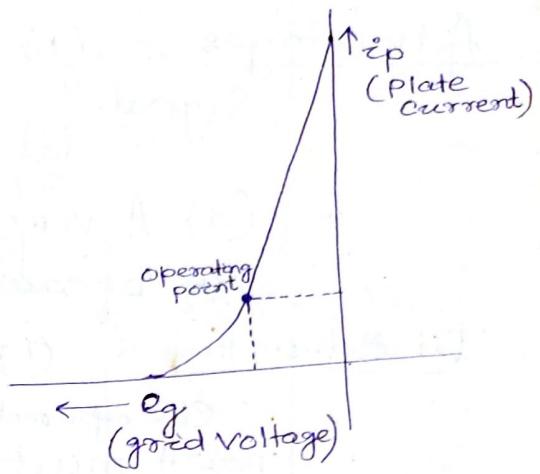
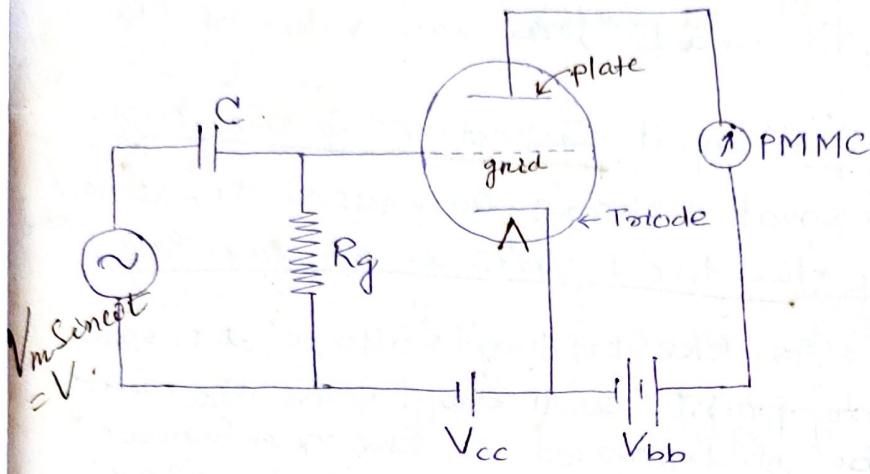
To measure the peak voltage the signal is connected across the input terminals. During the +ve half cycle diode conducts and current flows through the PMMC. During -ve half cycle, diode is reverse biased and no current flows through PMMC. The average current flows through the PMMC and PMMC gives a deflection. The jockey point S_1 is moved to the left until the PMMC just reads zero. Here the PMMC acts as a detector to convey the flow of

Current

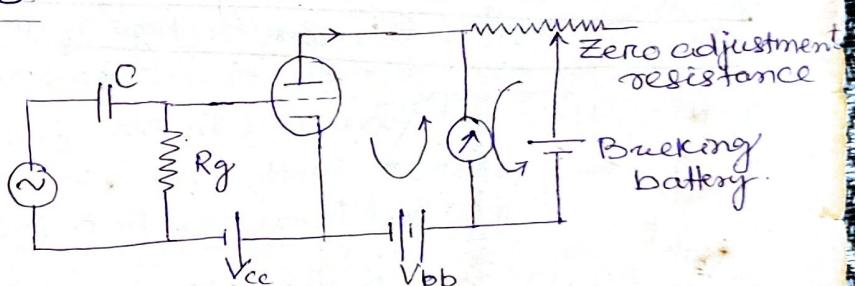
In Solid construction, there is no need of second potentiometer for Zero set & adjustment at short circuited.

 Transistor is called
Solid type instrument

TRIODE VTVM



Due to flow some current through PMMC by applying E_{cc} , another circuit containing variable resistance and breaking battery is connected across PMMC for Zero adjustment.



(grid voltage) $R_g = V_m \sin \omega t$

$$i_p = a e_g + b e_g^2 \quad (\text{at parabolic position}) \\ = a V_m \sin \omega t + b V_m^2 \sin^2 \omega t$$

$$I_{av} = \frac{1}{2\pi} \int_0^{2\pi} (a V_m \sin \omega t + b V_m^2 \sin^2 \omega t) dt \\ = \frac{1}{2\pi} \cdot b V_m^2 \int_0^{2\pi} \sin^2 \omega t dt \quad \left(\because \int_0^{2\pi} \sin \omega t dt = 0 \right)$$

$$= \frac{1}{4\pi} b V_m^2 \int_0^{2\pi} \frac{(1 - \cos 2\omega t)}{2} dt \\ = \frac{1}{4\pi} b V_m^2 \left[(2\pi - 0) - \frac{1}{2} \sin 2\omega t \Big|_0^{2\pi} \right]$$

$$= \frac{1}{4\pi} b V_m^2 \times 2\pi - 0 = \frac{b V_m^2}{2} = b \left(\frac{V_m}{\sqrt{2}} \right)^2 \\ = b V_{rms}^2$$

The deflection $\theta \propto I_{av}$

$$\text{or } \theta \propto V_{rms}^2$$

The PMMC reads exactly the rms value of the signal and it is called as True rms value instrument.

It measures all the harmonics of signal.

It is also called true rms reading Voltmeter.

Advantages :- (1) It reads true rms value of the signal.

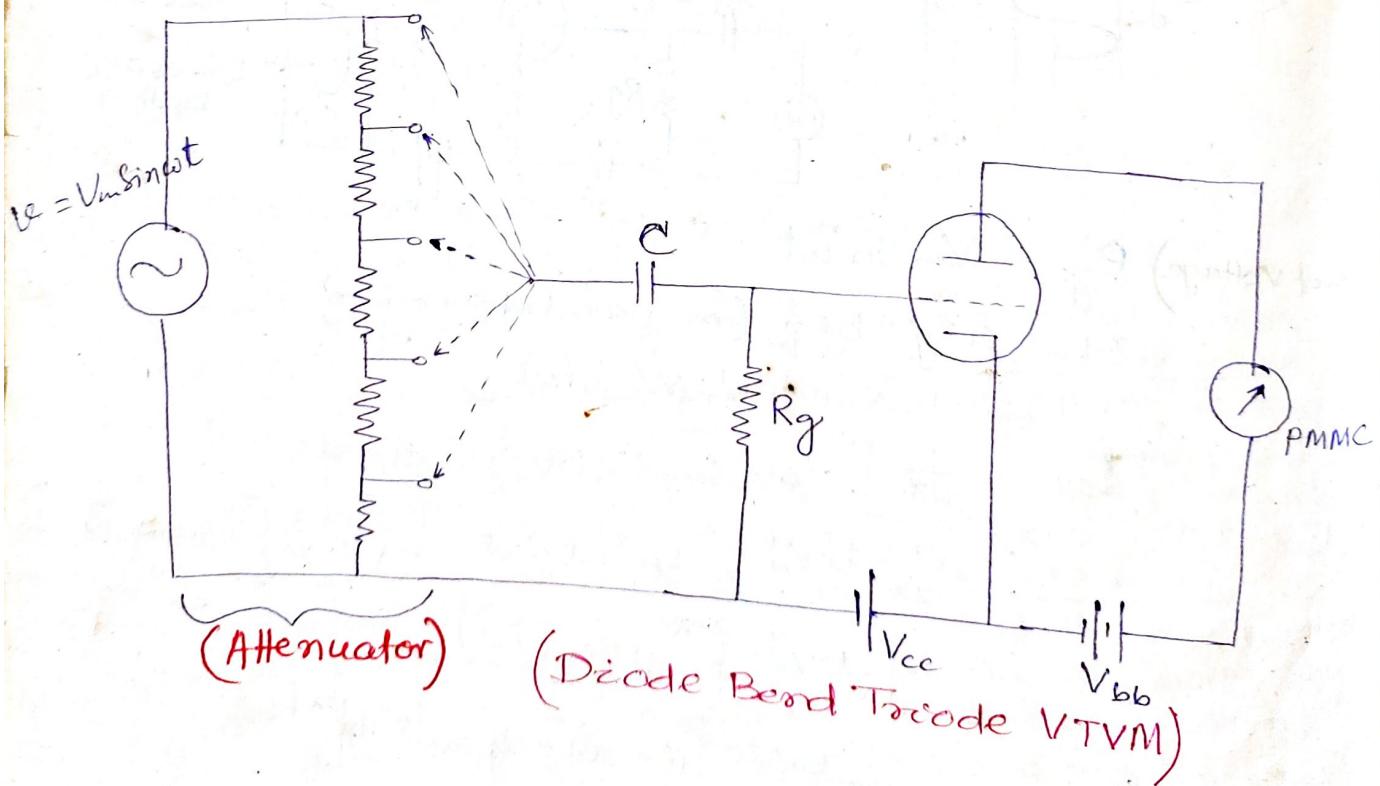
(2) The input impedance is very high.

(3) A very small voltage can also be measured.

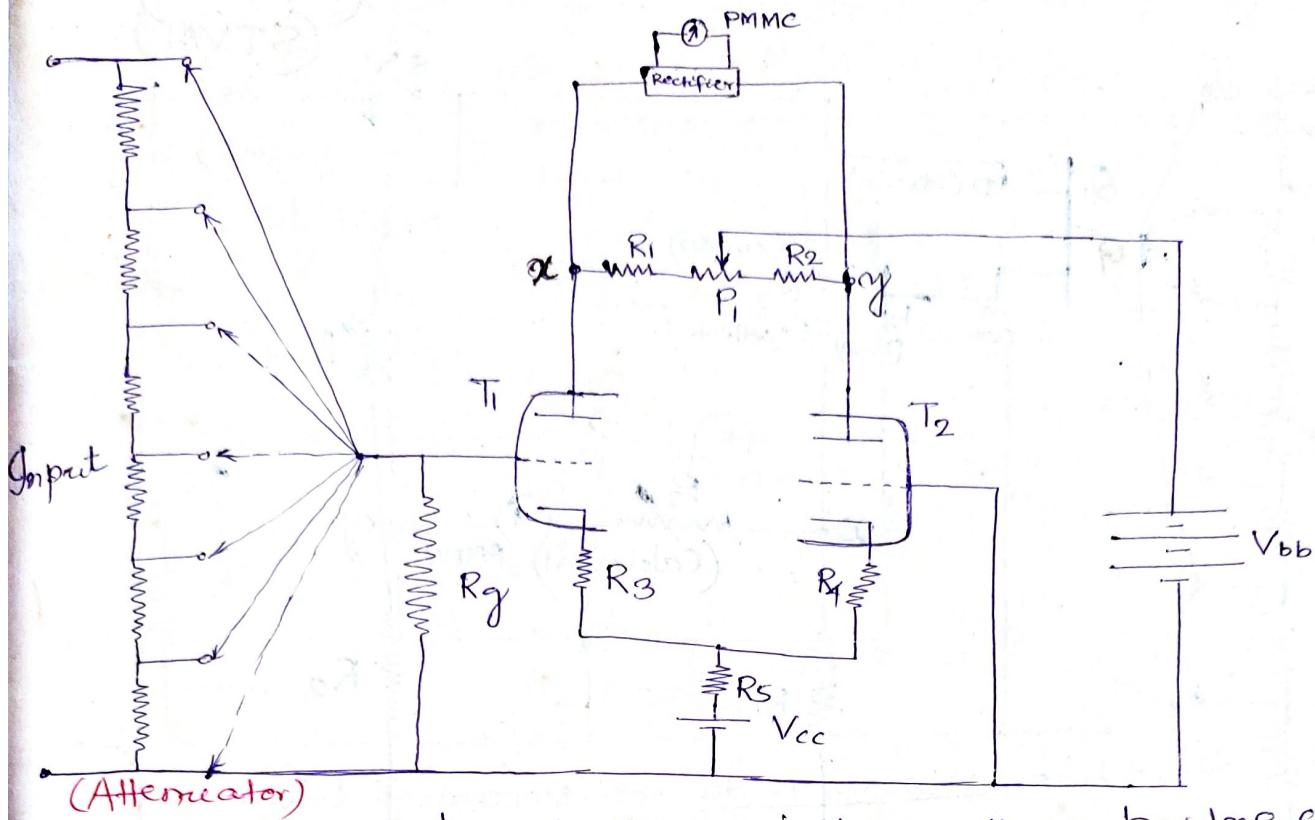
because the triode acts as an amplifier. Imp

Disadvantages (1) As the battery voltage decreases, the operating point will drift. The operating point must be maintained in the non-linear range or parabolic range. The input voltage must be small enough to operate within the non-linear range.

For large voltages, Attenuator is used and proper tappings are selected depending upon the input voltage such that the voltage applied to the grid will be in the non-linear range.



Balanced triode VTVM



(Attenuator)

The balanced triode VTVM is basically a bridge circuit. The bridge meter is adjusted ~~for~~ zero before using it. This is done by short circuiting the input terminals and varying P_i until the PMMC reads zero.

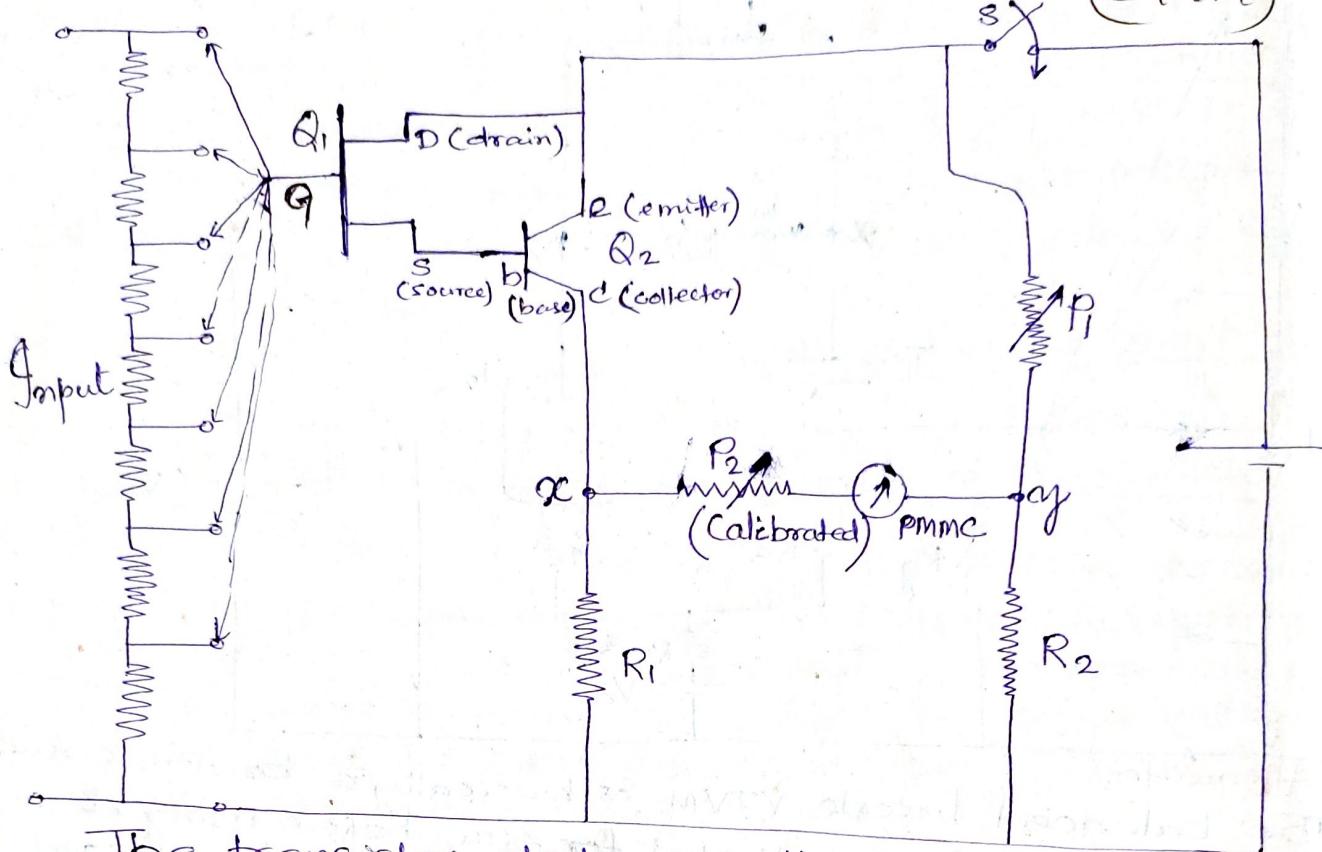
When the input voltage is +ve, P_i conducts more voltage at x , voltage decreases. The voltage across R_5 increases and this voltage acts as -ve signal to T_2 . The current through T_2 decreases and the voltage at y increases. This difference in voltage across x and y is applied to the rectifier and measured by PMMC.

When ac voltage is applied across the input terminals, this is amplified and made available across x and y . This amplified ac voltage is rectified and the average value is indicated by the PMMC.

When the plate supply or grid supply changes, the current through the tube change by same amount, voltages at x and y change by the same amount and the difference becomes zero. Here there is no problem of drift in this instrument.

Transistor Electronic Voltmeter

~~(STVM)~~
~~(SVTVM)~~
~~(STVM)~~

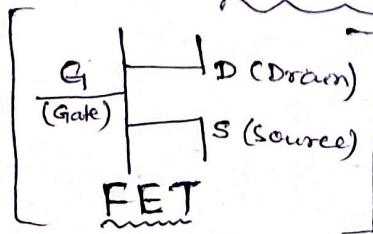


The transistor electronic voltmeter, basically is a bridge circuit consisting of P_1 , R_2 , R_1 & the transistors. The transistor Q_2 acts as a voltage controlled variable resistance. The instrument has to be adjusted for zero before using for measurement. For this the input terminals are short circuited and P_1 is varied until the PMMC reads zero.

When the input Voltage is negative, the drain to source resistance of Q_1 increases. The base collector and emitter of Q_2 increases causing a decrease in voltage at X whereas voltage at Y is constant and the meter deflects.

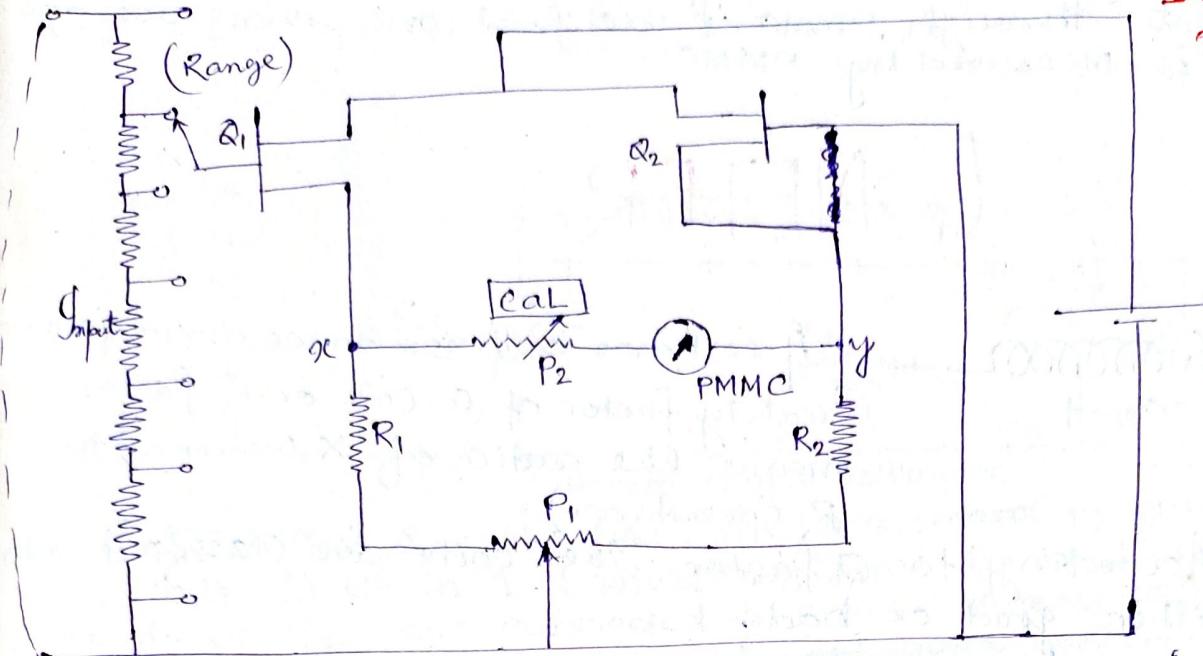
Calibrating Purpose (P_2): For calibrating the instrument apply a known voltage across the input terminals and P_2 is varied until the PMMC reads the exact value of the input voltage.

Sometimes this voltmeter is also called ~~as~~ as FET Input Transistor Voltmeter.



FET TYPE VOLTmeter

Important
for
exam.
points
view.

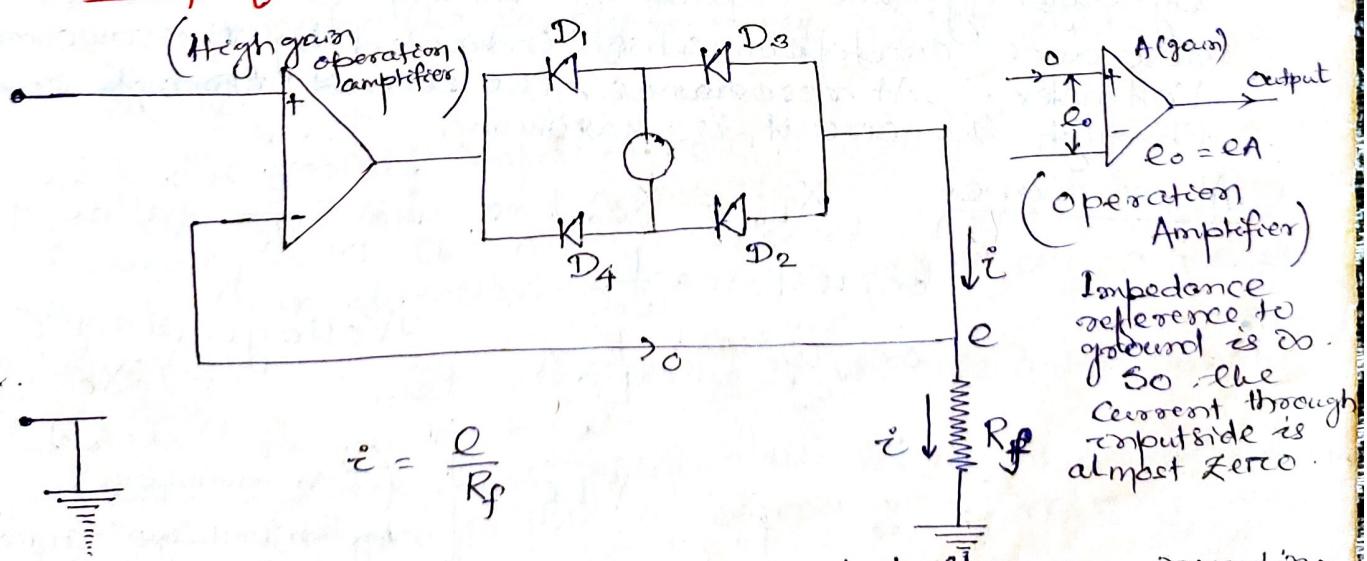


The input terminals are short circuited and P_1 is varied until the PMMC reads zero.

Q_1 and Q_2 are two matched pair of FET mounted on the same heat-sink to reduce the problem of temperature error.

When -ve voltage is applied across the input terminals the resistance of Q_1 increases, Voltage at x decreases whereas the voltage at y remains constant. This difference in voltage at x and y ~~is~~ circulates a current through PMMC and the meter deflects. The deflection θ is proportional to the magnitude of the input voltage.

Amplifier-rectifier Voltmeter :-



When the voltage applied to the non-converting terminal is e , the voltage across R_f is also equal to e . Current through R_f is $\frac{e}{R_f}$. This current is supplied by the operational amplifier which flows through

The dodees and through the PMMC. When input voltage increases, the current through PMMC also increases and deflection also increases. For an ac input, the output current of operational amplifier is also ~~is~~ ac but the current through PMMC is rectified one whose average value is measured by PMMC.

Q. METER

M o m m o m o

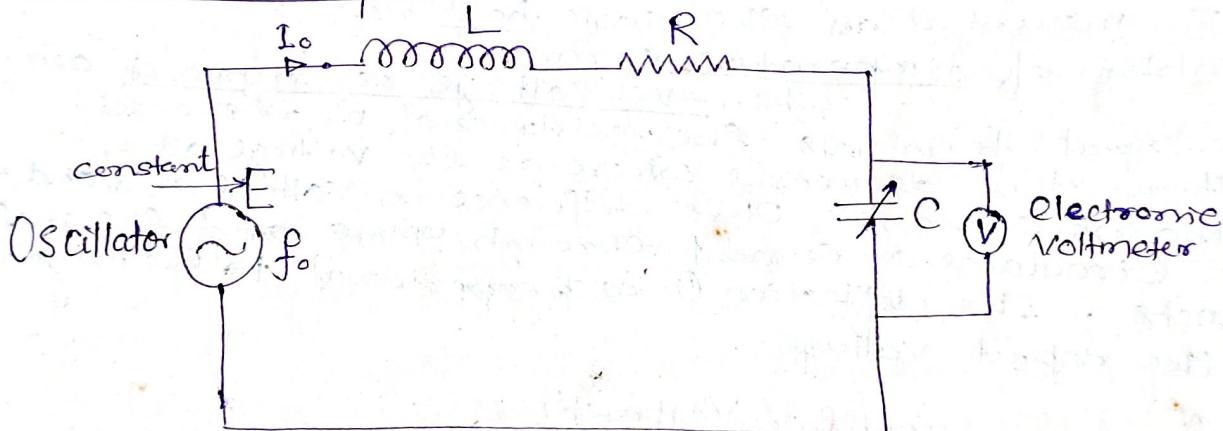
Coil

A coil has both resistance and inductance.
Quality factor of a coil or Q factor means the ratio of X (reactance) to R (resistance).

According to Q. factor, the coils are classified into either good or bad.

To measure Q. factor of a coil, Q meter is used straight way.

Basic Principle :-



By adjusting the capacitor, the ~~circuit~~ circuit comes to resonance condition which indicated by the electronic Voltmeter. At resonance, the current ~~through~~ ~~current~~ through the circuit is maximum.

$$\text{At resonance, } X_{L_0} = X_{C_0}, \therefore I_0 = \frac{E}{R} \quad (\text{at d.c input})$$

$$\text{or, } \omega_0 L = \frac{1}{\omega_0 C}$$

$$\text{or, } \omega_0^2 = \frac{1}{LC}$$

$$\text{or, } 2\pi f_0 = \frac{1}{\sqrt{LC}}$$

$$\text{or, } f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$\text{or, } V_C = \left(\frac{\omega_0 L}{R}\right) E = QE$$

$$\begin{aligned} \text{Voltage across } C. \\ &= I_0 X_{C_0} \end{aligned}$$

$$= I_0 \omega_0 L \quad (\because \text{At resonance } X_{L_0} = X_{C_0})$$

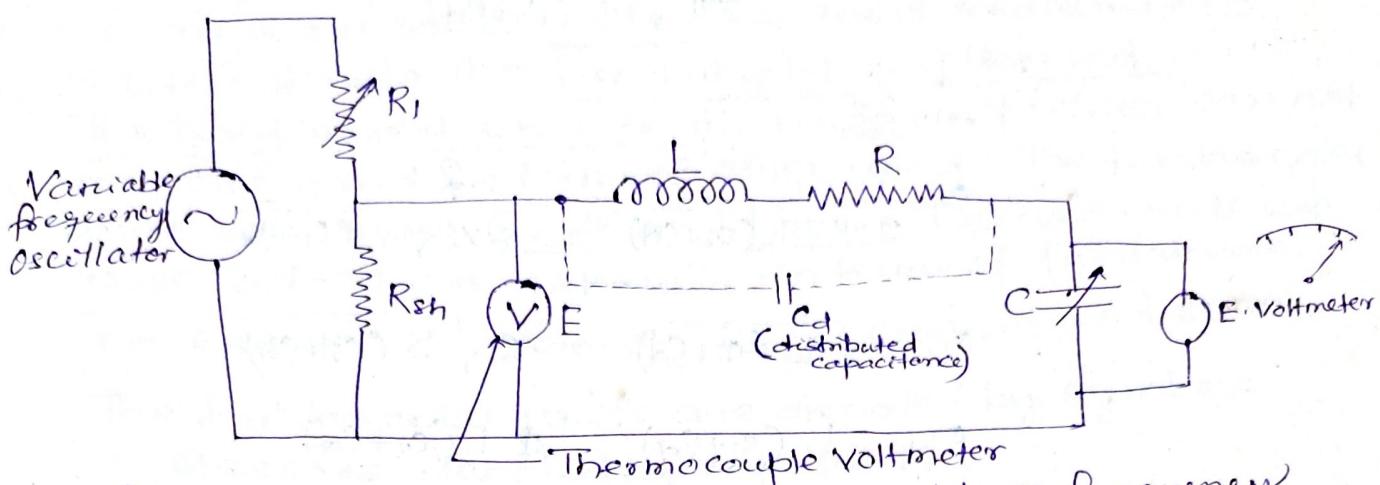
$$\text{Again voltage across } C = I_0 \omega_0 L$$

$$= \frac{E}{R} \cdot \omega_0 L$$

$$\text{or } V_C \propto Q \quad (\text{Quality factor})$$

The resistance of oscillator should be low as practicable as possible.

Practical Q-Meter



A practical Q-meter has variable frequency oscillator to which a Variable resistance R_1 and shunt resistance R_{sh} are connected in series. Across R_{sh} , the resonating circuit consisting of the coil and the variable capacitance are connected. Across the variable capacitance, an electronic voltmeter is connected across C . A thermocouple voltmeter is connected across R_{sh} . The variable capacitance is adjusted until the deflection shown by the voltmeter is maximum. Under this condition the voltage across C and the voltage across R_{sh} are noted. Then Q-factor of the coil is equal to the ratio of voltage across C to voltage across R_{sh} .

If the voltage across R_{sh} is kept constant at a particular value, then electronic voltmeter can be calculated in terms of Q-factor directly.

To measure the distributed capacitance (C_d):

Let us consider the supply frequency,

$$f_2 = 2f_1$$

Keep the variable capacitance C_1 has its max. value and adjust the frequency until the circuit resonates which is indicated by the max. deflection in the electronic voltmeter across the capacitance.

Let the value of variable capacitance be

C_1 and frequency be f_1 .

Next, the frequency of oscillator is doubled and the circuit is brought back to resonance by varying variable capacitance.

Under this condition, the frequency of oscillator be f_2 and capacitance value be C_2 .

Expression :- $f_1 = \frac{1}{2\pi\sqrt{L(C_1+C_d)}} \quad \dots (1)$

$$f_2 = \frac{1}{2\pi\sqrt{L(C_2+C_d)}} \quad \dots (2)$$

$$\therefore f_2 = 2f_1$$

$$\text{or}, \frac{1}{2\pi\sqrt{L(C_2+C_d)}} = \frac{2}{2\pi\sqrt{L(C_1+C_d)}}$$

$$\text{or}, \sqrt{L(C_1+C_d)} = 2\sqrt{L(C_2+C_d)}$$

$$\text{or}, L(C_1+C_d) = 4L(C_2+C_d)$$

$$\text{or}, C_1 + C_d = 4(C_2 + C_d)$$

$$\text{or}, 4C_d - C_d = C_1 - 4C_2$$

$$\text{or}, 3C_d = C_1 - 4C_2$$

$$\text{or}, C_d = \frac{C_1 - 4C_2}{3}$$

Correction factor for C_d :-

$$Q_{\text{measured}} = \frac{\omega_0 L}{R} = \frac{1}{R \omega_0 (C + C_d)} \quad \begin{matrix} \text{As } C_d \text{ is parallel to } C \\ (\because \text{At resonance } \omega_0 L = \frac{1}{\omega_0 C}) \end{matrix}$$

$$Q_{\text{true}} = \frac{\omega_0 L}{R} = \frac{1}{R \omega_0 C}$$

$$\frac{Q_{\text{true}}}{Q_{\text{measured}}} = \frac{R \omega_0 (C + C_d)}{R \omega_0 C} = \frac{C + C_d}{C}$$

$$\text{or, } Q_{\text{true}} = \left(1 + \frac{C_d}{C}\right) Q_{\text{measured}} \quad \text{correction factor for } C_d$$

Correction factor for R_{sh} :-

$$Q_{\text{measured}} = \frac{\omega_0 L}{R} = \frac{\omega_0 L}{R + R_{sh}} \quad (\text{neglect } C_d)$$

$$Q_{\text{true}} = \frac{\omega_0 L}{R}$$

$$\frac{Q_{\text{true}}}{Q_{\text{measured}}} = \frac{R + R_{sh}}{\omega_0 L} \times \frac{\omega_0 L}{R} = \frac{R + R_{sh}}{R}$$

$$\text{or, } Q_{\text{true}} = \left(1 + \frac{R_{sh}}{R}\right) Q_{\text{measured}} \quad \text{correction factor for } R_{sh}$$

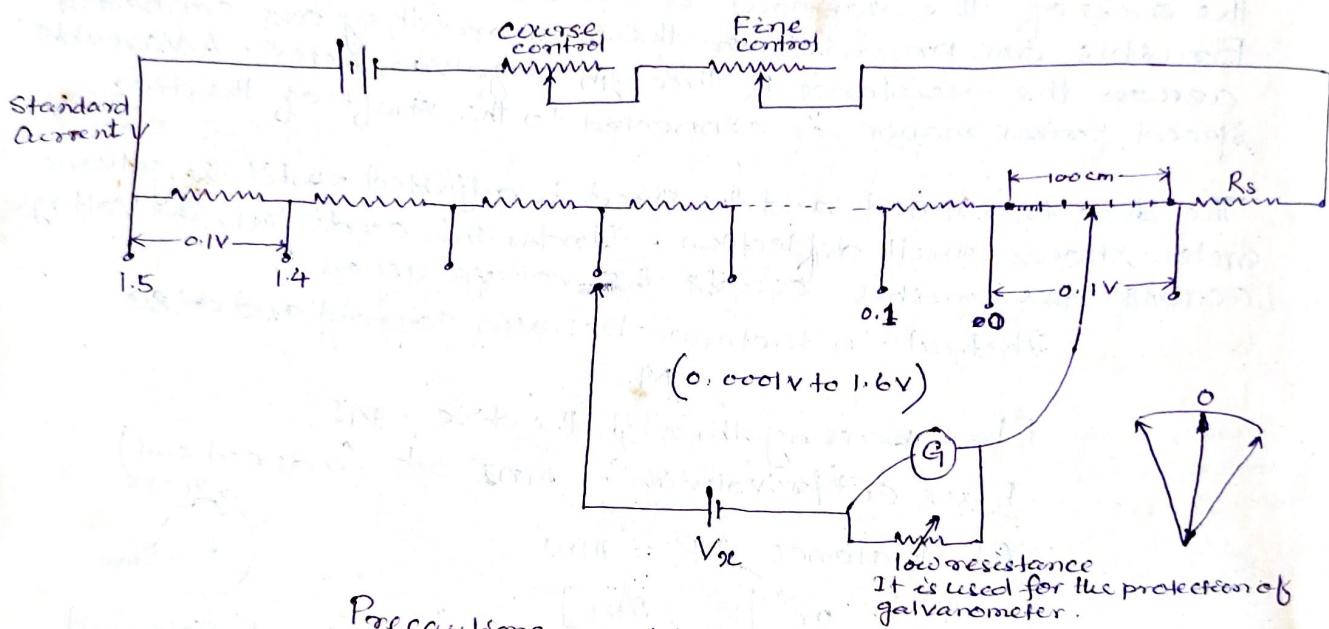
POTENTIOMETER

A Potentiometer is used to measure the voltage of the order of 1.5 Volts without drawing any current from the circuit i.e. it will not have any loading effect. Apart from measuring Voltage it is also used ~~indirectly~~ to measure Current, Resistance, Power etc.

There are two types of potentiometer

- ✓ (i) D.C Potentiometer
- ✓ (ii) A.C Potentiometer

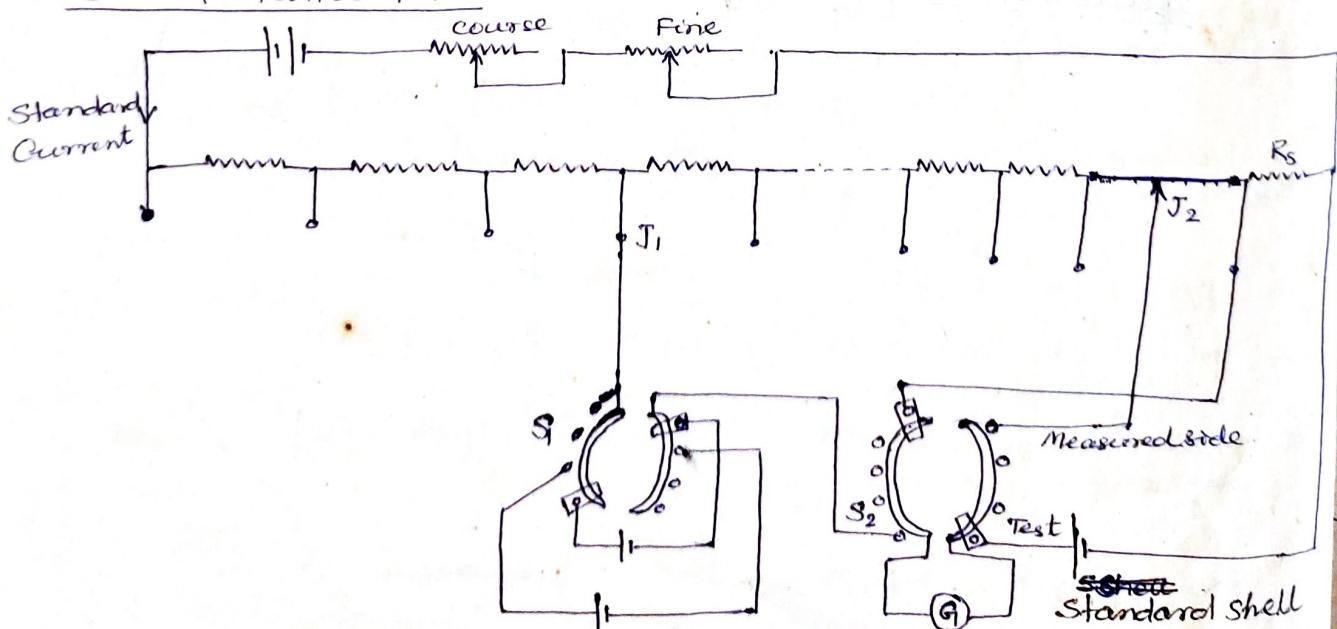
D.C POTENTIOMETER :-



Precautions :-

- (1) Polarity of battery
- (2) Load resistance

D.C Potentiometer



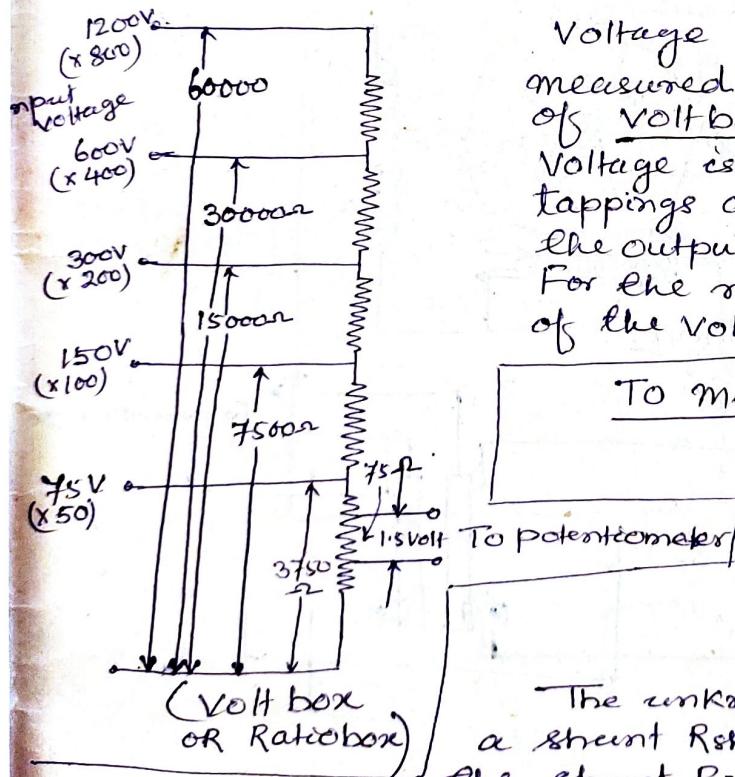
The potentiometer consists of 15 sections of resistances, each of value 10 ohms. In series with these resistances, a slide wire of 100 cm is connected. The jockey points J_1 & J_2 are provided on resistance.

coil and the slide wire. The resistances coils are marked as voltages in step of 0.1 Volt. The slide wire is of length 1 meter and of resistance 10Ω. The voltage across the slide wire is 0.1 Volt. When standard current flows through the wire, the smallest voltage that can be read from the slide wire is 0.0001 Volt. In series with slide wire, a standard calibrating resistance R_s is provided. The voltage across R_s when standard current flows is equal to the voltage across the standard shell. There are two rotary switches S_1 & S_2 . S_1 is used to select unknown voltage and S_2 is used to connect the galvanometer either on standard shell side or on the measurement side.

To Standardise the potentiometer :-

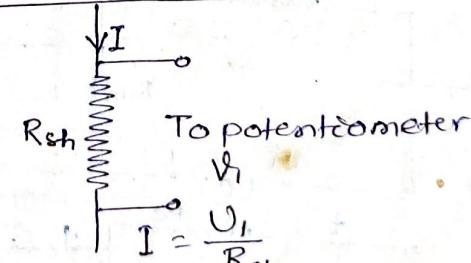
- S_2 is selected to test position.
- The course and fine resistance on the potentiometer circuit is varied until the galvanometer shows zero deflection. Under this condition, voltage across R_s is equal to standard shell. Now a standard current flows through the potentiometer wire and makes the potentiometer direct reading.
- The unknown voltage is selected by S_1 and galvanometer G is selected on measurement side using S_2 .
- Adjust Jockey Points J_1 and J_2 until the galvanometer reads zero value. Under this condition, the J_1 & J_2 gives the voltage of V_1 .

To measure large voltages :-



Voltage greater than 1.5 Volts can be measured using potentiometer with the help of voltbox or ratio box. The unknown voltage is given to appropriate input tappings and the voltage measured across the output terminals using potentiometer. For the reading, the multiplication factor of the voltbox, the voltage can be calculated.

To measure Current :-

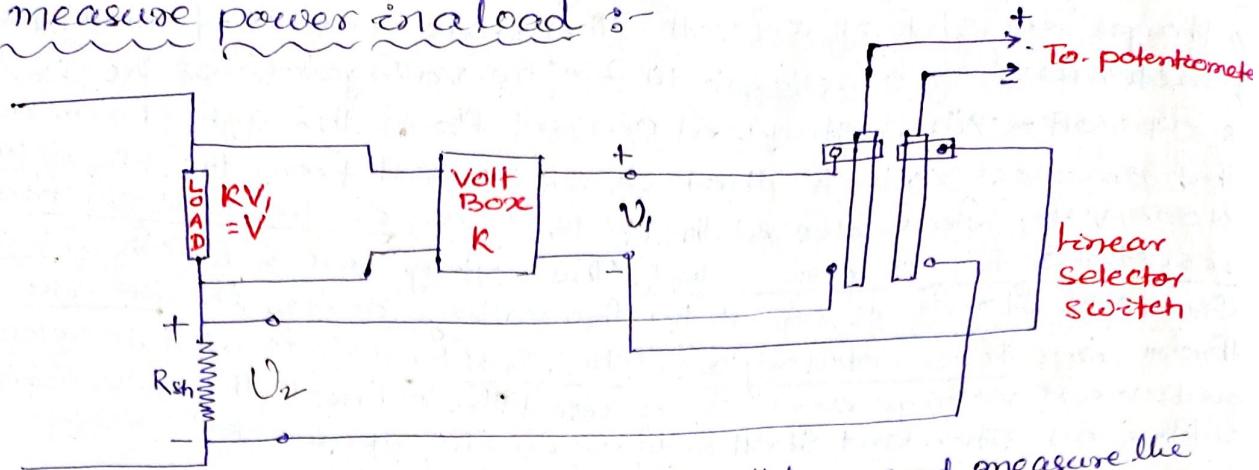


The unknown current is passed through a shunt R_{sh} and the voltage developed across the shunt R_{sh} is measured using potentiometer.

From the voltage, unknown current can be calculated as

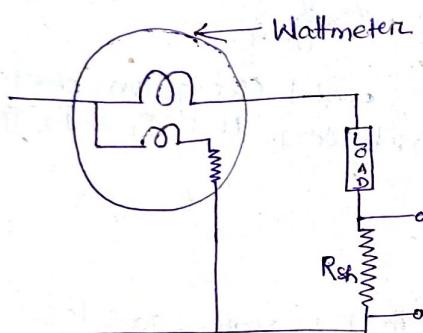
$$I = \frac{V_1}{R_{sh}}$$

To measure power in a load :-



At first select the voltage across volt box and measure the voltage using potentiometer. Next the voltage across R_{sh} is selected and measured. Let the voltage be V_2 . Then the current through the load $I = \frac{V_2}{R_{sh}}$. Power consumed by the load, $P = KV_1 \times \frac{V_2}{R_{sh}}$ watts. It is very accurate method.

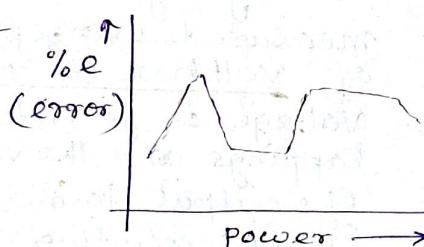
Calibrating of Wattmeter :-



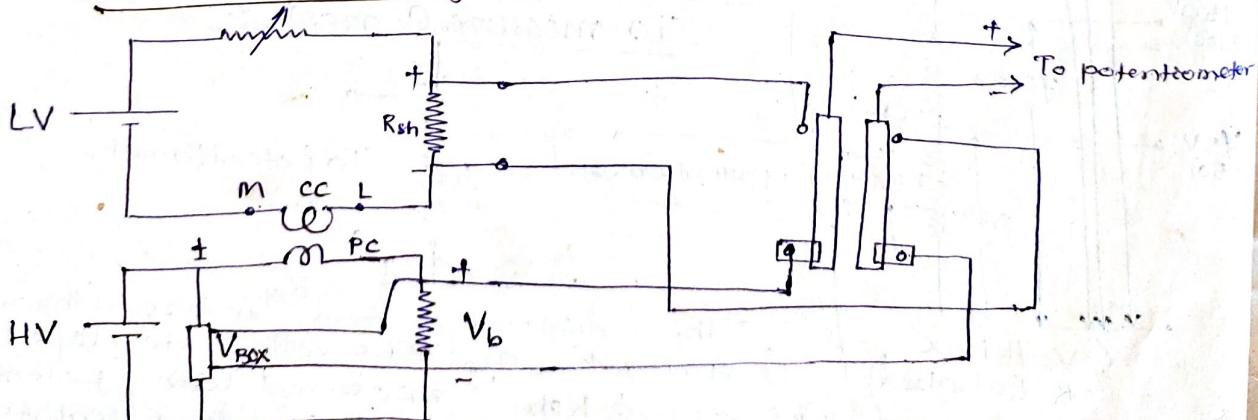
(+) There are two methods of calibrating of wattmeter.

- (1) Direct method
- (2) Phantom loading method

① Direct method :-



② Phantom loading method :-



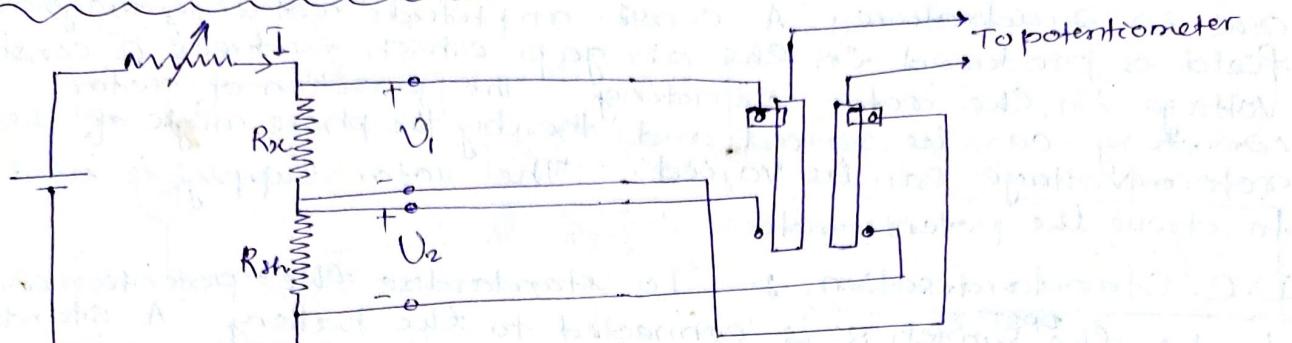
A low voltage battery is connected to current coil.

A high voltage battery is connected to pressure coil.

The current in the current coil is measured using a shunt and the voltage across the pressure coil is measured using volt box. From the voltages across the current and the volt box.

The true power can be calculated before using formulae $KIV_1 \times \frac{V_2}{R_{sh}}$. The actual reading shown by the wattmeter is read from the wattmeter. The difference between the actual power and the measured power gives the error of the wattmeter.

To measure resistance :

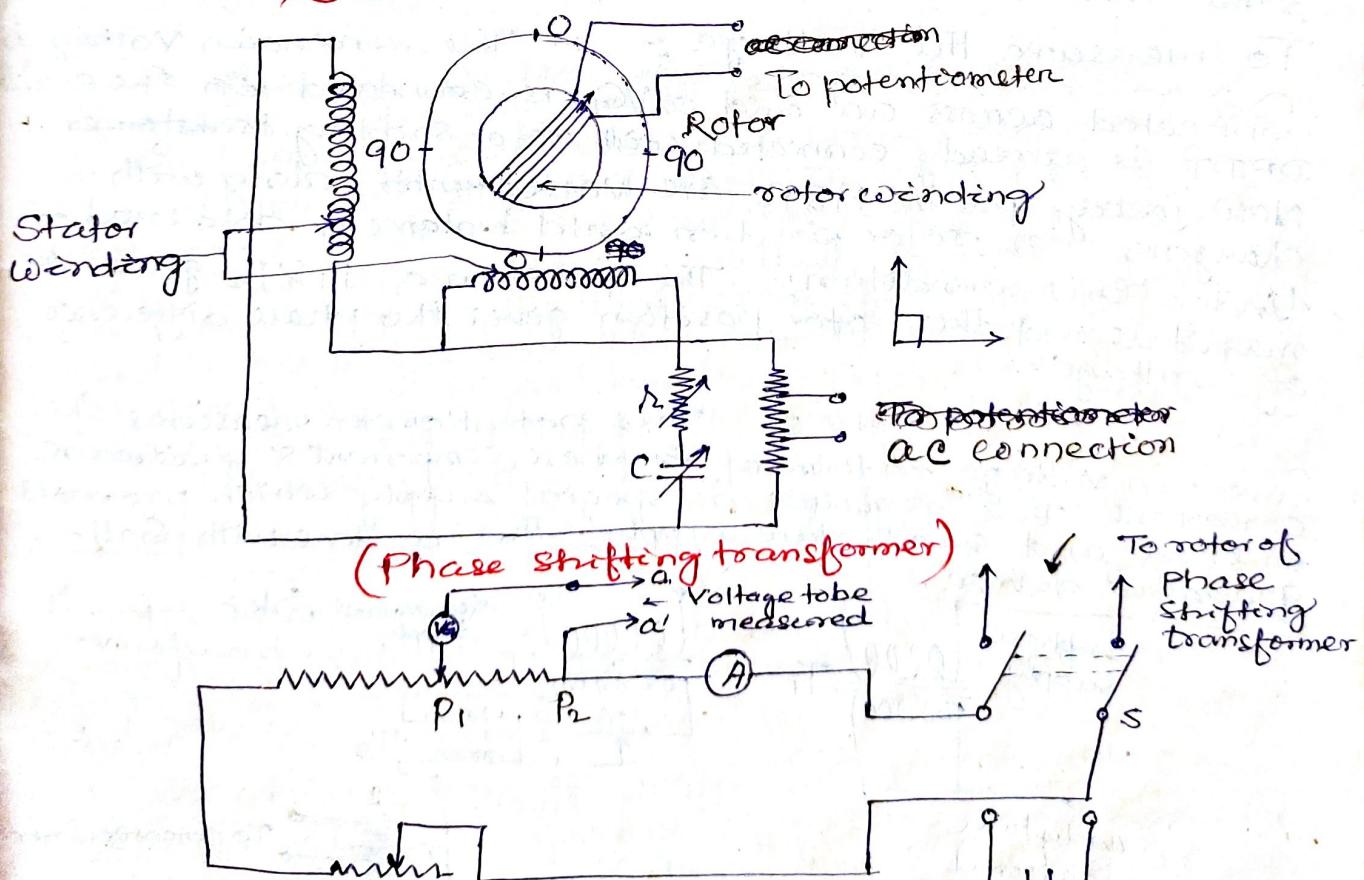


$$V_1 = IR_x, V_2 = IR_{sh},$$

$$\frac{V_1}{V_2} = \frac{R_x}{R_{sh}} \text{ or } R_x = \frac{V_1}{V_2} R_{sh}$$

Problem
W-91 A simple slide wire is used for the measurement of current on a circuit. The voltage drop across a standard resistance R of 1Ω is balanced at 75 cm . Find the magnitude of current of the standard shell having an emf of 145 Volt is balanced at 50 cm . (2.175A)

A.C. POTENTIOMETER (OR) A.C. POLAR POTENTIOMETER



In the case of polar potentiometer, the unknown voltage is measured in terms of the magnitude and phase angle. This potentiometer requires a phase shifting transformer which provides a constant

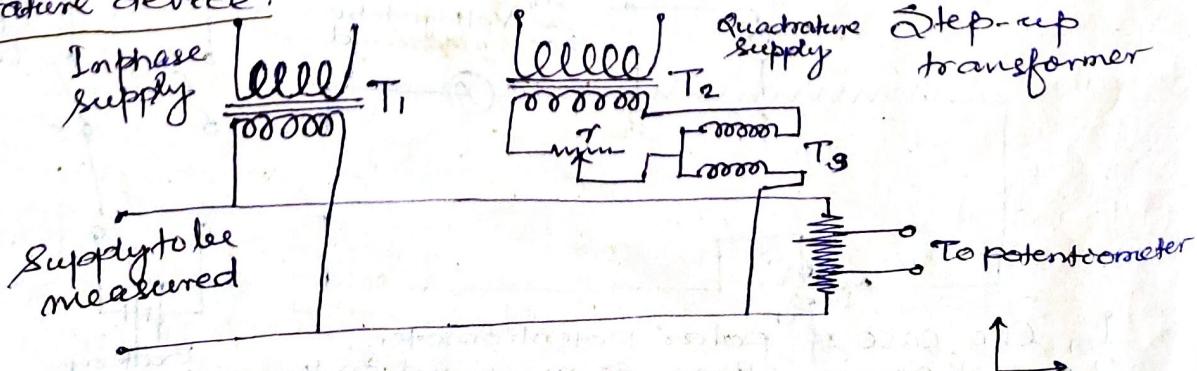
Magnitude and variable phase angle ac voltage. This is obtained using phase shifting transformer. This transformer has a stator and a rotor. The stator has two windings in quadrature. One winding is directly connected across single phase ac supply and other is connected through a variable ~~F~~ and C. When the current in two windings are in quadrature, a const. amplitude rotating magnetic field is produced in the air gap which induces a const. Voltage in the rotor winding. The position of rotor winding can be varied and thereby the phase angle of the motor voltage can be varied. The rotor supply is used to drive the potentiometer.

D.C. Standardisation :- To standardise the potentiometer to d.c., the switch, S is connected to the battery. A standard cell is connected across aa' . A vibration galvanometer is replaced by d.c. galvanometer. P_1 & P_2 jockey points are kept in position such that it reads the value of standard cell. Now R , is adjusted until the VG given null deflection. The current shown by ammeter is noted.

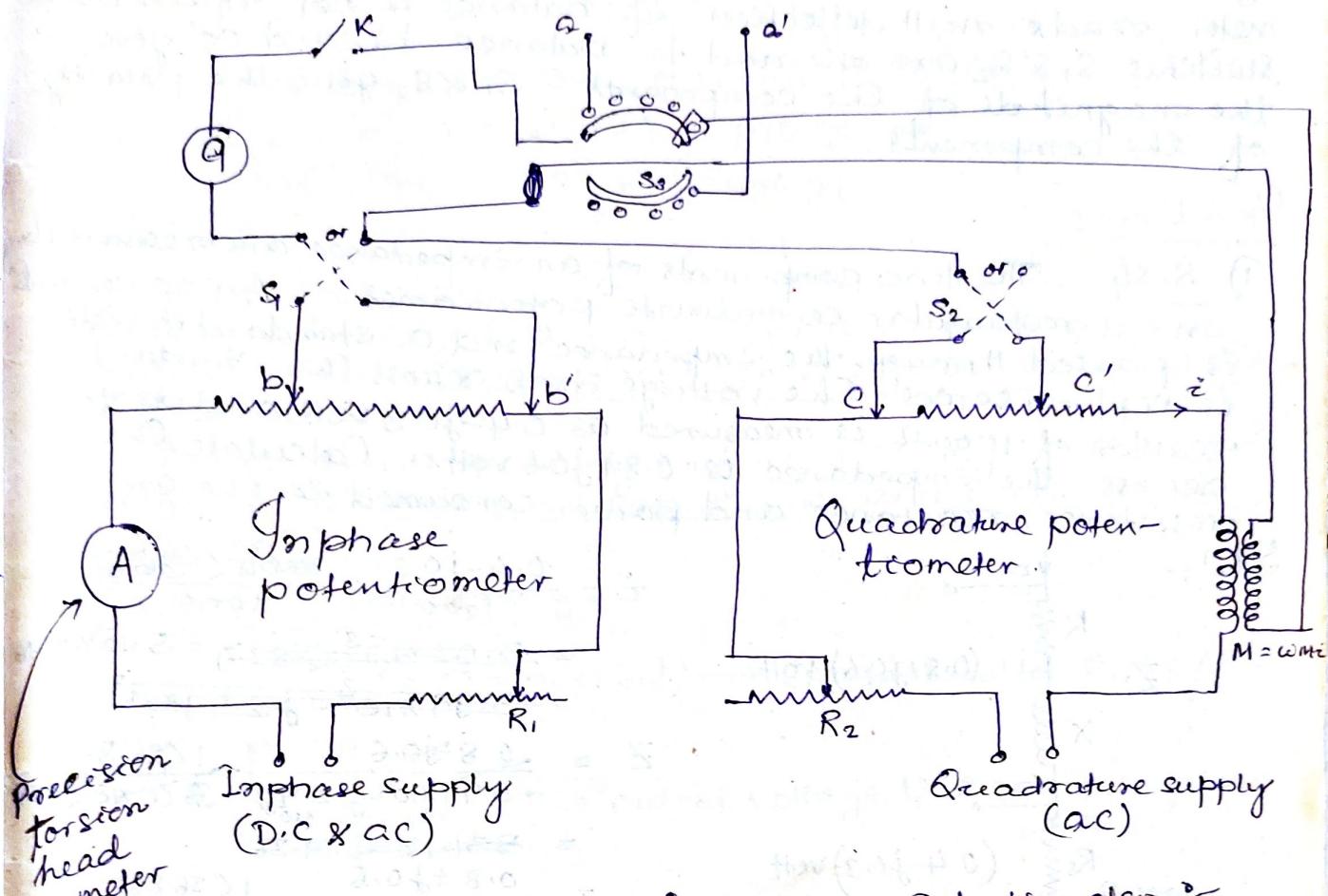
A.C. Standardisation :- Disconnect the standard cell and the d.c galvanometer and connect the DPDT to the rotor of phase shifting transformer. Vary the position of the rotor and note the reading shown by ammeter. If the ammeter reading changes with rotor position, vary R_1 and c, ammeter shows a constant value of irrespective of rotor position. Then adjust R_1 until the ammeter reads the same value as obtained in dc standardisation.

To measure the Voltage :- The unknown voltage is connected across aa' and a VG is connected in the circuit. DPDT is already connected with rotor shifting transformer. Now jockey points P_1 & P_2 are varied until balance is obtained - changing in rotor position until balance is obtained. Under this condition, the position of P_1 & P_2 gives the magnitude and the rotor position gives the phase difference of voltage.

Quadrature Potentiometer :- This potentiometer measures the unknown voltage in terms of inphase component & quadrature component. This requires a special supply which provides in phase and quadrature supply. This is done with Gall-quadrature device.



The Gall quadrature device has two transformers T_1 & T_2 . T_1 is connected to the supply which is to be measured. T_2 is connected across supply through variable resistance r and transformer T_3 . T_1 provides in-phase supply and T_2 provides quadrature supply. r is varied until the supply of T_2 is in quadrature to the supply of T_1 .



D.C Standardisation of Inphase potentiometer :-

A d.c battery is connected to inphase potentiometer. A standard cell is connected across aa' . D.C Gall galvanometer is connected across the circuit. bb' is kept to read the value of the standard cell. Adjust R_1 until the Galvanometer reads zero. The light spot of ammeter would have moved to one side, turn the torsion head until the spot of light comes back to zero position. From the next divisions turned in the torsion head, the current flowing through the ammeter is calculated.

A.C Standardisation of inphase potentiometer :-

Disconnect the standard cell and D.C Gall galvanometer. Connect the inphase supply to the inphase potentiometer and adjust R_1 again to bring the light spot back to zero position. Now it is ac standardisation.

A.C Standardisation of Quadrature potentiometer :-

Let the standard current of inphase potentiometer be i and if the same current flows through the quadrature potentiometer, a voltage $e = \omega M_i$ will be induced in the mutual conductance M . Now vibration Galvanometer is connected in the circuit and the induced voltage is selected by S_3 . bb' is kept in a position equal to the voltage e .

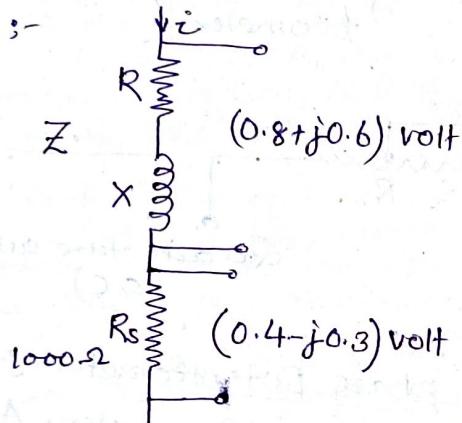
Now R_2 and r are varied until the vibration galvanometer reads null deflection. Now the Wheatstone potentiometer is standardised for AC.

To Measure Unknown Voltage :- The unknown voltage is connected across aa' and it is selected in the circuit using S_3 . Now bb' and cc' are varied until the vibration galvanometer reads null deflection. If balance is not obtained, the switches S_1 & S_2 are operated to balance. bb' and cc' give the magnitude of the components. S_1 & S_2 give the polarity of the components.

Problems :-

- ① S-84 The two components of an "impedance" are measured by an ac rectangular co-ordinate potentiometer. An ac current is passed through the impedance and a standard resistor is kept in series. The voltage drop across the standard resistor of 1000Ω is measured as $0.4 - j0.3$ volt and drop across the impedance is $0.8 + j0.6$ volts. Calculate the resistance, reactance and power consumed in the coil.

Soln:-



$$\begin{aligned} i &= \frac{0.4 - j0.3}{1000} = \frac{0.5}{1000} \angle -36.86^\circ \\ &= 3.02 \times 10^{-3} \angle -82.7^\circ = 5 \times 10^{-4} \angle -36.86^\circ \\ Z &\approx \frac{0.8 + j0.6}{3.99 \times 10^{-4} \angle 2.99^\circ} = 1 \angle 36.86^\circ \\ &\approx 3.31 \times 10^3 \angle 119.26^\circ \\ &= \frac{0.8 + j0.6}{5 \times 10^{-4} \angle -36.86^\circ} = \frac{1 \angle 36.86^\circ}{5 \times 10^{-4} \angle -36.86^\circ} \\ &= 2000 \angle 73.72^\circ = 560 + j1920 \Omega \end{aligned}$$

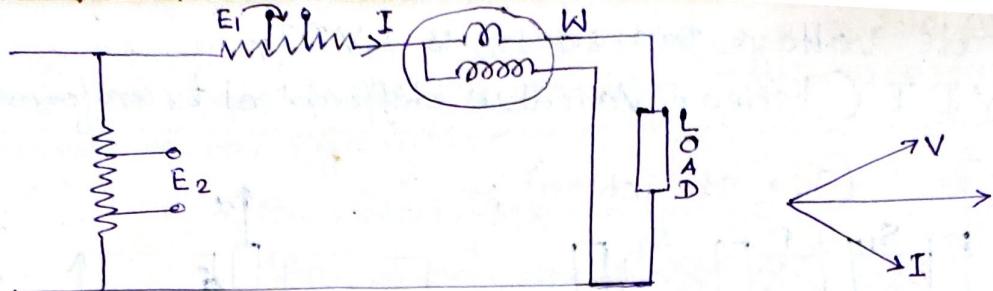
$$\text{Power} = |\bar{Z}|^2 \times 560 = 5 \times 10^4 \times 560 = 0.28 \text{ Watts}$$

- ② S-91. The current taken by a small iron core choke coil is measured by rectangular ac. potentiometer. A one Ω non-reactive resistance is connected in series with choke. The voltage measured across the resistance and coil R is $0.8 - j0.75$ volts and $1.2 + j0.3$ volt respectively. Assuming sinusoidal voltage and current. Determine the core loss.

Exactly same problem as (1) Ans:- 0.735 watts.

- ③ In calibrating a wattmeter with help of AC rectangular co-ordinate potentiometer as shown in figure, a 300Ω standard resistor was found to be $E_1 = 0.35 - j0.1$ volt. & voltage tapped of the voltage ratio box gave a reading of $E_2 = 0.8 + j0.15$ V. Determine the true reading of wattmeter and load power factor.

Soln :-



$$I = \frac{E_1}{R_1} = \frac{0.35 - j0.1}{0.1} = |I| \angle \phi_1$$

$$\bar{V} = 300E_2 = 300(0.8 + j0.15) = |V| \angle \phi_2$$

$$\text{Power} = VI \cos(\phi_2 + \phi_1) = 795.01 \text{ watts}$$

$$\cos(\phi_1 + \phi_2) = 0.8944 \text{ (lagging)}$$

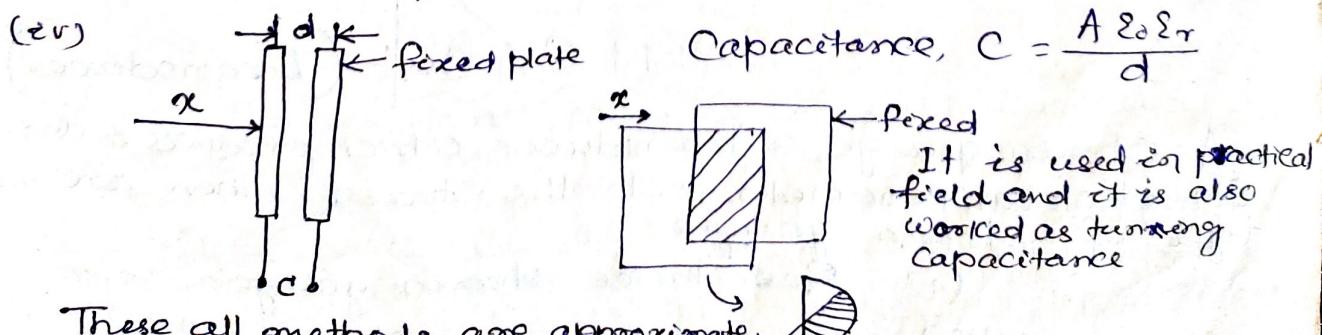
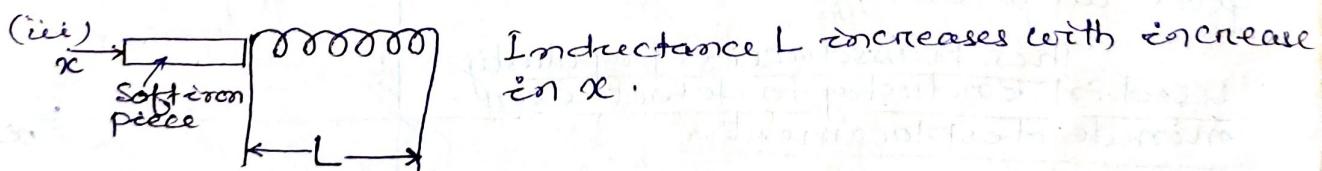
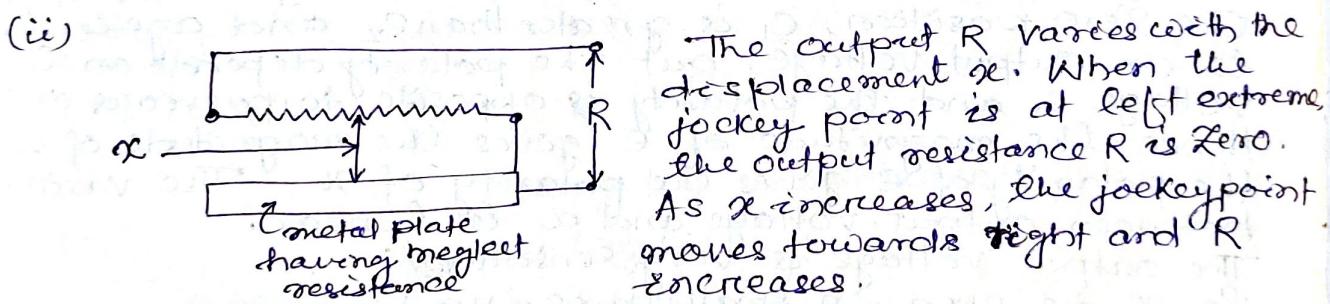
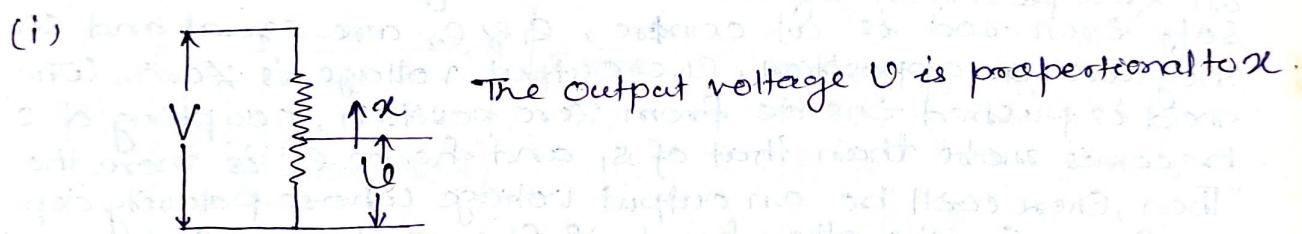
Transducers & Strain Gauge

A transducer is a device which converts physical quantities like temp., displacement, pressure, speed into electrical quantities.

The electrical quantity may be a voltage, a change in resistance, a change in capacitance and a change in inductance.

Linear transducers :-

x (displacement) \rightarrow (proportional to), E , ΔR , ΔL , ΔC

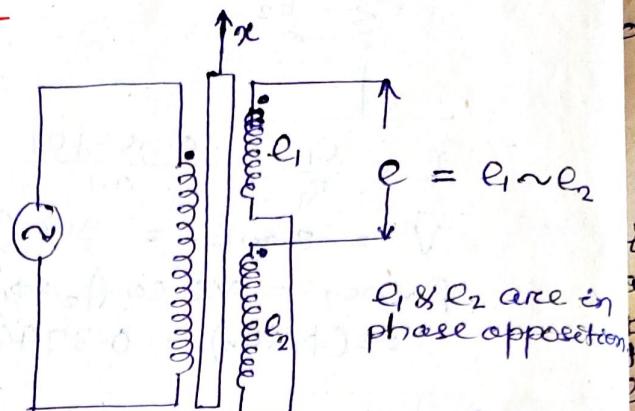
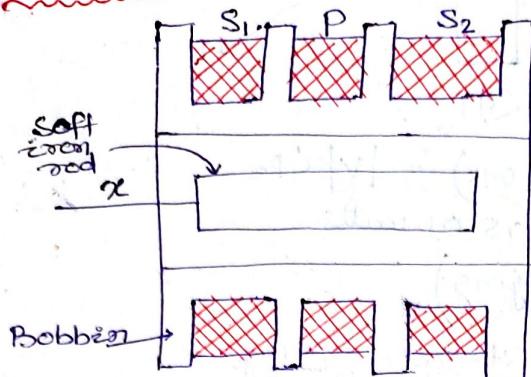


These all methods are approximate.

Accurate voltage transducer is LVDT.

LVDT (Linear Variable differential transformer)

LVDT (construction) :-



The LVDT is used for converting (Circuit diagram) the displacement x into voltage. It has an insulated bobbin over which a primary and two identical secondary core are wound symmetric to the primary. Along the axis of the coil, a movable soft iron rod is provided.

The primary is connected to an ac voltage source. The secondaries are connected differentially i.e. they are connected in phase opposition. The primary winding produces a flux and links with both the secondary windings.

The flux linkage between the primary and secondary depend on the position of the movable soft iron rod. When the soft iron rod is at centre, e_1 & e_2 are equal and since they are in opposition, the output voltage is zero. When the rod is pushed inside from zero position, coupling of S_2 becomes more than that of S_1 , and hence e_2 is more than e_1 . Then, there will be an output voltage whose polarity depends on e_2 . On the other hand if the rod is pulled out from the zero position, e_1 is greater than e_2 and again there is an output voltage, but the polarity depends on the voltage e_1 , and the polarity is opposite to previous one. Hence, the magnitude of e gives the magnitude of x and the polarity of e gives the polarity of x . The variation between output voltage and x is linear.

The output voltage is very sensitive for x i.e. even a small change in x gives large voltage.

This transducer is popularly used in industry to detect even minute displacement.

Linear

x

STRAIN GAUGE (transducer)

A strain gauge is transducer which produces a change in electrical parameter with the strain. There are two types of strain gauges.

(1) Resistance strain gauge

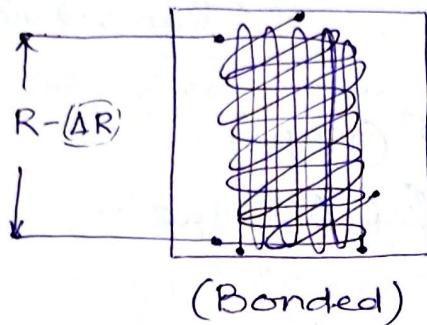
(2) Semi-conductor strain gauge

Resistance strain gauge uses a resistive wire whose resistance changes with the strain. There are two types of resistance strain gauge.

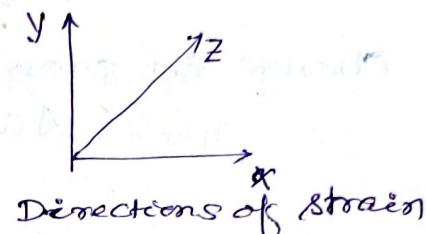
- (i) Bonded strain gauge
- (ii) Unbonded strain gauge

In the case of bonded strain gauge, a grill of fine wire is connected to a thin paper sheet or a thick bakelite sheet and covered with a protective covering of paper, felt or thin bakelite. This paper sheet is bonded with an adhesive material to the structure under study. In unbonded strain gauge, the resistance wire is directly wound round the structure under study.

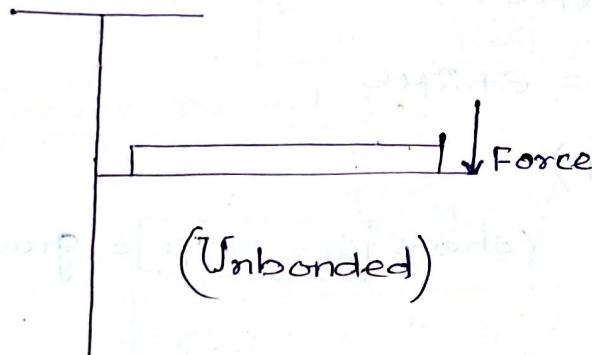
(i)



$$R = \frac{\rho l}{\alpha d}$$



(ii)



Expression the relation between ΔR & ϵ :

$$\text{Strain, } \epsilon = \frac{\Delta L}{L}, \quad R = \frac{\rho L}{\alpha d}$$

Taking log on both sides,

$$\log R = \log \rho + \log L - \log \alpha$$

$$\text{Differentiating, } \frac{d \log R}{dR} = \frac{d \log \rho}{dp} + \frac{d \log L}{dL} - \frac{d \log \alpha}{d\alpha}$$

$$\text{or, } \frac{1}{R} \Delta R = 0 + \frac{1}{L} \Delta L - \left(\frac{\Delta \alpha}{\alpha} \right)$$

$$\text{or, } \frac{\Delta R}{R} = \frac{\Delta L}{L} + \frac{\Delta \alpha}{\alpha}$$

$$\text{or, } \frac{\Delta R}{R} = \epsilon + \frac{\Delta \alpha}{\alpha}$$

Let the resistance wire is elongated, this causes an increase in length and decrease in diameter.

Both these changes cause an increase in resistance.

Let L be the length, d be the diameter, R be the resistance of the wire before elongation. After elongation, let the above be $L + \Delta L$, $d - \Delta d$ & $R + \Delta R$

Change in diameter of the wire = μe
 where, μ = poisson's ratio
 e = strain
 d = diameter

$$\text{New diameter} = d - \Delta d = d - \mu e d$$

$$\text{Old area} = \frac{\pi d^2}{4}$$

$$\begin{aligned}\text{new area} &= \frac{\pi}{4} (d - \mu e d)^2 \\ &= \frac{\pi}{4} (d^2 + \mu^2 e^2 d^2 - 2\mu e d^2) \\ &= \frac{\pi}{4} d^2 (1 + \mu^2 e^2 - 2\mu e) \quad (\text{As } e \text{ is very small, so } \mu^2 e^2 \text{ can be neglected}) \\ &= \frac{\pi}{4} d^2 (1 - 2\mu e)\end{aligned}$$

Change in cross section,

$$\begin{aligned}\Delta a &= \frac{\pi d^2}{4} - \frac{\pi d^2}{4} (1 - 2\mu e) \\ &= \frac{\pi d^2}{4} - \frac{\pi d^2}{4} + \frac{\pi d^2}{4} \cdot 2\mu e\end{aligned}$$

$$\text{or, } \Delta a = Q \cdot 2\mu e$$

$$\text{or, } \frac{\Delta a}{a} = 2\mu e$$

$$\text{Now } \frac{\Delta R}{R} = e + \frac{\Delta a}{a} = e + 2\mu e$$

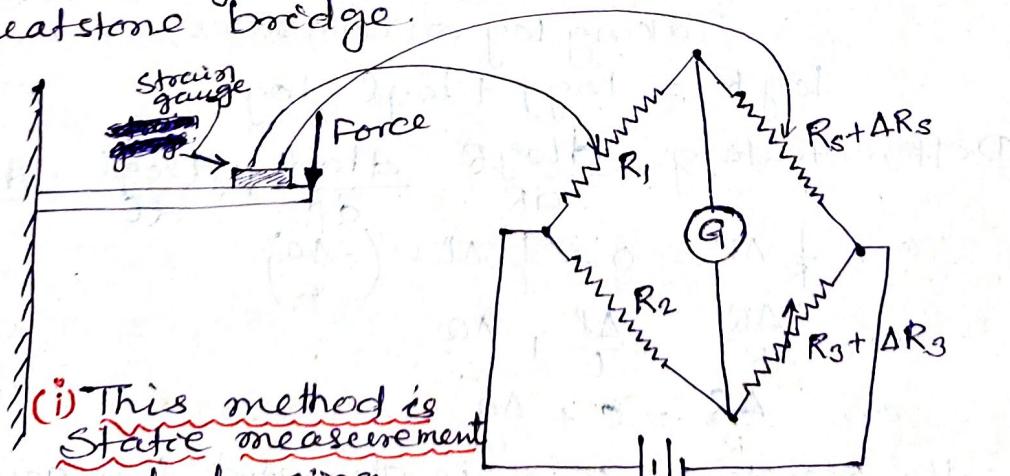
$$\text{or, } \frac{\Delta R}{R} = e(1 + 2\mu)$$

$$\text{or, } \frac{\Delta R}{R} = Qe \quad \text{where } [Q = 1 + 2\mu] = \text{gauge factor}$$

$$\text{or, } \boxed{\Delta R = R Q e}$$

To measure the change in Resistance :-

The change in the resistance can be measured using Wheatstone bridge.



(i) This method is static measurement or balancing.

Connect the strain gauge to one of the arms of wheatstone bridge. R_3 is a variable resistance before applying strain. The bridge is balanced by varying R_3 .

Let R_3 be the resistance at balance.

$$\text{Before strain, } \frac{R_1}{R_2} = \frac{R_s}{R_3}$$

$$\text{After strain, } \frac{R_1}{R_2} = \frac{R_s + \Delta R_s}{R_3 + \Delta R_3}$$

$$\therefore \frac{R_s + \Delta R_s}{R_3 + \Delta R_3} = \frac{R_s}{R_3}$$

$$\text{or, } \frac{R_s + \Delta R_s}{R_s} = \frac{R_3 + \Delta R_3}{R_3}$$

$$\text{or, } \frac{R_s + \Delta R_s - R_s}{R_s} = \frac{R_3 + \Delta R_3 - R_3}{R_3}$$

$$\text{or, } \frac{\Delta R_s}{R_s} = \frac{\Delta R_3}{R_3}$$

$$\text{or, } \boxed{\Delta R_s = R_s \cdot \frac{\Delta R_3}{R_3}}$$

We know previously that $\Delta R = R G e$

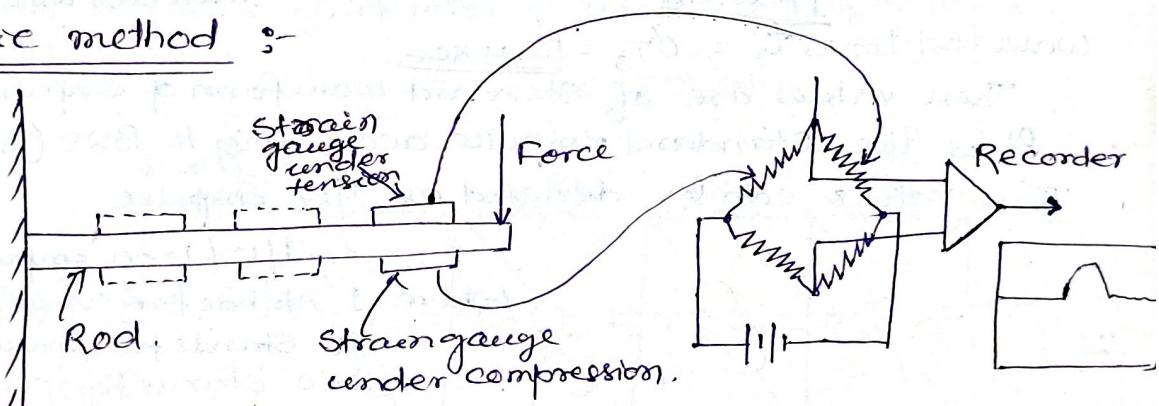
$$\text{or, } \Delta R_s = R_s G e$$

$$\therefore \frac{R_s \cdot \Delta R_3}{R_3} = R_s G e$$

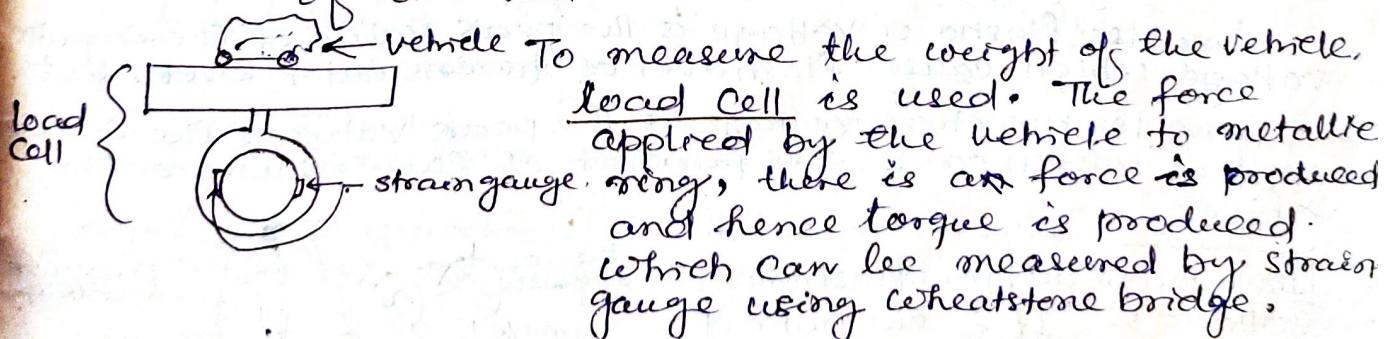
$$\text{or, } G e = \frac{\Delta R_3}{R_3}$$

$$\text{or, } \boxed{e = \frac{\Delta R_3}{R_s G}}$$

(ii) Dynamic method :-



To have better sensitivity, there are many strain gauges can be applied on either side of the rod.



Another method can be adopted to measure the change in resistance. The galvanometer is calibrated in terms of strain directly.

Rectifier type Instruments

Rectifier instruments are used for a.c measurements. By using a rectifier to convert a.c to d.c.

To indicate the value of rectified a.c., a d.c. meter ~~has~~ is to be used. It is very attractive, because a PMMC instrument has a higher sensitivity than an electrodynamometer or M.T. instruments. For ~~high~~ low voltages and high resistances, rectifier instruments are suited on communication.

Rectifier instruments are suited to measurements on communication circuits and for other light current works where the voltages are low and resistances high.

A rectifier element is used to convert the alternating current to direct current before it flows through the meter. The rectifier elements are copper oxide, ~~selenium~~, doped Germanium and Silicon crystal diode.

~~obsolete~~ Copper oxide element has a peak inverse voltage of the order of 2V.

~~Selenium~~ elements

PIV is ~~approx~~ about 10V. and they can handle only limited amounts of current and they are becoming ~~obsolete~~ (out of date).

~~Germanium~~

PIV is of order of 300V and current rating of ~~100~~ mA approximately.

~~Low current~~
Silicon diode
rectifiers

PIV is up to 1000V and a current rating of 500 mA.

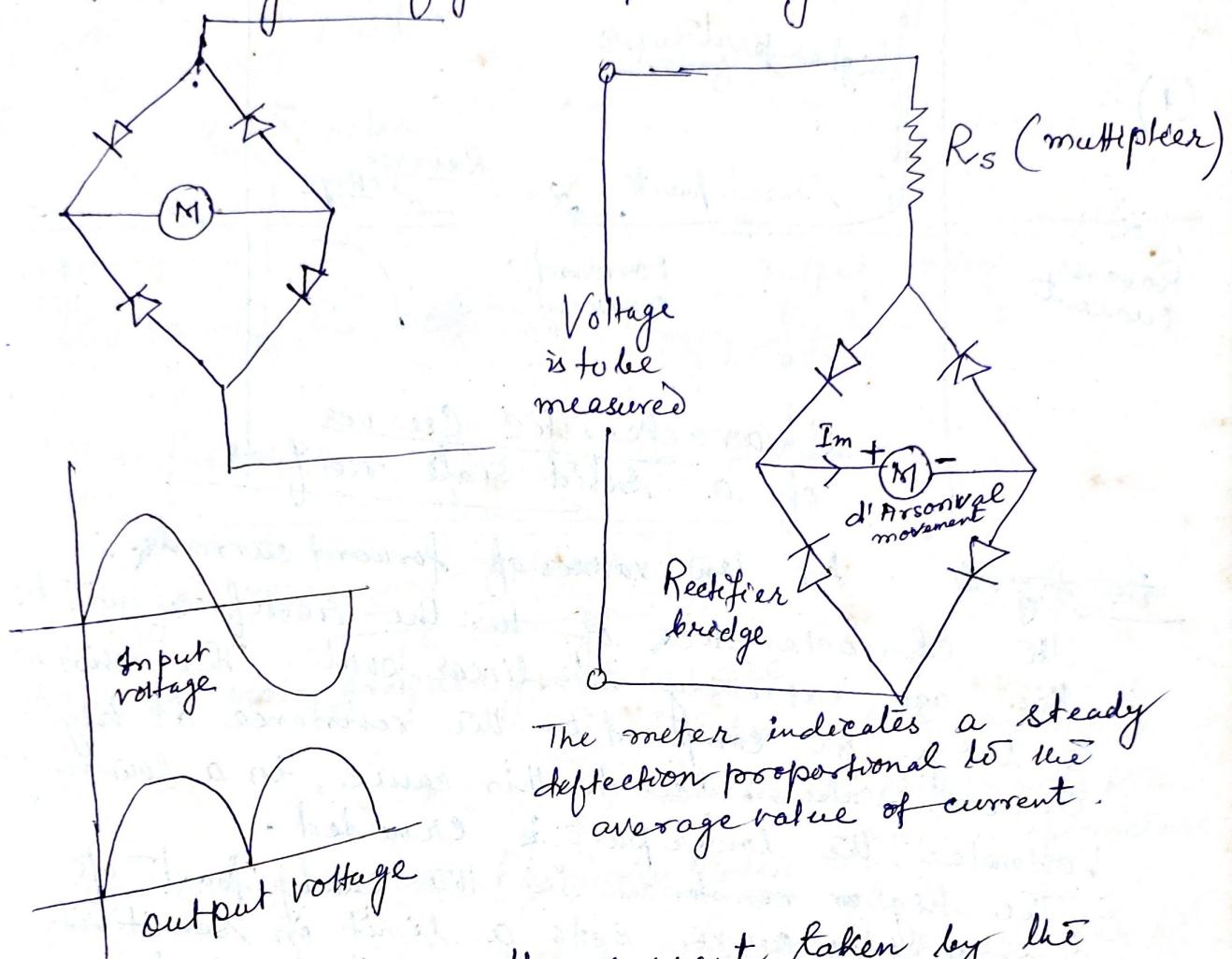
~~High current~~
Silicon diodes
rectifiers

are seldom used in indicating instruments but they are used in power applications requiring currents as high as 85 A. For these reasons, Germanium and silicon diodes are used in modern rectifier instruments.

Rectifier Voltmeters

Full wave Rectifier circuit :- Rectifiers

Consist of four rectifying elements connected in a bridge configuration providing full wave rectification



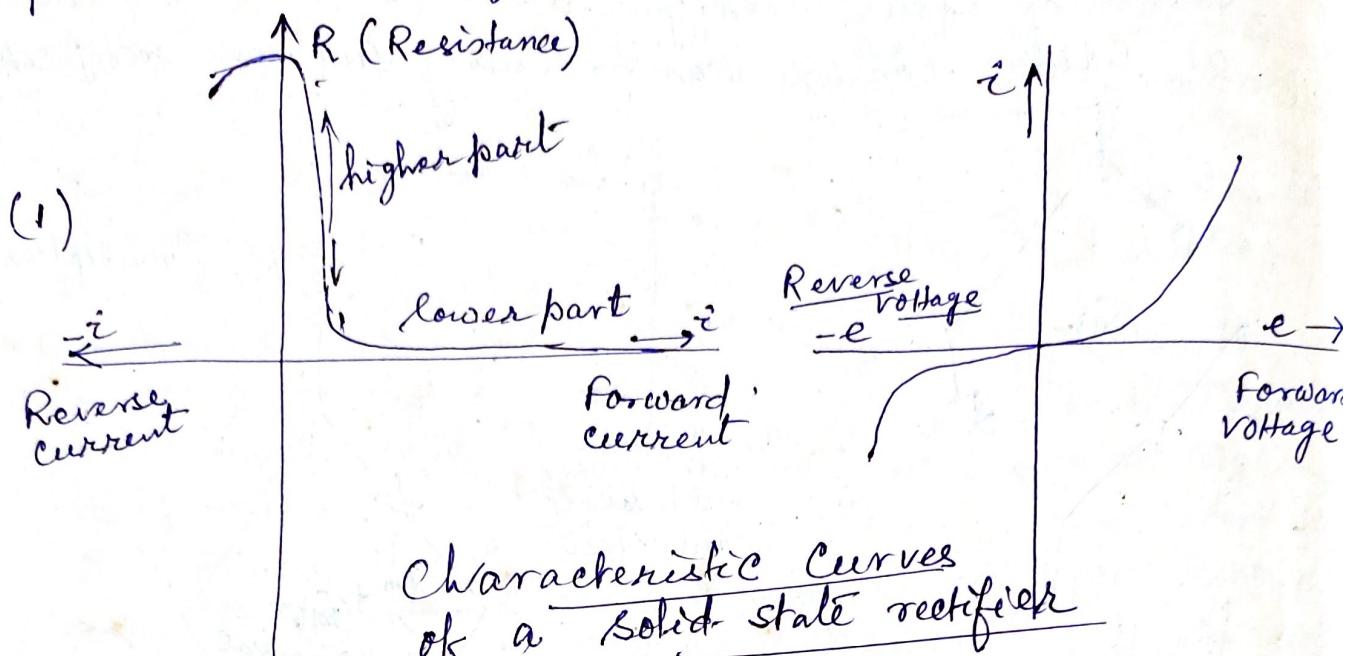
The meter indicates a steady deflection proportional to the average value of current.

For these applications, the current taken by the voltmeters should not exceed 1 mA (say) in order that there are no loading effects. Rectifier instruments have a range of $1000 \Omega/V$ to $2000 \Omega/V$.

The bridge rectifier produces a pulsating unidirectional current through the meter movement over complete cycle of input voltage. Because of inertia of moving coil, the meter will indicate a steady deflection proportional to the average value of the current. Due to rms values of current and voltage (alternating), the meter scale is calibrated in terms of rms value of a sinusoidal waveform.

$$I_{\text{av.}} = \frac{2}{\pi} I_m = 0.636 I_m, I_{\text{rms}} = \frac{1}{\sqrt{2}} I_m = 0.707 I_m.$$

The ideal rectifier element should have zero forward and infinite reverse resistance. In ~~its practice~~, practice, the rectifier is a non-linear device.



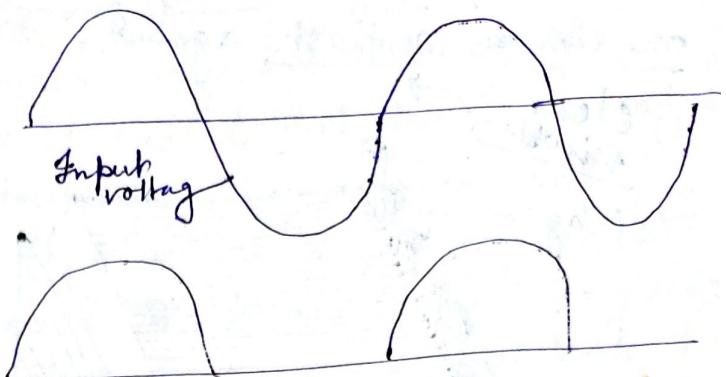
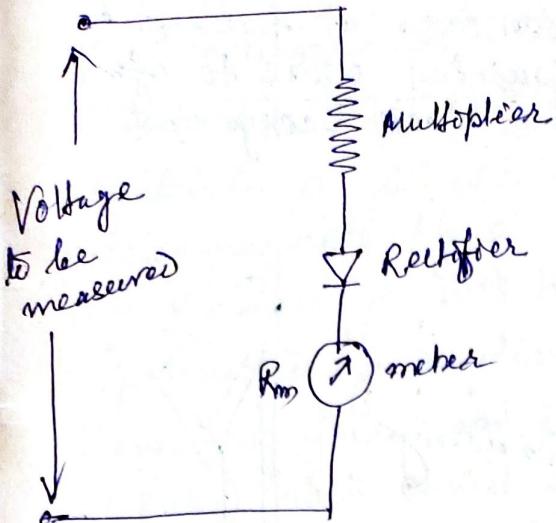
~~At first~~ At low values of forward current, the characteristics of the rectifier will be ~~be~~ an extremely non-linear part. The resistance is large as compared to the resistance at higher current values. Due to this cause, in a low-range Voltmeter, the lower part is crowded.

The higher resistance in the early part of characteristic curve sets a limit on sensitivity which can be obtained in ~~micro~~ ammeters and voltmeters.

~~The major drawback of rectifier-type ac instrument is that the resistance of rectifier element changes with varying temperature.~~

The meter accuracy is usually satisfactory under normal operating conditions at room temperature and is generally on the order of $\pm 5\%$ of full scale reading for sinusoidal waveforms. The total resistance of measuring circuit causes ~~the~~ error in reading at ~~very much~~ higher or lower temperatures. For large temperature variations, ~~the~~ meter should be enclosed in a temperature-controlled case.

~~✓ of frequency also affects the operation of rectifier elements. The rectifier exhibits capacitive properties and tends to bypass the higher frequencies. Meter reading may be in error by as much as 0.5% decrease for every 1-Khz increase.~~



$$V_{av} = \frac{1}{\pi} V_m = \text{[redacted]} V_{rms} \quad V_{rms} = \frac{V_m}{\sqrt{2}}$$

$$\text{Form factor} = \frac{V_{rms}}{V_{av}} = \frac{1}{0.45} = 2.22.$$

Frequency Meters.

The two main frequency meters are

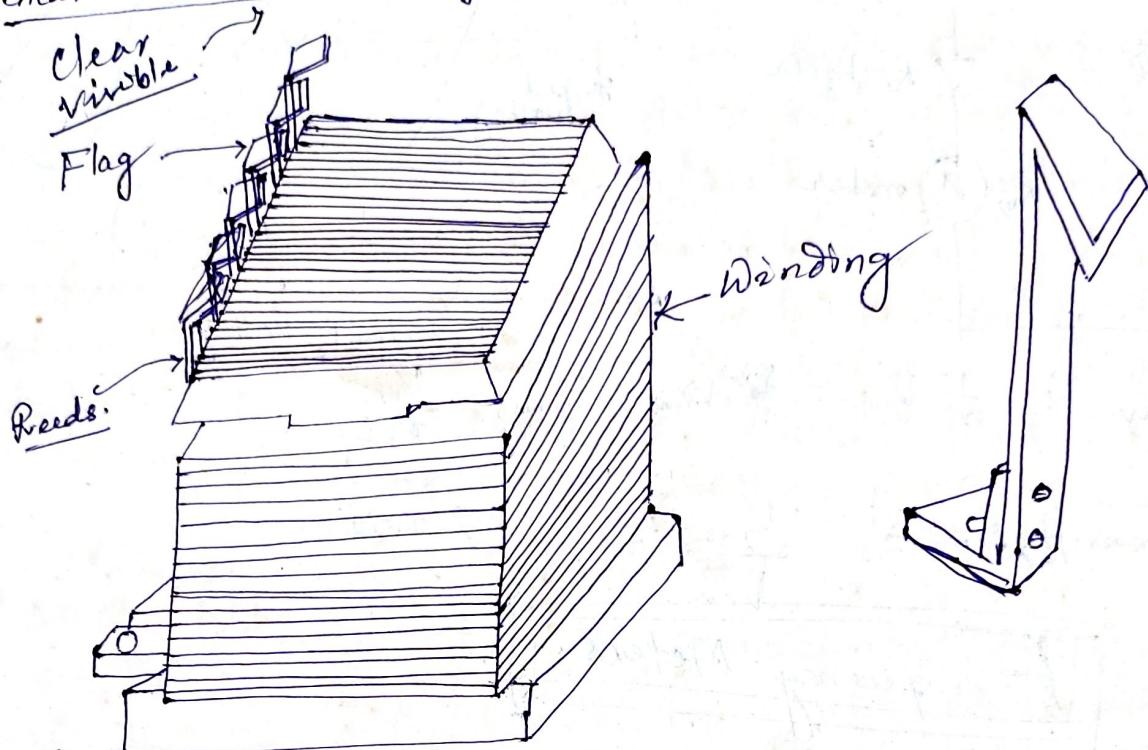
- i) Mechanical Resonance type
- ii) Electrical Resonance type

Mechanical Resonance Frequency Meter: - frequency meters.

Construction: - This meter consists of a number of reeds made by thin steel strips and an electromagnetic. These reeds are placed ~~in a row alongside~~ close to the electromagnet in a row alongside. The electromagnet has a laminated iron core and its coil is connected in series with a resistance, across the supply whose frequency is to be measured. All the reeds are not similar to each other. A reed is of ~~promote~~ width 4mm and 1/2 mm thick. The reeds ~~are~~ either slightly different dimensions to carry different weights of flags at their tops.

The natural frequency of vibration of the reeds depends upon ~~the~~ their weights and dimensions. The reeds are arranged in ascending order of natural frequency frequencies of vibration as 1/2 Hz difference frequency. The natural frequency of 1st reed may be 47 Hz, of 2nd 47.5 Hz, of the next 48 Hz and so on. The reeds are fixed at the bottom ends and are free at the top ends.

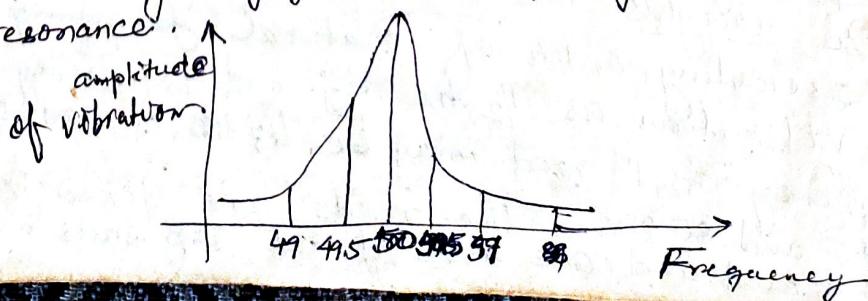
For the frequency meter, the reeds are arranged to be viewed over at free end to serve as a flag. The flags are painted white to afford maximum contrast against their black background.



(Vibrating reed type frequency meter)

Operation. When the frequency meter is connected across the supply whose frequency is to be measured, the coil of electromagnet carries an alternating current i . The force of attraction between reeds and electromagnet is proportional to i^2 and this force varies at twice the supply frequency. Thus the force is exerted on the reeds every half cycle.

All the reeds vibrate and ~~will be in resonance~~. The reed whose natural frequency is equal to twice the frequency of supply will be in resonance and will vibrate most. Normally the vibration of other reeds is so slight as to be unobservable. For sharpness of tuning of meters when the excitation frequency departs from the resonant frequency, the amplitude of vibration decreases rapidly and is becoming negligible for a frequency 1 to 2% away from resonance.



In the resonant condition.

The frequency of these meters is six hz say 47 Hz to 53 Hz. The frequency range of a set of reeds may be doubled in a simple manner. By polarization of electromagnets by a direct current (d.c), the range of the frequency will be doubled. The electromagnet is polarized by a direct flux and it superposed on the alternating flux and is equal ~~to~~ ^(mixed) in magnitude. The fields (a.c & d.c) will cancel each other in one half cycle and reinforce during other half cycle and so the reeds will be attracted only once in a cycle. Thus a reed whose natural frequency is 100Hz will respond to 50 Hz when the electromagnet is unpolarized.

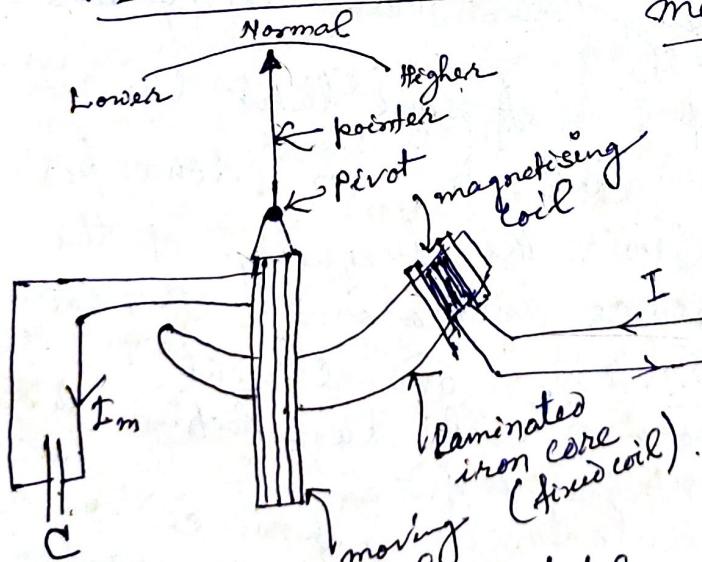
The polarization is accomplished by using a d.c winding in addition to a.c winding or by using a permanent magnet.

Advantage

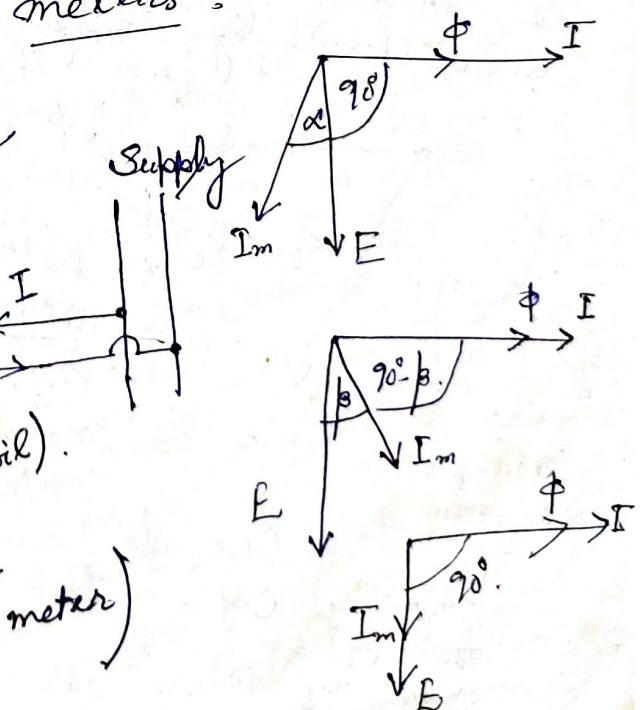
The indication in case of reed type frequency meter is independent of the waveform of the supply voltage.

Disadvantage :- The instruments cannot be much closer than half the frequency difference between adjacent reeds. Thus they can not be used for precision measurements.

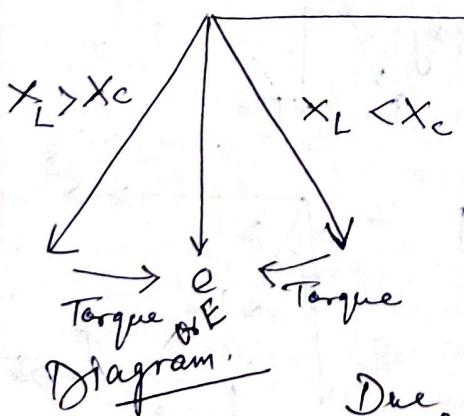
Electrical Resonance type frequency meters



(Ferrodynamic type frequency meter)



A ferrodynamometer or ferroresonance type frequency meter is meant for measuring frequencies upto about 200 Hz. The instrument consists of a horn shaped laminated core. At the end where the area of cross section of the core is large, a magnetising coil is provided. This coil is connected to the source whose frequency is to be measured. The moving system has a coil and the ends of the coil are connected to a capacitor. The moving coil and a pointer are connected to a pivot. The pointer moves on a calibrated scale. The instrument does not have mechanism for producing control torque. The magnetising coil produces a flux along the laminated core. This flux passes through the moving coil and induces an emf. Since the coil circuit is closed, a current flows through the coil. This current interacts with the flux and produces a flux torque on the moving coil which makes the coil to move to a position until the torque produced on the coil becomes zero. When the coil moves to a position such that $X_C = X_L$ (i.e.), the moving coil circuit is under resonance, the torque becomes zero. The direction of magnitude of torque produced can be easily understood by studying the vector diagrams shown.



Due to this, a torque is produced on the moving coil. This moves in the direction of reducing X_L i.e. the coil moves towards a position where the area of core reduces such that X_L decreases.

Assume that the moving coil is in some position.

Let the frequency of the source increases. This causes $X_L > X_C$ and current in the moving coil lags behind the voltage induced in the coil.

Due to this, a torque is produced on the moving coil. This moves in the direction of reducing X_L i.e. the coil moves towards a position where the area of core reduces such that X_L decreases.

This movement continues until X_L becomes equal to X_C . Under this condition, the moving coil circuit is under electrical series resonance. The current in the moving coil circuit lags the flux by 90° and the torque on the moving coil reduces to zero. On the other hand when frequency decreases, X_L decreases and X_C increases. So after the coil moves towards the larger area of the core such that L increases until $X_L = X_C$. Thus the moving coil circuit is always kept at series resonance whatever may be the frequency.

$$L = \frac{\mu N^2 A}{l}$$

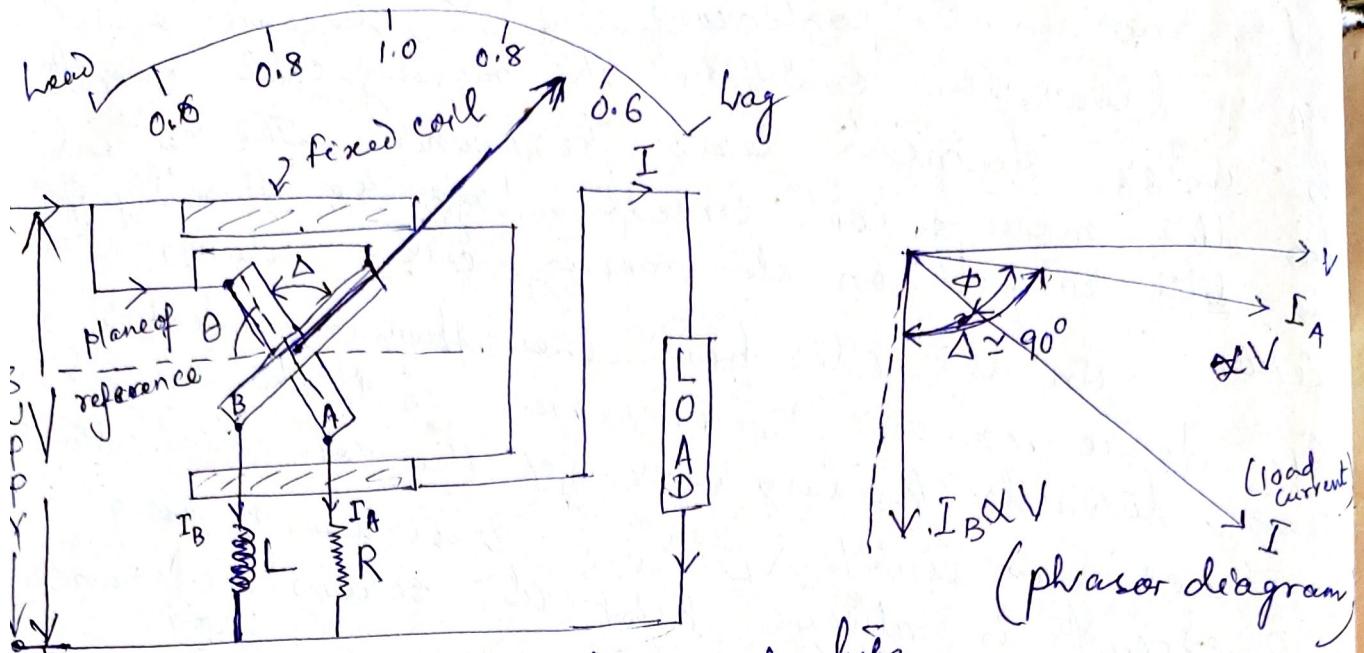
A = Area of cross section

Power factor Meters

Power factor can be calculated by the relation $\cos \phi = \frac{P}{VI}$. Like Wattmeter, power factor meter consists of a current circuit and a pressure circuit. The current circuit carries the current in the circuit whose power factor is to be measured. The pressure circuit is connected across the circuit whose power factor is to be measured and is usually split up into two parallel paths - one inductive and other non-inductive. The deflection of the instrument depends upon the phase difference between the main current and the currents in the two paths of the pressure circuit. It is upon phase angle or power factor of the circuit. The moving system is perfectly balanced by two opposing forces and there is no need for a controlling torque. There are two types of power factor meters. They are

1. Electrodynamometer type
2. Moving iron type

Standings



(Single phase electrodynamic type power factor meter)

It consists of a fixed coil which acts as current coil. This coil is split up into two parts and carries the current of the circuit under test. Two identical pressure coils A, & B pivoted on a spindle constitute on the moving system. Pressure coil A has a non-inductive resistance R connected in series with it and coil B has a highly inductive choke coil L connected in series with it. The values of L and R are so adjusted that the two coils carry the same value of current at normal frequency, i.e. $R = \omega L$. Current through the coil A is in phase with the circuit voltage while the current through coil B lags behind the voltage by a angle Δ ($\approx 90^\circ$). The angle between the planes of the coils is also made equal to Δ . There is no controlling device.

Construction to moving coils are made through thin silver or gold ligaments which are extremely flexible and thus give a minimum control effect on the moving system. There will be two deflecting torques acting on A and B. The windings are so arranged that the torques are opposite in direction, Thus The pointer will take up a position where these two torques are equal.

For case of lagging power factor of $\cos \phi$,

Deflecting torque, $T_A = KVI \cos \theta \cos \phi \sin \theta$

θ = angular deflection from the plane of reference.

M_{max} = maximum mutual inductance between two coils

(torque in clockwise direction)

$$T_B = KVI M_{max} \cos(90^\circ - \phi) \cdot \sin(90^\circ + \theta)$$

$$= KVI M_{max} \sin \phi \cos \theta$$

(torque in anticlockwise direction)

The two coils will take up such a position that the two torques are equal.

At equilibrium, $T_A = T_B$

$$\Rightarrow KVI M_{max} \cos \phi \sin \theta$$

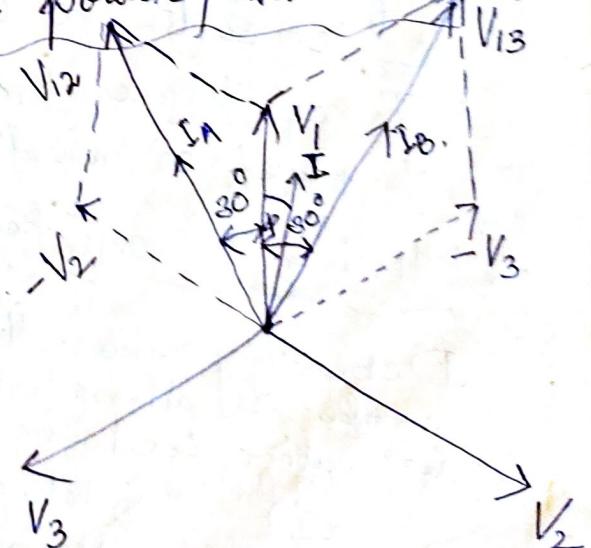
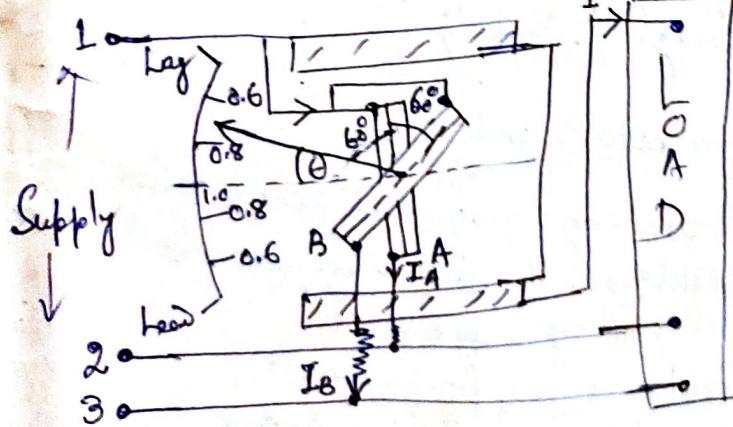
$$= KVI \sin \phi \cdot \cos \theta$$

$$\Rightarrow \tan \phi = \tan \theta$$

$$\Rightarrow \phi = \theta$$

Therefore, the deflection gives the measurement of phase angle of the circuit. The scale of the instrument can be calibrated directly in terms of power factor. For harmonics supply frequency, it will give rise to serious errors in the indication due to change of value of choke coil.

∴ Three phase power factor meter :-



This meter is only useful for balanced loads. The moving coils are so placed that the angle between their planes is 120° . They are connected across the two different phases of supply circuit. Each coil has series resistance.

ϕ = phase angle of the circuit

θ = angular deflection from the plane of reference.

$$V_1 = V_2 = V_3 = V$$

$$\text{Torque, } T_A = KV_1 I M_{\max} \cos(30^\circ + \phi) \cdot \sin(60^\circ + \theta)$$

$$= \sqrt{3} K VI M_{\max} \cdot \cos(30^\circ + \phi) \cdot \sin(60^\circ + \theta)$$

$$T_B = \sqrt{3} K VI M_{\max} \cdot \cos(30^\circ - \phi) \cdot \sin(120^\circ + \theta)$$

$$T_A = T_B \quad (\text{acting opposite directions})$$

$$\Rightarrow \cos(30^\circ + \phi) \cdot \sin(60^\circ + \theta) = \cos(30^\circ - \phi) \cdot \sin(120^\circ + \theta)$$

$$\Rightarrow \frac{\cos(30^\circ + \phi)}{\cos(30^\circ - \phi)} = \frac{\sin(120^\circ + \theta)}{\sin(60^\circ + \theta)} = \frac{\sin(60^\circ - \theta)}{\sin(60^\circ + \theta)}$$

$$\Rightarrow \frac{2 \cos 30^\circ \cos \phi}{2 \sin 30^\circ \sin \phi} = \frac{2 \sin 60^\circ \cos \theta}{2 \cos 60^\circ \sin \theta}$$

$$\Rightarrow - \frac{\sqrt{3}}{2} \times \frac{2}{1} \cot \phi = - \frac{\sqrt{3}}{2} \times \frac{2}{1} \cot \theta$$

$\Rightarrow \phi = \theta$ Therefore Thus the angular deflection of the pointer from the plane of reference is equal to the phase angle of the circuit to which meter is connected.

Three phase ~~frequency~~ power factor meter is independent of waveforms and frequency of supply.

Moving Iron Power factor meters :-

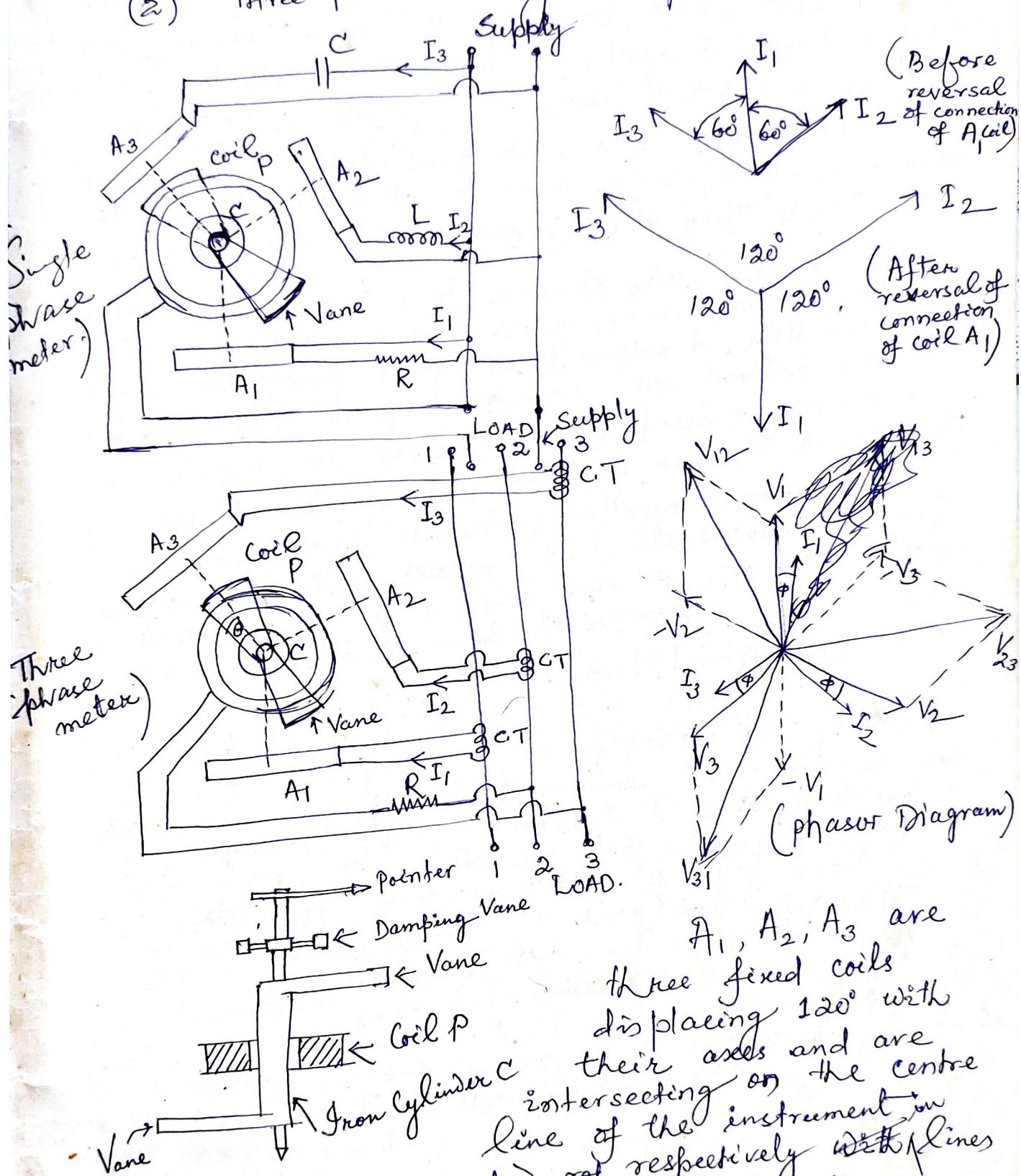
Depending upon the rotating magnetic field and number of alternating fields, these instruments can be categorised into two. They are

- (i) Rotating field power factor meter
- (ii) Alternating field power factor meter.

→ : Rotating Field Power Factor Meter :

According to supply it is of two types.

- (1) Single phase moving iron power factor meter.
- (2) Three phase moving iron power factor meter



A_1 , A_2 , A_3 are three fixed coils displacing 120° with their axes and are intersecting on the centre line of the instrument respectively with lines.

They are connected to 1, 2, 3 of a three phase supply. Usually current transformers are used for the purpose.

P is a fixed coil connected in series with a high resistance ' R ' across one pair of lines.

There is an iron cylinder C inside coil P. Two sector shaped iron Vanes V are fixed to this cylinder. The two Vanes are 180° apart in space. The spindle carries damping Vanes and a pointer. There are no control springs.