PKAIET BARGARH

3^{RD} SEMESTER ELECTRICAL ENGINEERING

TH-2 (CIRCUIT AND NETWORK THEORY)

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I.BASIC CONCEPTS

An Electric circuit is an interconnection of various elements in which there is at least one closed path in which current can flow. An Electric circuit is used as a component for any engineering system.

Elements of an Electric circuit:

An Electric circuit consists of following types of elements.

Active elements:

Active elements are the elements of a circuit which possess energy of their own and can impart it to other element of the circuit.

Active elements are of two types

a)Voltage source b)Current source

A Voltage source has a specified voltage across its terminals, independent of current flowing through it.
A current source has a specified current through it independent of the voltage appearing across it.

Passive Elements:

The passive elements of an electric circuit do not possess energy of their own. They receive energy from the sources. The passive elements are the resistance, the inductance and the capacitance.When electrical energy is supplied to a circuit element, it will respond in one and more of the followingways.

If the energy is consumed, then the circuit element is a pure resistor.

If the energy is stored in a magnetic field, the element is a pure inductor.

And if the energy is stored in an electric field, the element is a pure capacitor.

Linear and Non-Linear Elements.

Linear elements show the linear characteristics of voltage & current. That is its voltage-current characteristics are at all-times a straight-line through the origin.

For example, the current passing through a resistor is proportional to the voltage applied through its and the relation is expressed as $V \propto I$ or $V = IR$. A linear element or network is one which satisfies the principle of superposition, i.e., the principle of homogeneity and additivity.

Resistors, inductors and capacitors are the examples of the linear elements and their properties do not change with a change in the applied voltage and the circuit current.

Non linear element's V-I characteristics do not follow the linear pattern i.e. the current passing through it does not change linearly with the linear change in the voltage across it. Examples are the semiconductor devices such as diode, transistor.

Bilateral and Unilateral Elements:

An element is said to be bilateral, when the same relation exists between voltage and current for the current flowing in both directions.

Ex: Voltage source, Current source, resistance, inductance & capacitance.

The circuits containing them are called bilateral circuits.

An element is said to be unilateral, when the same relation does not exist between voltage and current when current flowing in both directions. The circuits containing them are called unilateral circuits.

Ex: Vacuum diodes, Silicon Diodes, Selenium Rectifiers etc.

The circuits containing them are called unilateral circuits.

Lumped and Distributed Elements

Lumped elements are those elements which are very small in size & in which simultaneous actions takes place. Typical lumped elements are capacitors, resistors, inductors.

Distributed elements are those which are not electrically separable for analytical purposes.

For example a transmission line has distributed parameters along its length and may extend for hundreds of miles.

Types of Sources:

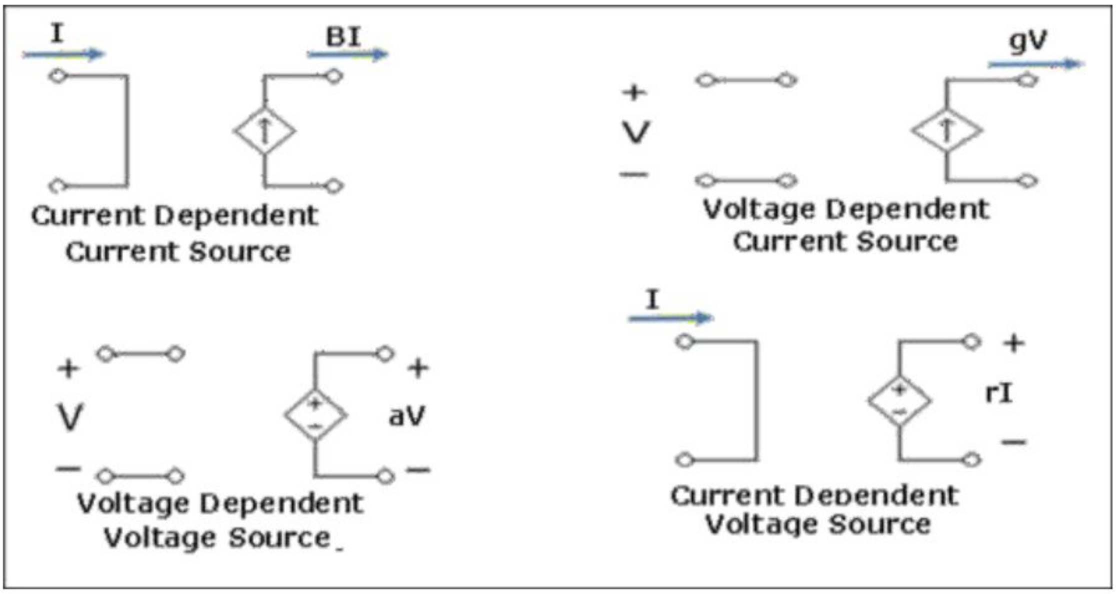
Independent & Dependent sources:

If the voltage of the voltage source is completely independent source of current and the current of the current source is completely independent of the voltage, then the sources are called as independent sources.

The special kind of sources in which the source voltage or current depends on some other quantity in the circuit which may be either a voltage or a current anywhere in the circuit are called Dependent sources or Controlled sources.

There are four possible dependent sources:

a. Voltage dependent Voltage source
b. Current dependent Current source
c. Voltage dependent Current source
d. Current dependent Current source



The constants of proportionalities are written as B, g, a, r in which B & a has no units, r has units of ohm & g units of mhos.

Independent sources actually exist as physical entities such as battery, a dc generator & analternator. But dependent sources are used to represent electrical properties of electronic devices such as OPAMPS & Transistors.

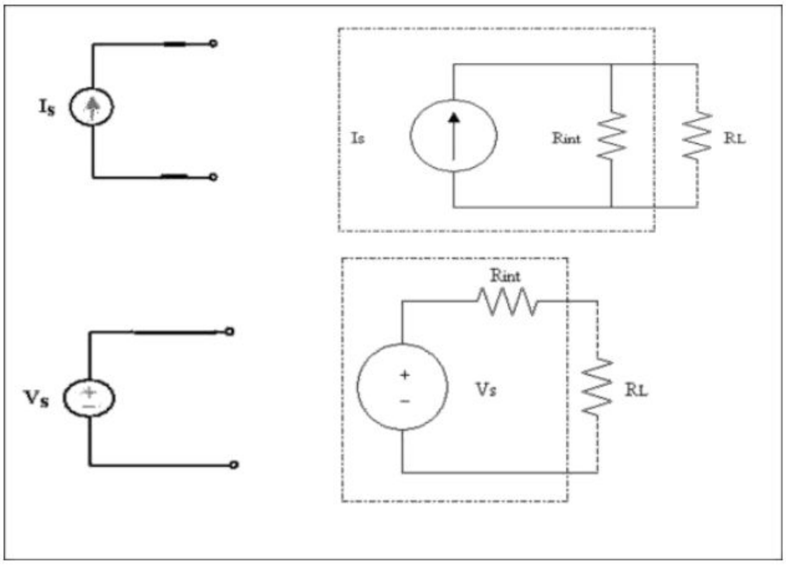
Ideal & Practical sources:

1. An ideal voltage source is one which delivers energy to the load at a constant terminal voltage, irrespective of the current drawn by the load.

2. An ideal current source is one, which delivers energy with a constant current to the load, irrespective of the terminal voltage across the load.

3. A Practical voltage source always possesses a very small value of internal resistance r. The internal resistance of a voltage source is always connected in series with it & for a current source; it is always connected in parallel with it. As the value of the internal resistance of a practical voltage source is very small, its terminal voltage is assumed to be almost constant within a certain limit of current flowing through the load.

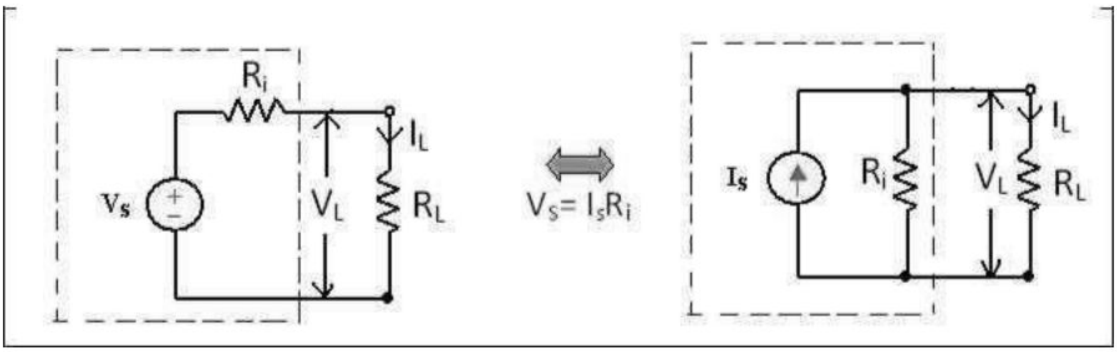
4. A practical current source is also assumed to deliver a constant current, irrespective of the terminal voltage across the load connected to it.



Source transformation:

A current source or a voltage source drives current through its load resistance and the magnitude of the current depends on the value of the load resistance.

Consider a practical voltage source and a practical current source connected to the same load resistance RL as shown in the figure



R1's in figure represents the internal resistance of the voltage source VS and current source IS.

Two sources are said to be identical, when they produce identical terminal voltage VL and load current IL.

The circuit in figure represents a practical voltage source & a practical current source respectively, with load connected to both the sources.

The terminal voltage VL and load current IL across their terminals are same.

Hence the practical voltage source & practical current source shown in the dotted box of figure are equal.

The two equivalent sources should also provide the same open circuit voltage & short circuit current.

Fromfig(a)

IL=$\frac{Vs}{R+R_L}$

Fromfig(b)

IL= I $\frac{r}{R+R_L}$

$\therefore \frac{Vs}{R+R_L}$ IR$\frac{IR}{R+R_L}$

VS=IR or I=$\frac{Vs}{R}$

Henceavoltagesource Vsinserieswithitsinternalresistance Rcanbeconvertedintoacurrentsource $I = \frac{Vs}{R}$, with its internal resistance R connected in parallel with it. Similarly a current source I In parallel with its internal resistance R can be converted in to a voltage source V=IR in series with its internal resistance R.

Parameters:

1. **Resistance:**

Resistance is that property of a circuit element which opposes the flow of electric current and in doing so converts electrical energy in to heat energy.

It is the proportionality factor in ohm's law relating voltage and current.

Ohm's law states that the voltage drop across a conductor of given length and area of cross section is directly proportional to the current flowing through it.

$$V \alpha I$$

$$V=Ri$$

$$i=\frac{V}{R}=GV$$

Where the reciprocal of resistance is called conductance G. The unit of resistance is ohm and the unit of conductance is mho or Siemens.

When current flows through any resistive material, heat is generated by the collision of electrons with other atomic particles. The power absorbed by the resistor is converted to heat and is given by the expression

P=vi=i2R where i is the current through the resistor in amps, and v is the voltage across the resistor in volts.

Energy lost in a resistance in time t is given by

$$W=\int_0^t p\,dt = pt = i^2Rt = \frac{v2}{R}$$

2. **Inductance:**

Inductance is the property of a material by virtue of which it opposes any change of magnitude and direction of electric current passing through conductor. A wire of certain length, when twisted into acoil becomes a basic conductor. A change in the magnitude of the current changes the electromagneticfield.

This induces a voltage across the coil according to Faradays laws of Electromagnetic Induction.

Induced Voltage V=L $\frac{di}{dt}$

V= Voltage across inductor in volts

I= Current through inductor in amps

di= $\frac{1}{L}$ vdt

Integratingbothsides,

$$\int_0^t di = \frac{1}{L}\int_0^t vdt$$

Power absorbed by the inductor P=VI=Li $\frac{di}{dt}$

Energy stored by the inductor

W= $\int_0^t P\,dt$ = $\int_0^t Li\,\frac{di}{dt}$ dt= $\frac{Li^2}{2}$

W= $\frac{Li^2}{2}$

Conclusions:

1) V=L$\frac{di}{dt}$

The induced voltage across an inductor is zero if the current through it is constant. That means an inductor acts as short circuit to dc.

2) For minute change in current within zero time (dt = 0) gives an infinite voltage across the inductor which is physically not at all feasible. In an inductor, the current cannot change abruptly. An inductor behaves as open circuit just after switching across dc voltage.

3) The inductor can store finite amount of energy, even if the voltage across the inductor is zero.

4) A pure inductor never dissipates energy, it only stores it. Hence it is also called as a non–dissipative passive element. However, physical inductor dissipates power due to internal resistance.

3. Capacitance:

1) A capacitor consists of two metallic surfaces or conducting surfaces separated by a dielectric medium.
2) It is a circuit element which is capable of storing electrical energy in its electric field.
3) Capacitance is its capacity to store electrical energy.
4) Capacitance is the proportionality constant relating the charge on the conducting plates to the potential.

Charge on the capacitor $q \propto V$

$$q=CV$$

Where `C` is the capacitance in farads, if q is charge in coulombs and V is the potential difference across the capacitor in volts.
The current flowing in the circuit is rate of flow of charge

$$i=\frac{dq}{dt} \quad \therefore i = C\frac{dv}{dt}$$

The capacitance of a capacitor depends on the dielectric medium & the physical dimensions. For a parallel plate capacitor, the capacitance

$$C=\frac{\epsilon A}{D}=\epsilon_0\epsilon_r\frac{A}{D}$$

A is the surface area of plates D is the separation between plates

ϵ is the absolute permeability of medium ϵ_0 is the absolute permeability of free space ϵ_r is the relative permeability of medium

$$i=\frac{dq}{dt}=C\frac{dv}{dt}$$

$$\frac{dv}{dt}=\frac{i}{c}$$

$$V=\frac{1}{c}\int idt$$

The power absorbed by the capacitor $P= vi = vc\frac{dv}{dt}$

Energy stored in the capacitor $W=\int_0^t Pdt=\int_0^t VC\frac{dv}{dt}dt$

$$=C\int_0^t vdv=\frac{1}{2}cv^2 \text{ Joules}$$

This energy is stored in the electric field set up by the voltage across capacitor.

Conclusions:

1. The current in a capacitor is zero, if the voltage across it is constant, that means the capacitor acts as an open circuit to dc
2. A small change in voltage across a capacitance within zero time gives an infinite current through the capacitor, which is physically impossible.

   ∴In a fixed capacitor, the voltage can not change abruptly

   ∴ A capacitor behaves as short circuit just after switching across dc voltage.
3. The capacitor can store a finite amount of energy, even if the current through it is zero.
4. A pure capacitor never dissipates energy but only stores it hence it is called non-dissipative element.

Kirchhoff`s Laws:

Kirchhoff's laws are more comprehensive than Ohm's law and are used for solving electrical networks which may not be readily solved by the latter.

Kirchhoff`s laws, two in number, are particularly useful in determining the equivalent resistance of a complicated network of conductors and for calculating the currents flowing in the various conductors.

1. **Kirchhoff`s Current Law (KCL)**

In any electrical network, the algebraic sum of the currents meeting at a point (or junction) is Zero.

That is the total current ***entering*** a junction is equal to the total current ***leaving*** that junction.

Consider the case of a network shown in Fig (a).

$I1+(-I2)+(I3)+(+I4)+(-I5)=0$

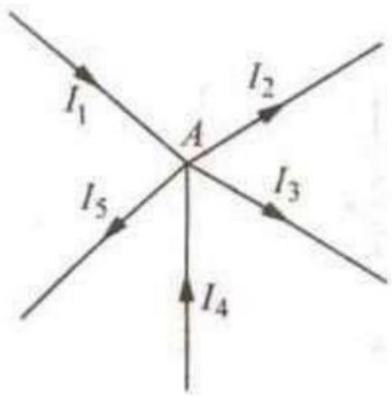
$I1+I4-I2-I3-I5=0$

Or

$I1+I4=I2+I3+I5$

Or

Incoming currents = Outgoing currents



2. Kirchhoff's Mesh Law or Voltage Law (KVL)

In any electrical network, the algebraic sum of the products of currents and resistances in each of the conductors in any closed path (or mesh) in a network plus the algebraic sum of the e.m.f.'s. in that path is zero.

That is, $\sum IR + \sum e.m.f = 0$ round a mesh

It should be noted that algebraic sum is the sum which takes in to account the polarities of the voltage drops.

That is, if we start from a particular junction and go round the mesh till we come back to the starting point, then we must be at the same potential with which we started.

Hence, it means that all the sources of emf met on the way must necessarily be equal to the voltage drops in the resistances, every voltage being given its proper sign, plus or minus.

DeterminationofVoltageSign

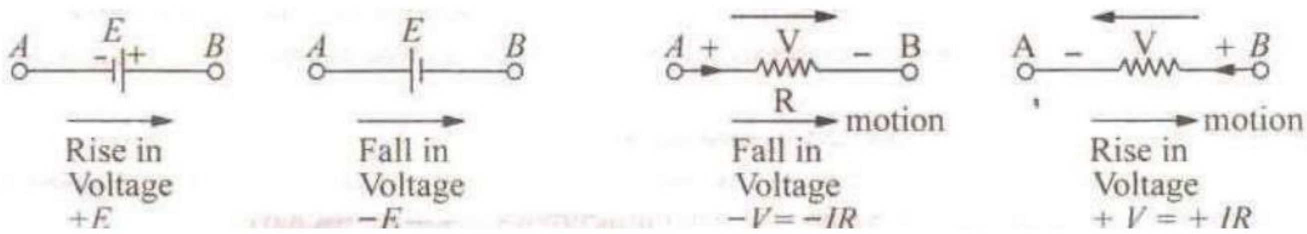
In applying Kirchhoff's laws to specific problems, particular attention should be paid to the algebraic signs of voltage drops and e.m.fs.

(a) Sign of Battery E.M.F.

A *rise* in voltage should be given a +ve sign and *a fall* in voltage a –ve sign.That is, if we go from the -ve terminal of a battery to its +ve terminal there is a *rise* in potential, hence this voltage should be given a +ve sign.

And on the other hand, we go from +ve terminal to -ve terminal, then there is *a fall* in potential, hence this voltage should be preceded by a –ve sign.

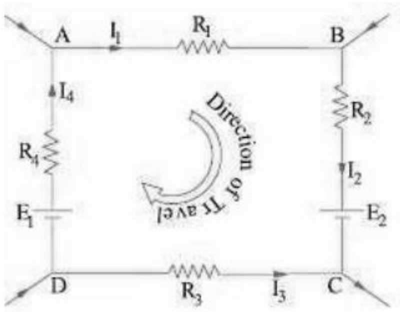
The sign of the battery e.m.f is independent of the direction
Of the current through that branch.



***(b)* Sign of *IR* Drop**

Now, take the case of a resistor (Fig.below). If we go through a resistor in the *same* direction as the
current, then there is a fall in potential because current flows from a higher to a lower potential..
Hence, this voltage fall should be taken -ve. However, if we go in a direction opposite to that of the
current, then there is a *rise* in voltage.Hence, this voltage rise should be given a positive sign.

Consider the closed path *ABCDA* in Fig.



As we travel around the mesh in the clockwise direction, different voltage drops will have the following signs :

$I1R1$ is -ve (fall in potential)
$I2R2$ is -ve (fall in potential)
$I3R3$ is + ve (rise in potential)
$I4R4$ is - ve (fall in potential)
$E2$ is - ve (fall in potential)
$E1$ is+ ve (rise in potential)

Using Kirchhoff's voltage law, we get

$$-I_1R_1 - I_2R_2 - I_3R_3 - I_4R_4 - E_2 + E_1 = 0$$

$$\text{Or} \qquad I_1R_1+I_2R_2-I_3R_3+I_4R_4=E_1-E_2$$

Assumed Direction of Current:

In applying Kirchhoff's laws to electrical networks, the direction of current flow may be assumed either clockwise or anticlockwise. If the assumed direction of current is not the actual direction, then on solving the question, the current will be found to have a minus sign.

If the answer is positive, then assumed direction is the same as actual direction. However, the important point is that once a particular direction has been assumed, the same should be used throughout the solution of the question.

Kirchhoff's laws are applicable both to d.c. and a.c. voltages and currents. However ,in the case of alternating currents and voltages, any e.m.f. of self-inductance or that existing across a capacitor should be also taken in to account.

II. NETWORK ANALYSIS

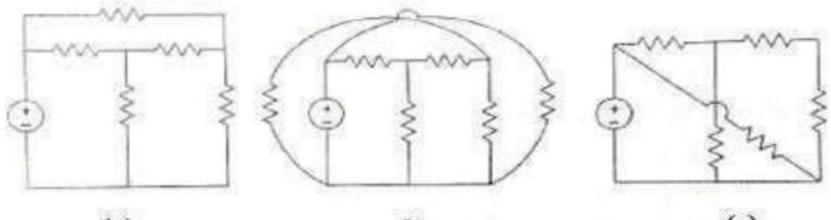
Mesh Analysis and Nodal Analysis are two important techniques used in network analysis to find out different branch currents and Node voltages. The suitability of each analysis depends mainly on the number of voltage/current sources in the given network. If the voltage sources are more Mesh analysis is suitable and if current sources are more Nodal analysis is more suitable.

Mesh Analysis:

Mesh analysis provides general procedure for analyzing circuits using mesh currents as the circuit variables. Mesh Analysis is applicable only for planar networks. It is preferably useful for the circuits that have many loops. This analysis is done by using KVL and Ohm's law.

Planar circuit: A planar circuit is one that can be drawn in a plane with no branches crossing one another. In the figure below (a) is a planar circuit.

Non-Planar circuit: A non planar circuit is one that cannot be drawn in a plane without the branches crossing one another. In the figure below (b) is a non-planar circuit and (c) is a planar circuit but appears like a non-planar circuit



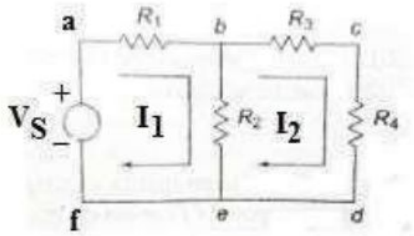
(a) (b) (c)

:

Loop: It is a closed path along the circuit elements.

Mesh: Mesh is a loop which does not contain any loop within it.

Mesh analysis with example:
Determination of mesh currents:



Solution:

Step (1): Identify the no. of meshes in the given circuit.

There are two meshes.. Mesh(1).....abef and

Mesh(2).....bcde

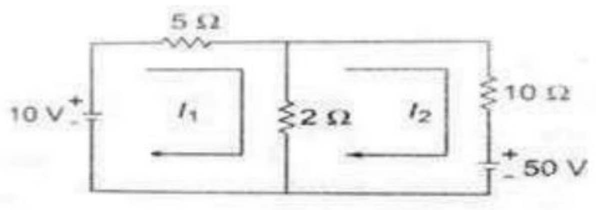
Step(2): Apply the KVL to the all meshes.

For mesh(1) by applying KVL…Vs-I1 .R1+(I1-I2).R2=0. ....(1)

For mesh(2) by applying KVL**....**I2.(R3)+I2(R4)+(I2-I1).R2=0. ....(2)

Step(3):solve the above equations for mesh currents.

Problem: Write down the mesh current equations for the circuit shown in the figure below and determine the currents I1andI2.



Solution:

By applying KVL to the two meshes, we get

5I1+2(I1-I2)=10

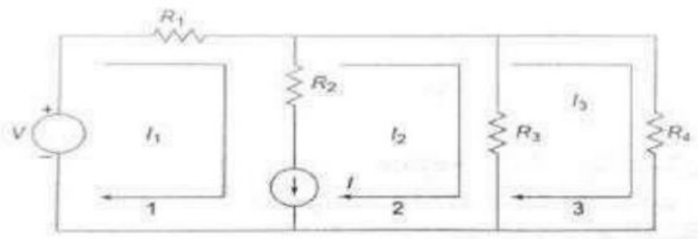
10I2+2(I2-I1)=-50.

Solving the above equations gives....I1 =0.25 A and I2 =-4.25 A. The negative sign for the Current I2 indicates that it flows in the opposite direction to that assumed in the loop two.

Super Mesh Analysis: If there is only current source between two meshes in the given network then it is difficult to apply the mesh analysis. Because the current source has to be converted in to a voltage source in terms of the current source, write down the mesh equations and relate the mesh currents to the current source. But this is a difficult approach .This difficulty can be avoided by creating super mesh which encloses the two meshes that have common current source

Super Mesh: A super mesh is constituted by two adjacent meshes that have a common currentsource.

Let us illustrate this method with the following simple generalized circuit.



Solution:

Step(1): Identify the position of current source.

Here the current source is common to the two meshes 1and 2. so, super mesh is nothing but the combination of meshes 1and 2.
Step(2): Apply KVL to super mesh and to other meshes
Applying KVL to this super mesh (combination of meshes1and2) we get

R1.I1+R3(I2–I3)=V ....(1)

Applying KVL to mesh 3, we get

R3(I3–I2)+R4.I3= 0 .... (2)

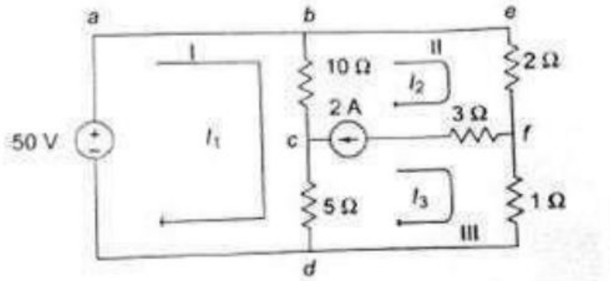
Step(3): Make the relation between mesh currents with current source to get third equation.

Third equation is nothing but the relation between I, I1 and I2 which is

I1-I2=I .... (3)

Step(4):Solve the above equations to get the mesh currents.

Example(1):Determine the current in the 5Ω resistor shown in the figure below.



Solution:

Step(1): Here the current source exists between mesh(2) and mesh(3).Hence, super mesh is the combination of mesh(2) and mesh(3). Applying KVL to the super mesh (combination of mesh2 and mesh 3 after removing the branch with the current source of 2 A and resistance of 3 Ω we get:

10(I2–I1)+2.I2+I3+5(I3–I1)=0

-15.I1+12I2+6.I3=0....................... (1)

Step(2): Applying KVL first to the normal mesh1 weget:

10(I1–I2)+5(I1–I3)=50

15.I1–10.I2–5.I3=50 ....(2)

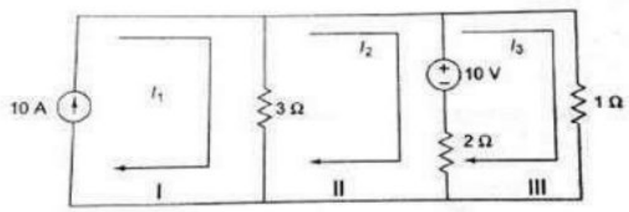
Step (3): We can get the third equation from the relation between the current source of 2 A, and currents I2 & I3 as :

I2-I3=2A...................... (3)

Step (4): Solving the above three equations for I1, I2 and I3 we get I1=19.99A I2 =17.33 A and I3=15.33A

The current in the 5Ω resistance = I1-I3= 19.99-15.33= 4.66A

Example(2): Write down the mesh equations for the circuit shown in the figure below and find out the values of the currents I1,I2 and I3



Solution: In this circuit the current source is in the perimeter of the circuit and hence the first mesh is ignored. So, here no need to create the super mesh.

Applying KVL to mesh 2 we get:

3(I2–I1)+2(I2–I3)=-10

-3.I1+5.I2 –2.I3=-10.................(1)

Next applying KVL to mesh 3 we get:

I3+ 2(I3–I2)=10

-2.I2+3.I3=-10.................(2)

And from the first mesh we observe that.......I1=10A.................(3)

And solving these three equations we get :I1=10A,I2 = 7.27 A,I3=8.18 A

Nodal analysis:

Nodal analysis provides another general procedure for analyzing circuits nodal voltages as the circuit variables. It is preferably useful for the circuits that have many no. of nodes. It is applicable for the both planar and non planar circuits. This analysis is done by using KCL and Ohm's law.

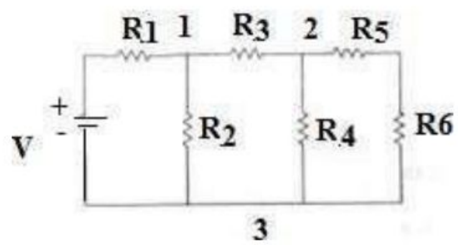
Node: It is a junction at which two or more branches are interconnected.

Simple Node: Node at which only two branches are interconnected.

Principal Node: Node at which more than two branches are

interconnected. **Nodal analysis with example:**

Determination of node voltages:

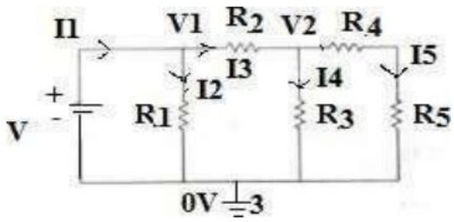


Procedure:

Step(1): Identify the no. nodes, simple nodes and principal nodes in the given circuit. Among all the nodes one node is taken as reference node. Generally bottom is taken as reference node. The potential at the reference node is 0v.

In the given circuit there are 3 principal nodes in which node (3) is the reference node.

Step(2): Assign node voltages to the all the principal nodes except reference node and assign branch currents to all branches.



Step(3): Apply KCL to those principal nodes for nodal equations and by using ohm's law express the node voltages in terms of branch current.

Applying KCL to node (1)-----1=I2+I3

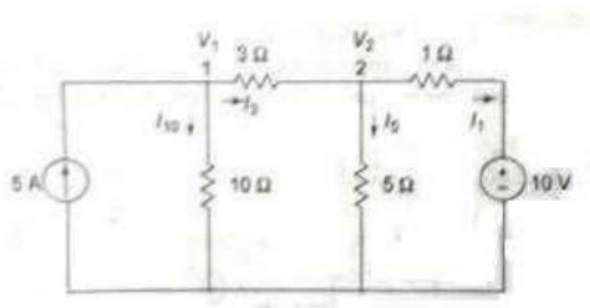
Using ohm's law, we get (V-V1)/R1=(V2-0)/R2+(V1-V2)/R3.............(1)

Applying KCL to node (2)-----3=I4 +I5

Using ohm's law, we get (V1-V2)/R3=(V4-0)/R4 +(V5-0)/R5**.............. (2)**

Step(4): Solve the above nodal equations to get the node voltages.

Example: Write the node voltage equations and find out the currents in each branch of the circuit shown in the figure below.



Solution:

The node voltages and the directions of the branch currents are assigned as shown in given figure. Applying KCL to node1, we get: 5=I10+I3

5=(V1-0)/10+(V1-V2)/3

V1(13/30) -V2(1/3)=5............(1)

Applying KCL to node 2, we get: I3=I5+I1

(V1-V2)/3=(V2-0)/5+(V2-10)/1

V1(1/3)-V2(23/15)=-10.................(2)

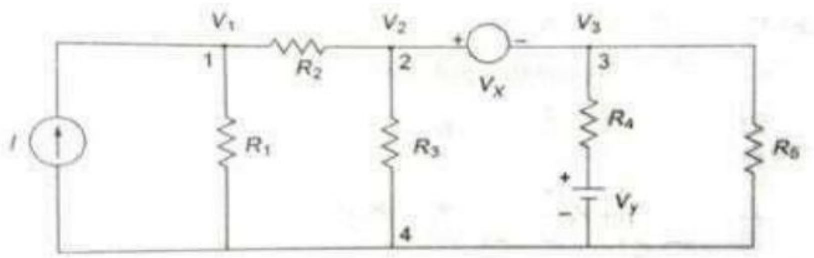
Solving these two equations for V1 and V2 we get:

V1=19.85 VandV2 =10.9Vand the currents are:
I10=V1/10=1.985A
I3 = (V1-V2)/3 = (19.85-10.9)/3 =
2.98AI5=V2/5=10.9/5=2.18A
I1=(V2-10)= (10.9-10)/1=0.9A

Super Node Analysis: If there is only voltage source between two nodes in the given network then it is difficult to apply the nodal analysis. Because the voltage source has to be converted in to a current source in terms of the voltage source, write down the nodal equations and relate the node voltages to the voltage source. But this is a difficult approach .This difficulty can be avoided by creating super node which encloses the two nodes that have common voltage source.

Super Node: A super node is constituted by two adjacent nodes that have a common voltage source.

Example: Write the nodal equations by using super node analysis.



Procedure:

Step(1): Identify the position of voltage source. Here the voltage source is common to the two nodes 2 and 3. so, super node is nothing but the combination of nodes 2and3.

Step(2): Apply KCL to super node and to other nodes.
Applying KCL to this super node(combination of meshes 2 and 3), we get

$(V2-V1)/R2 + V2/R3+(V3-Vy)/R4+ V3/R5=0$.................. (1)

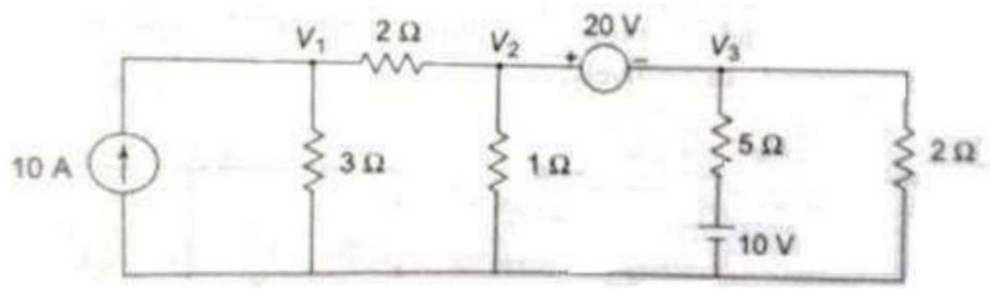
Applying KVL to node 1,we get

$I= V1/R1+(V1-V2)/R2$.................. (2)

Step(3): Make the relation between node voltages with voltage source to get third equation.

Third equation is nothing but the relation between VX, V2 andV3 which is
$V2 -V3=Vx$............... (3)
Step(4): Solve the above nodal equations to get the node voltages.

Example: Determine the current in the 5Ω resistor shown in the circuit below



Solution:

Applying KCL to node 1 :$10=V1/3+(V1-V2)/2$
$V1[1/3+1/2]-V2/2=10$

$0.83V1-0.5V2=10$..................... (1)

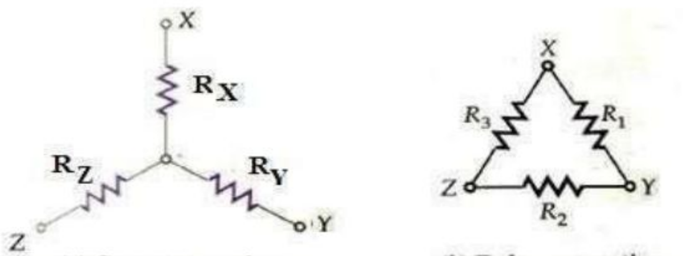
Next applying KCL to the super node
2&3: $(V2-V1)/2+V2/1+(V3-10)/5+V3/2=$
0
$-V1/2 +V2(1/2 +1)V3(1/5 +1/2)=2$
$0.5V1+1.5V2+0.7V3=2$.............................. (2)
And the third and final equation
is : $V2 -V3=20$ .... (3)

Solving the above three equations we get $V3= -8.42V$

The current through the 5 Ω resistor $I5 = [-8.42 - 10] / 5 = -3.68A$
The negative sign indicates that the current flows towards the node 3.

III.NETWORK THEOREMS

Delta to Star Transformation



(a) Star connection (b) Delta connection

The circuit configurations are identical provided the net resistances across the terminal pairs XY, YZ and ZX in both connections are the same. In Star Connection they are:

RX-Y=RX+RY ....(1)

RY-Z =RY+RZ......................... (2)

RZ-X =RZ+RX..............................(3)

Similarly in Delta connection they are:

$$R_{X-Y} = R1//(R2+R3) = \frac{R1(R2+R3)}{R1+R2+R3} \quad .... (4)$$

$$R_{Y-Z} = R2//(R1+R3) = \frac{R2(R1+R3)}{R1+R2+R3} \quad .... (5)$$

$$R_{Z-X} = R3//(R1+R2) = \frac{R3(R1+R2)}{R1+R2+R3} \quad ....(6)$$

By equating the respective equations, we get

$$RX+RY=\frac{\mathbf{R1(R2+R3)}}{\mathbf{R1+R2+R3}}\text{...........(7)}$$

$$RY+RZ=\frac{\mathbf{R2(R1+R3)}}{\mathbf{R1+R2+R3}}\text{.............(8)}$$

$$RZ+RX=\frac{\mathbf{R3(R1+R2)}}{\mathbf{R1+R2+R3}}\text{.............(9)}$$

By subtracting equation8 from equation7 given above, we get

$$RX-RZ=\frac{R1R2+R1R3}{R1+R2+R3}-\frac{R2R1+R2R3}{R1+R2+R3}\text{...........(10)}$$

Then adding this equation to equation9 above i.e.(RZ+RX) we get:

$$2RX=\frac{R1R2+R1R3-R2R1-R2R3+R3R1+R3R2}{R1+R2+R3}$$

$$=\frac{2R1R3}{R1+R2+R3}$$

$$RX=\frac{R1R3}{R1+R2+R3}$$

And in a similar way we can get:

$$RY=\frac{R1R2}{R1+R2+R3}$$

$$R2R3$$

$$RZ = \frac{}{R1+R2+R3}$$

Where RX, RY and RZ are the equivalent resistances in the Star connection corresponding to the Delta connection with resistances R1, R2 and R3.

Star to DeltaTransformation:

Now we have to get the equivalent values of R1,R2 and R3 in Delta connection in terms of the three resistances RX, RYand RZ in Star connection.

Let us use the equations we got earlier i.e. RX,RY and RZ in terms of R1,R2 and R3 and get the sum of the three product pairs i.e.RXRY+ RYRZ+RZRXas :

$$RXRY+RYRZ+RZRX = \frac{R1^2R2R3 + R2^2R1R3 +}{R3^2R1R2(R1+R2+R3)^2}$$

Now let us divide this equation by RX to get:

$$RY + RZ + \frac{RYRZ}{RX} = \frac{R1R2R3(R1+R2+R3)}{(R1+R2+R3)^2}$$

$$= \frac{R1R2R3}{(R1+R2+R3)}$$

Now substituting the value of RX=(R1+R2+R3)/R1.R3 from the earlier equations in to the above equation we get:

$$RY+RZ+\frac{RYRZ}{RX} = \frac{R1R2R3}{(R1+R2+R3)} \times \frac{(R1+R2+R3)}{R1R3} = R2$$

Then similarly dividing the same equation by RYand RZwe get the other two relations as:

$$RX+RZ+\frac{RXRZ}{RY} = R3$$

$$RY+RX+\frac{RXRY}{RZ} = R1$$

Thus we get the three equivalent resistances R1,R2and R3 in Delta connection interms of the three resistances RX,RYandRZ in Star connection as:

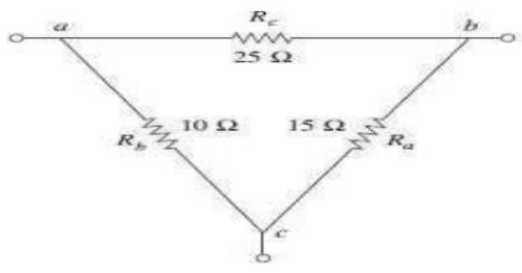
$$RY+RX+\frac{RXRY}{RZ}=R1$$

$$RY+RZ+\frac{RYRZ}{RX}=R2$$

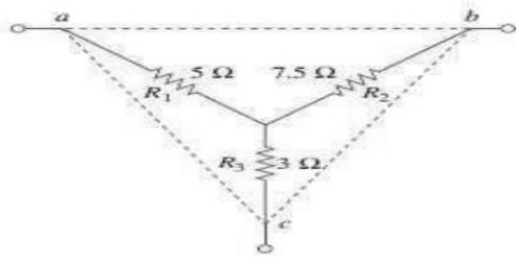
$$RX+RZ+\frac{RXRZ}{RY}=R3$$

Example problems:

1) Convert the Delta network in a)Fig.(a) to an equivalent star network



(a)



(b)

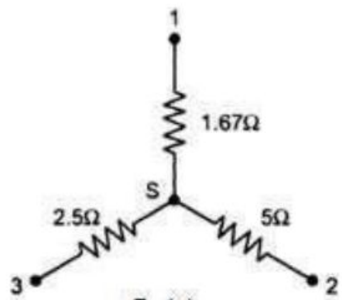
Solution:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{10 \times 25}{15 + 10 + 25} = \frac{250}{50} = 5\ \Omega$$

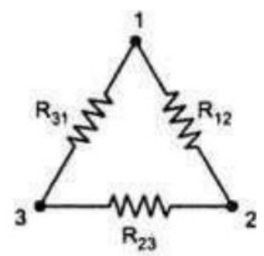
$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c} = \frac{25 \times 15}{50} = 7.5\ \Omega$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{15 \times 10}{50} = 3\ \Omega$$

2) Convert the star network in fig(a) to delta network



fig(a)



fig(b)

Solution: The equivalent delta for the given star is shown in fig(b), where

$$R_{12} = 1.67 + 5 + \frac{1.67 \times 5}{2.5} = 1.67 + 5 + 3.33 = \mathbf{10\ \Omega}$$

$$R_{23} = 5 + 2.5 + \frac{5 \times 2.5}{1.67} = 5 + 2.5 + 7.5 = \mathbf{15\ \Omega}$$

$$R_{31} = 2.5 + 1.67 + \frac{2.5 \times 1.67}{5} = 2.5 + 1.67 + 0.833$$

$$= \mathbf{5\ \Omega}$$

Superposition Theorem

Statement:

Any linear, bilateral two terminal network consisting of more than one sources,The total current or voltage in any part of a network is equal to the algebraic sum of the currents or voltages in the required branch with each source acting individually while other sources are replaced by their ideal internal resistances. (i.e. Voltage sources by a short circuit and current sources by open circuit)

Steps to Apply Superposition Principle:

1. Replace all independent sources with their internal resistances except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.
2. Repeat step1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Example: By Using the superposition theorem find I in the circuit shown in figure?

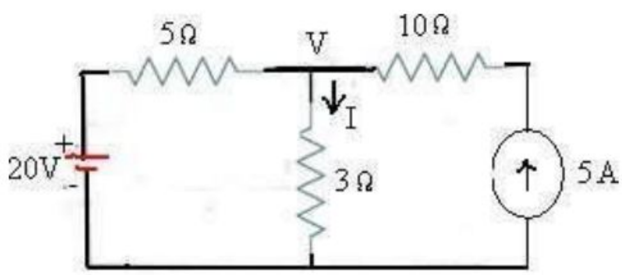


Fig.(a)

Solution: Applying the superposition theorem, the current **I2** in the resistance of 3 Ω due to the voltage source of 20V alone, with current source of 5A open circuited [as shown in the figure.1below]is given by:

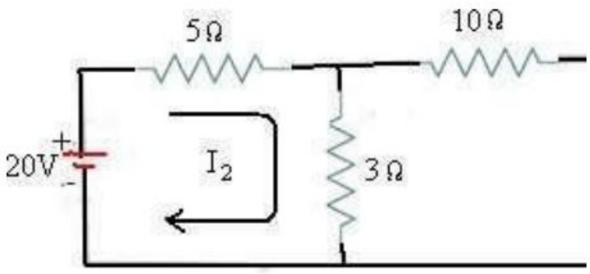


Fig1

I2=20/(5+3)=2.5A

Similarly the current I5 in the resistance of 3 Ω due to the current source of 5A alone with voltage source of 20V short circuited [as shown in the figure.2 below] is given by:

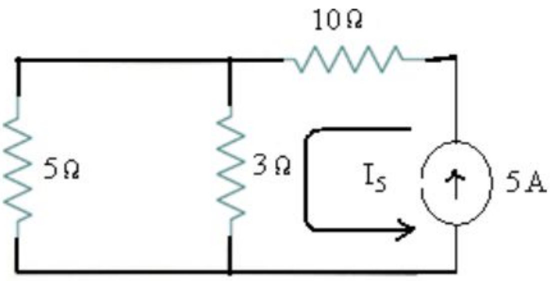


Fig.2

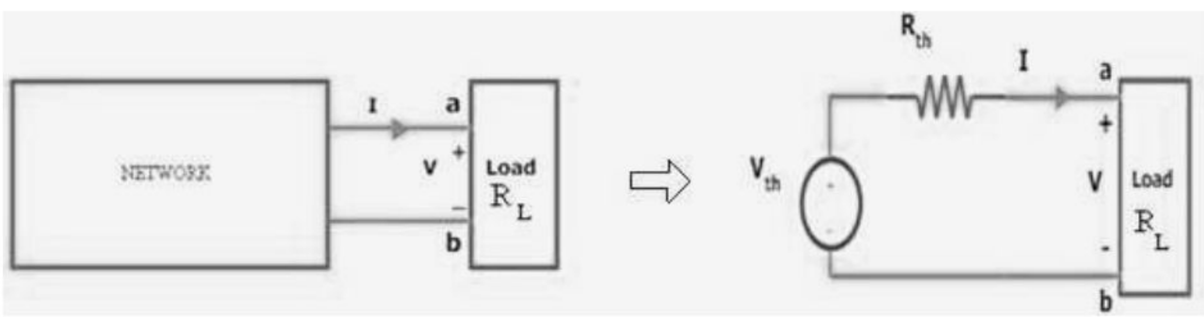
I5= 5x5/(3+5)=3.125A

The total current passing through the resistance of 3Ω is then I=I2+ I5= 2.5+3.125=**5.625A**

Thevenin's Theorem

Statement :

Any linear, bilateral two terminal network consisting of sources and resistors(Impedance), can be replaced by an equivalent circuit consisting of a voltage source in series with a resistance(Impedance). The equivalent voltage source **VTh** is the open circuit voltage looking into the terminals (with concerned branch element removed) and the equivalent resistance **RTh** while all sources are replaced by their internal resistors at ideal condition i.e. voltage source is short circuit and current source is open circuit.

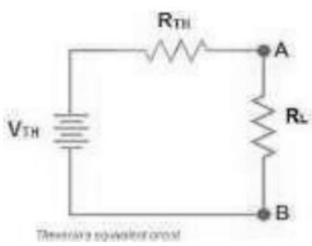


(a) (b)

Figure(a) shows a simple block representation of a network with several active/passive elements with the load resistance **RL** connected across the terminals **'a & b'** and figure (b) shows the **Thevenin equivalent circuit** with **VTh** connected across **RTh** & **RL.**

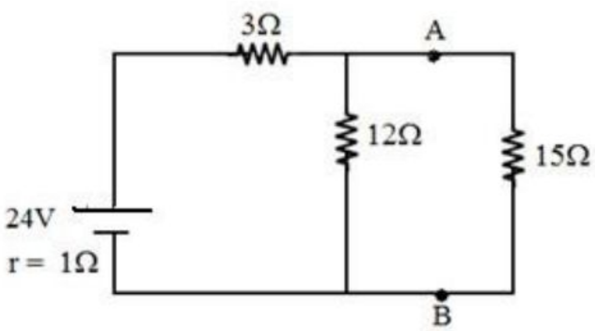
steps to find out VTh and RTh:

1. The terminals of the branch/element through which the current is to be found out are marked as say **a&b** after removing the concerned branch/element.
2. Open circuit voltage **VOC** across these two terminals is found out using the conventional network mesh/node analysis methods and this would be **VTh.**
3. **Thevenin resistance RTh** is found out by the method depending upon whether the network contains dependent sources or not.
   a. With dependent sources: **RTh=Voc/Isc**
   b. Without dependent sources: **RTh=Equivalent resistance looking in to the concerned terminals** with all voltage & current sources replaced by their internal impedances (i.e. ideal voltage sources short circuited and ideal current sources open circuited)
4. Replace the network with **VTh** in series with **RTh** and the concerned branch resistance**(or)** Load resistance across the load terminals(A&B) as shown in below fig.



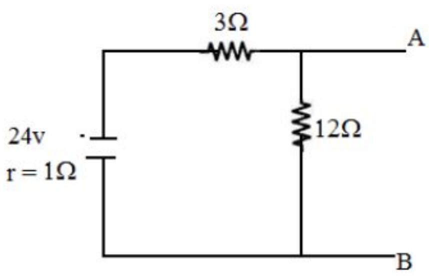
Example01- Applying thevenin theorem find the following from given figure

(i) The Current in the load resistance R_L of 15 □



Solution:(i) Finding Voc

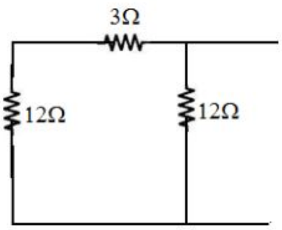
Remove15Ω resistance and find the Voltage across A and B



V_{oc} is the voltage across 12Ω resister

$$V_{oc}= \frac{24x12}{3+1+12}\square 18V^{12\square}$$

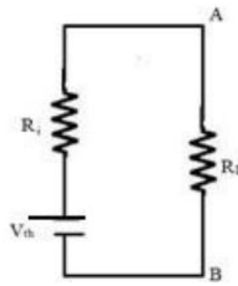
(ii) Finding R_{th}



R_{th} is calculated from the terminal A&B in to the network. The1Ω resister and 3Ω are series and then parallel 12 Ω

$R_{th}=(3+1) // 12 = 8/3$

(iii)

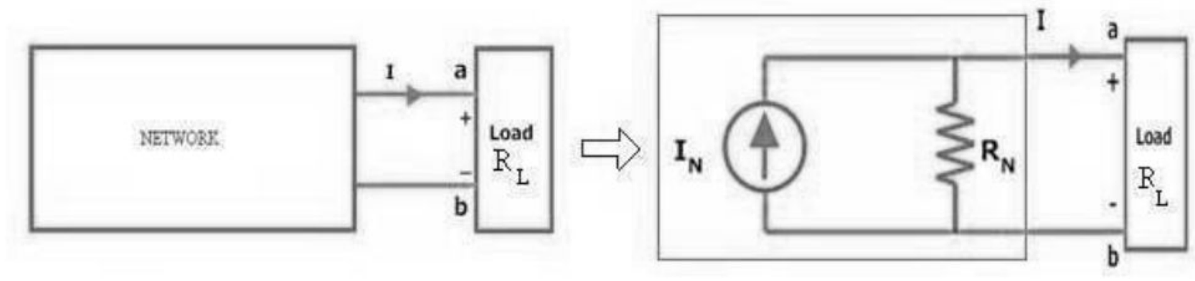


$I_L = V_{th} / (R_L+R_{th}) = 18/(15+8/3) = 1.018$ A

Norton'sTheorem

Statement:

Any linear, bilateral two terminal network consisting of sources and resistors(Impedance),can be replaced by an equivalent circuit consisting of a current source in parallel with a resistance (Impedance),the current source being the short circuited current across the load terminals and the resistance being the internal resistance of the source network looking through the open circuited load terminals.



(a) (b)

Figure(a) shows a simple block representation of a network with several active/passive elements with the load resistance **RL** connected across the terminals **'a & b'** and figure (b) shows the **Norton equivalent circuit** with **IN** connected across **RN** & **RL.**

steps to find out IN and RN:

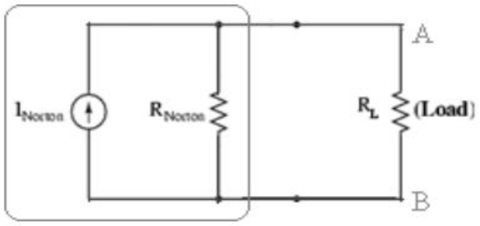
1. The terminals of the branch/element through which the current is to be found out are marked as say **a&b** after removing the concerned branch/element.
2. Open circuit voltage **VOC** across these two terminals and **ISC** through these two terminals are found out using the conventional network mesh/node analysis methods and they are same as what we obtained in Thevenin's equivalent circuit**.**

3. Next **Norton resistance RN** is found out depending upon whether the network contains dependent sources or not.

   a) With dependent sources: **RN = Voc/Isc**

b) Without dependent sources: **RN**=Equivalent resistance looking in to the concerned terminals with all voltage & current sources replaced by their internal impedances (i.e. ideal voltage sources short circuited and ideal current sources open circuited)

4. Replace the network with **IN** in parallel with **RN** and the concerned branch resistance across the load terminals (A&B) as shown in below fig

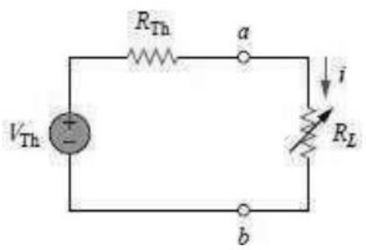


Maximum Power Transfer Theorem:

In many practical situations, a circuit is designed to provide power to a load.While for electric utilities, minimizing power losses in the process of transmission and distribution is critical for Efficiency and economic reasons, there are other applications in area ssuch as communications where it is desirable to maximize the power delivered to a load.electrical applications with electrical loads such as Loud speakers, antennas, motors etc. it would be required to find out the condition under which maximum power would be transferred from the circuit to the load.

Statement:

Any linear, bilateral two terminal network consisting of a resistance load, being connected to a dc network, receives maximum power when the load resistance is equal to the internal resistance (Thevenin's equivalent resistance) of the source network as seen from the load terminals.



According to Maximum Power Transfer Theorem, for maximum power transfer from the network to the load resistance ,**RL** must be equal to the source resistance i.e. Network's Thevenin equivalent resistance **RTh. i.e. RL=RTh**

$$I = \frac{VTH}{RTH+RL}$$

The power delivered by the

circuit to the load:

$$P = I^2 R = \frac{V_{TH}^2}{(R_{TH}+R_L)^2} R_L$$

The condition for maximum power transfer can be obtained by differentiating the above expression for power delivered with respect to the load resistance (Since we want to find out the value of **RL** for maximum power transfer) and equating it to zero as:

$$\frac{\partial P}{\partial R_L} = 0 = \frac{V_{TH}^2}{(R_{TH}+R_L)^2} - \frac{2V_{TH}^2 R_L}{(R_{TH}+R_L)^3} = 0$$

Simplifying the above equation, we get:

$$(R_{TH}+R_L) - 2R_L = 0 \Rightarrow R_L = R_{TH}$$

Under the condition of maximum power transfer, the power delivered to the load is given by:

$$P_{MAX} = \frac{V_{TH}^2}{(R_L+R_L)^2} \times R_L = \frac{V_{TH}^2}{4R_L}$$

Under the condition of maximum power transfer, the efficiency $\boldsymbol{\eta}$of the network is then givenby:

$$P_{LOSS} = \frac{V_{TH}^2}{(R_L+R_L)^2} \times R_{TH} = \frac{V_{TH}^2}{4R_L}$$

$$\boldsymbol{\eta} = \frac{\text{output}}{\text{input}} = \frac{\frac{V_{TH}^2}{4R_L}}{\left(\frac{V_{TH}^2}{4R_L} + \frac{V_{TH}^2}{4R_L}\right)} = 0.50$$

For maximum power transfer the load resistance should be equal to the Thevenin equivalent resistance (or Norton equivalent resistance) of the network to which it is connected . Under the condition of maximum power transfer the efficiency of the system is 50%.

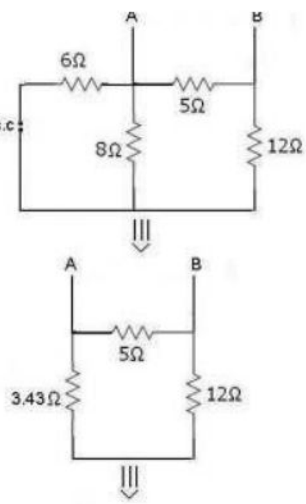
Example

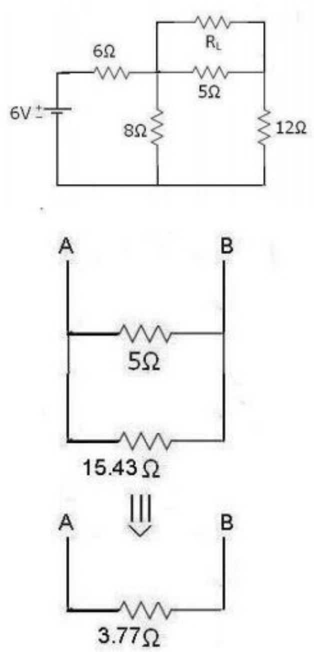
Find the value of RL for the given network below that the power is maximum? And also find the Max Power through load-resistance RL by using maximum power transfer theorem?

Solution

Fortheabovenetwork,weare goingtofind-outthevalueofunknownresistancecalled“R_L ”.Inprevious post, I already show that whenpowerismaximumthroughload-resistanceisequalstotheequivalentresistanc ebetweentwoendsofload-resistanceafterremoving.

So,forfindingload-resistance R_L. We have to find-out theequivalentresistancelikethatforthiscircui t.





So, $R_{AB}/R_L = 3.77\Omega$

Now, For finding Maximum Power through load-resistance we have to find-out the value of Vo.c. Here,Vo.c is known as voltage between open circuits. So, steps are

For this circuit using Mesh-analysis. We get

Applying Kvl in loop 1st:- 6-6I1-8I1+8I2=0

-14I1+8I2=-6 .... (1)

Again, Applying Kvl in loop 2nd:-

-8I2-5I2-12I2+8I1=0

8I1-25I2=0 .... (2)

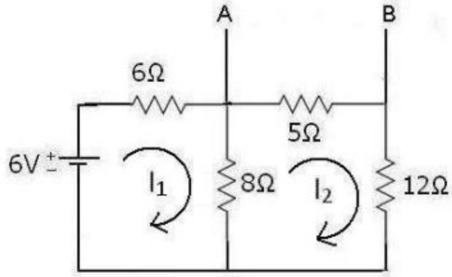
On solving ,eqn (1) &eqn (2), We get

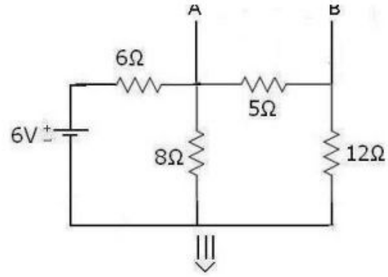
I1 = 0.524 A I2 = 0.167 A

Now, From the circuit Vo.c is VA-5I2- VB = 0

Vo.c/ VAB= 5I2 = 5X0.167 = 0.835v

So, the maximum power through the RL is given by:-





$$P_{max} = \frac{V_{o.c}^2}{4R_L}$$

$$P_{max} = \frac{0.835^2}{4X\ 3.77}$$

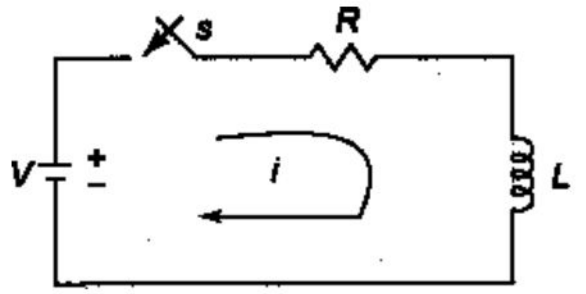
$$\boxed{P_{max} = 0.046 \text{ watt}}$$

IV.TRANSIENTS

Whenever a network containing energy storage elements such as inductor or capacitor is switched from one condition to another, either by change in applied source or change in network elements, the response current and voltage change from one state to the other state. The time taken to change from an initial steady state to the final steady state is known as the *transient period*. This response is known as *transient response* or *transients.* The response of the network after it attains a final steady value is independent of time and is called the steady-state response. The complete response of the network is determined with the help of a differential equation.

DC RESPONSE OF A R-L CIRCUIT

Consider a circuit consisting of a resistance and inductance as shown in figure. The inductor in the circuit is initially uncharged and is in series with the resistor. When the switch S is closed ,we can find the complete solution for the current. Application of kirchoff's voltage law to the circuit results in the following differential equation.



$$V = Ri + L\frac{di}{dt} \quad \ldots\ldots 1$$

$$\text{Or } \frac{di}{dt} + \frac{R}{L}i = \frac{V}{L} \quad \ldots\ldots 1.2$$

In the above equation, the current I is the solution to be found and V is the applied constant voltage. The voltage V is applied to the circuit only when the switch S is closed. The above equation is a linear differential equation of first order. comparing it with a non-homogeneous differential equation

$$\frac{dx}{dt} + Px = K \quad \ldots\ldots 1.3$$

Whose solution is

$$X = e^{-Pt}\int Ke^{+Pt}\,dt + c.e^{-Pt} \quad \ldots\ldots 1.4$$

Where c is an arbitrary constant. In a similar way,we can write the current equation as i=c $e^{-\left(\frac{R}{L}\right)t} + e^{-\left(\frac{R}{L}\right)t} \int \frac{V}{L} e^{\left(\frac{R}{L}\right)t}$dt

Hence,i = c $e^{-\left(\frac{R}{L}\right)t}$ +....$\frac{V}{R}$..................................... 1.5

To determine the value of c in equation 1.5 , we use the initial conditions .In the circuit shown in Fig.1.1, the switch s is closed at t=0. at t=0-,i.e. just before closing the switch s , the current in the inductor is zero. Since the inductor does not allow sudden changes in currents, at t=o+ just after the switch is closed, the current remains zero.

Thus at t=0,i=0

Substituting the above condition in equation c , we

have $0=\frac{V}{R}$c+

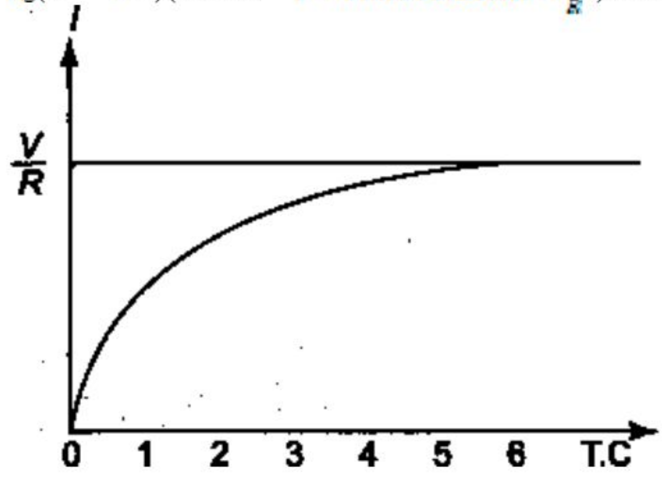
Substituting the value of c in equation c, we get

$$i= \frac{V}{R} - \frac{V}{R} e^{\frac{-Rt}{L}}$$

$$i = \frac{V}{R} (1-e^{\frac{-Rt}{L}})$$

$$i= I_o (1-e^{\frac{-Rt}{L}}) \text{ (where } I_o = \frac{V}{R})$$

i=I_o(1- $e^{\frac{-t}{\tau}}$)(where $\tau = Time\,constant = \frac{L}{R}$) 1.6



Equation 1.6 consists of two parts, the steady state part (I_o =V/R) and the transient part$I_o\, e^{\frac{-Rt}{L}}$.

When switch S is closed , the response reaches a steady state value after a time interval as shown in above figure

Here the transition period is defined as the time taken for the current to reach its final or steady state value from its initial value. In the transient part of the solution, the quantity L/R is important in describing the curve since L/R is the time period required for the current to reach its initial value of zero to the final value I_oV/R. The time =

constant of a function $I_o\, e^{\frac{-Rt}{L}}$ is the time at which the exponent of e is unity, where e is the base of the natural logarithms. The term L/R is called the time constant and is denoted by τ .

$$\text{So,}\tau = \frac{L}{R} \text{ sec}$$

Hence,the transient part of the solution is

$$i= -\frac{V}{R}e^{\frac{-Rt}{L}} = \frac{V}{R}e^{\frac{-t}{\tau}}$$

At one Time constant, the transient term reaches 36.8 percent of its initial value.

$$i(\tau)= -\frac{V}{R}e^{\frac{-\tau}{\tau}} = \frac{V}{R}e^{-1} =-0.368\frac{V}{R}$$

Similarly,

$$i(2\tau)=-\frac{V}{R}e^{-2} =-0.135\frac{V}{R}$$

$$i(3\tau)=-\frac{V}{R}e^{-3} =-0.0498\frac{V}{R}$$

$$i(5\tau)=-\frac{V}{R}e^{-5} =-0.0067\frac{V}{R}$$

After 5TC the transient part reaches more than 99 percent of its final value.

In figure A we can find out the voltages and powers across each element by using the current. Voltage across the resistor is

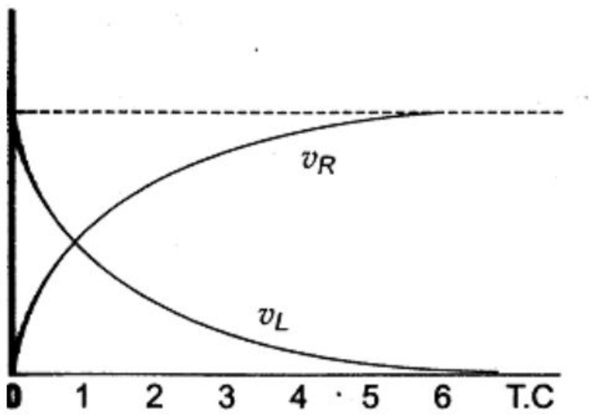
$$v_R = Ri = R\left(1-e^{\frac{-Rt}{L}}\right)\times\frac{V}{R}$$

Hence, $v_R = V\left(1-e^{\frac{-Rt}{L}}\right)$

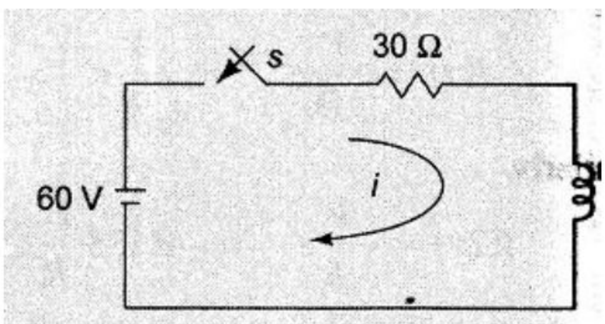
Similarly, the voltage across the inductance is

$$v_L = L\frac{di}{dt} = L\,\frac{V}{R}\times\frac{R}{L}e^{\frac{-Rt}{L}} = Ve^{\frac{-Rt}{L}}$$

The responses are shown in Figure below



Problem:



A series R-L circuit with R = 30Ω and L = 15 H has a constant voltage V = 50 V applied at t=0 as shown in above Fig..determine the current i,the voltage across resistor and across inductor.

Solution:

By applying Kirchoff's voltage Law,

we get $15\frac{di}{dt}+30i=60$

$+2i=4\frac{di}{dt}$

The general solution for a linear differential equation

is $i=c\ +e^{-pt}\int Ke^{pt}dt$

where P=2,K=4

putting the values

$i=c\ +e^{-2t}\int 4e^{2t}dt$

$i=c\ \Rightarrow +2\ e^{-2t}$

At t=0, the switch s is closed.

Since the inductor never allows sudden change in currents. At t= 0^{+} the current in the circuit is zero. Therefore at t=0^{+}, i =0

0=c+2

c=-2

Substituting the value of c in the current equation, we

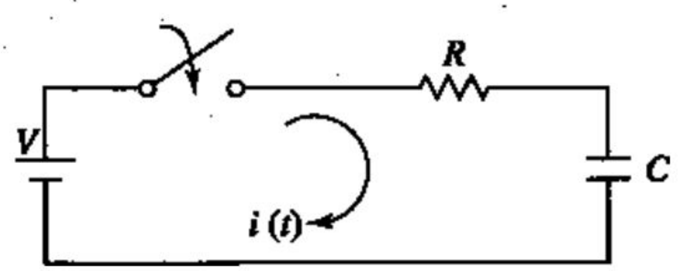
have $i=2(1-e^{-2t})A$

voltage across resistor (V_R)=iR =2(1- e^{-2t})x30=60(1- e^{-2t})v

voltage across inductor (V_L) = L $\frac{di}{dt}$=15 x $\frac{d}{dt}$2(1- e^{-2t})=30 x $2e^{-2t}$ = $60e^{-2t}$ v

DC RESPONSE OF A R-C CIRCUIT

Consider a circuit consisting of a resistance and capacitance as shown in figure. The capacitor in the circuit is initially uncharged and is in series with the resistor. When the switch S is closed at t=0 , we can find the complete solution for the current. Application of kirchoff's voltage law to the circuit results in the following differential equation.



$V = Ri + \frac{1}{C}\int i\,dt$………………1.7

By differentiating the above equation, we get

$0 = R\frac{di}{dt} + \frac{i}{C}$……………1.8

Or

$\frac{di}{dt} + \frac{1}{RC}i = 0$ ………………1.9

Equation 1.9 is a linear differential equation with only the complementary function. The particular solution for the above equation is zero. The solution for this type of differential equation is

$i = c\,e^{-\left(\frac{t}{RC}\right)}$ …………………………………..1.10

To determine the value of c in equation 1.10 , we use the initial conditions .In the circuit shown inFig. the switch s is closedat t=0. Since the capacitor does not allow sudden changes in voltage, it will act as a short circuit at t=o+ just after the switch is closed.

So the current in the circuit at t=0+ is $\frac{V}{R}$

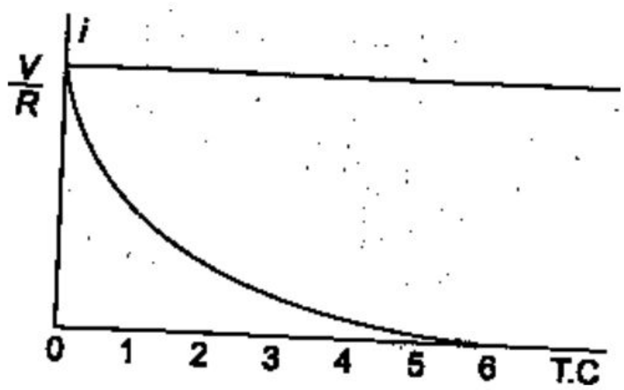
Thus at t=0, the current $i = \frac{V}{R}$

Substituting the above condition in equation c ,we have

$= c\ \frac{V}{R}$

Substituting the value of c in equation c, we get

$i = \frac{V}{R}\ e^{-\left(\frac{t}{RC}\right)}$ ……………………………………………1.11



When switch S is closed , the response decays as shown in

figure. The term RC is called the time constant and is denoted by

τ.

So, $\tau = RC$ sec

After 5TC the curve reaches 99 percent of its final value.

In figure A we can find out the voltage across each element by using the current equation.

Voltage across the resistor is

$$v_R = Ri = R \times \frac{V}{R} e^{\frac{-t}{RC}}$$

Hence, $v_R = V e^{\frac{-t}{RC}}$

Similarly, voltage across the capacitor is

$$v_C = \frac{1}{C}\int i \, dt$$

$$= \frac{1}{C}\int \frac{V}{R} e^{\frac{-t}{RC}} \, dt$$

$$= -\left(\frac{V}{RC} \times RC \, e^{\frac{-t}{RC}}\right) + c$$

$$= -V e^{\frac{-t}{RC}} + c$$

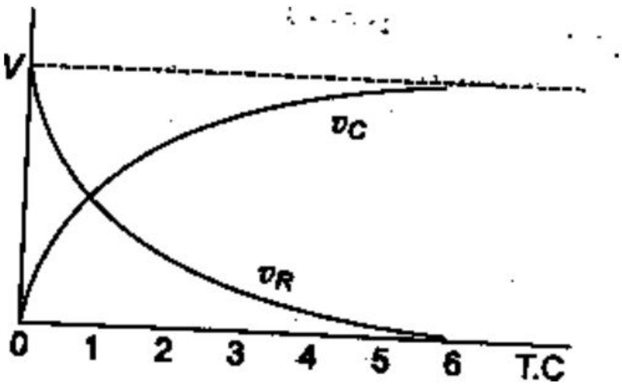
At t=0, voltage across capacitor is

zero So, c = V

And

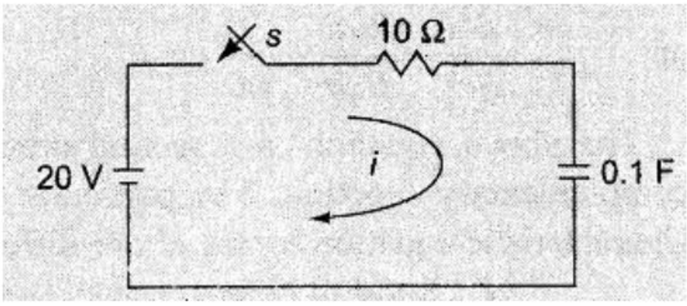
$$V_C = V\left(1 - e^{\frac{-t}{RC}}\right)$$

The responses are shown in Figure below.



Problem:

A series R-Ccircuit with R = 10Ω and C =0.1 Fhas a constant voltage V = 20 V applied at t=0 as shown inFig.determine the current i, the voltage across resistor and across capacitor.



Solution:

By applying Kirchoff's voltage Law,

we get $10i + \frac{1}{0.1}\int i\,dt = 20$

Differentiating w.r.t.t we

get $10\frac{di}{dt} + \frac{i}{0.1} = 0$

$\frac{di}{dt} + i = 0$

The solution for above equation is

$i = c\,e^{-t}$

At t=0, the switch s is closed.

Since the capacitor never allows sudden change in voltages.At $t=0^{+}$ the current in the circuit is i= V/R=20/10 =2 A

.Therefore at t=0,i=2A

the current equation is $i = 2e^{-t}$

voltage across resistor $(V_R) = iR = 2e^{-t} \times 10 = 20e^{-t}$ v

voltage across capacitor$(V_C) = V(1 - e^{\frac{-t}{RC}}) = 20(1 - e^{-t})$ V

DC RESPONSE OF A R-L-C CIRCUIT

Consider a circuit consisting of a resistance, inductance and capacitance as shown in figure. The capacitor and inductor in the circuit is initially uncharged and are in series with the resistor. When the switch S is closed at t=0 , we can find the complete solution for the current. Application of kirchoff's voltage law to the circuit results in the following differential equation.

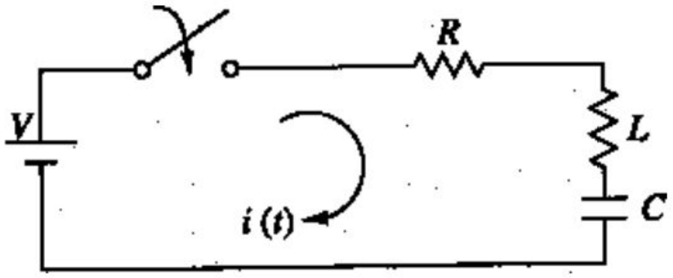


Figure1.11

V= Ri +L $+ \frac{1}{c}\int \frac{di}{dt} dt$

…………………………………………………………..1.12

By differentiating the above equation, we get

0= R $\frac{di}{dt}$+ $L\, d^2i/dt^2 + \frac{i}{C}$..1.13

Or

d^2i/dt^2+ + $\frac{R}{L}\frac{di}{dt}$ $\frac{1}{LC}$ i=0...1.14

The above equation c is a second order linear differential equation with only the complementaryfunction. The particular solution for the above equationis zero. The characteristics equation for this type of differential equation is

D^2+ D+ $\frac{R}{L}$ $\frac{1}{LC}$=0.. 1.15

The roots of equation 1.15

are D_1, D_2=- $\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$

By assuming =- K_1 and $\frac{R}{2L}$ = $\sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}}$ K_2

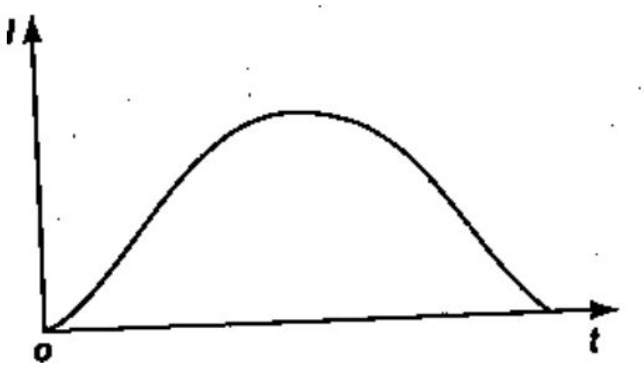
$D_1 = K_1 + K_2$ and D_2= $K_1 - K_2$

Here K_2 may be positive, negative or zero.

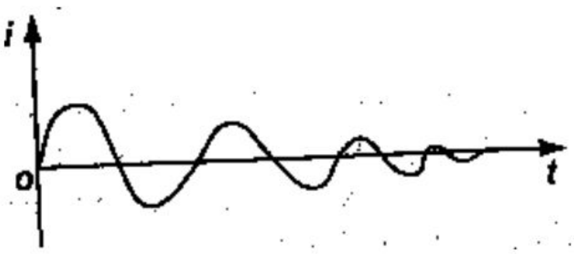
CaseI: K_2 is Po $\left(\frac{R}{2L}\right)^2 > \frac{1}{LC}$

Then , the roots are Real and Unequal and give an over damped Response as shown in figure below.

The solution for the above equation is :i= $C_1\, e^{(K_1+K_2)t}$ $C_2\, e^{(K_1-K_2)t}$



CaseII: K_2 is Negative $\left(\frac{R}{2L}\right)^2 < \frac{1}{LC}$

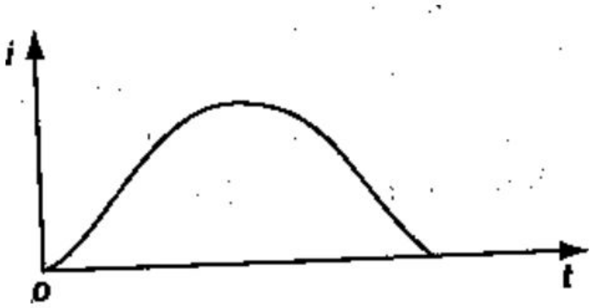


Then , the roots are Complex Conjugate, and give an under-damped Response as shown in figure below

The solution for the above equation is:i= $e^{K_1 t}(C_1 \cos K_2 t + C_2 \sin K_2 t)$

CaseIII: K_2 is Zero $\left(\frac{R}{2L}\right)^2 = \frac{1}{LC}$

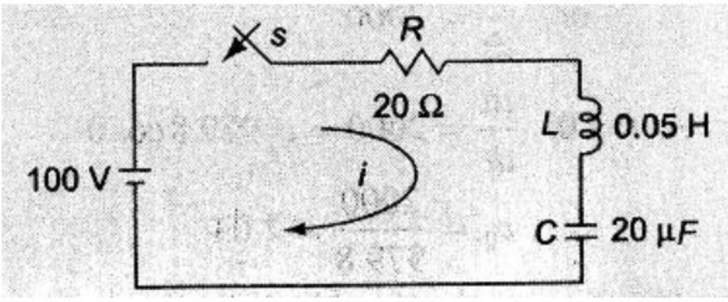
Then,the roots are Equal and give an Critically-damped Response as shown in figure below



The solution for the above equation is: i= $e^{K_1 t}(C_1 + C_2 t)$

Problem

A series R-L-C circuit with R = 20Ω , L = 0.05Hand C = 20 μF has a constant voltage V = 100 V applied at t=0 as shown in Fig.determine the transient current i.



Solution:

By applying Kirchoff's voltage Law,

we get $100=30i+0.05\frac{di}{dt}+\frac{1}{20\times10^{-6}}\int i\,dt$

Differentiating w.r.t.t weget

$$0.05\frac{d^2i}{dt^2}+20\frac{di}{dt}+\frac{1}{20\times10^{-6}}i=0$$

$$\Rightarrow \frac{d^2i}{dt^2}+400\frac{di}{dt}+10^6 i=0$$

$$\Rightarrow (D^2+400D+10^6)i=0$$

The roots of equation are

$$D_1, D_2=-\frac{400}{2}\pm\sqrt{\left(\frac{400}{2}\right)^2-10^6}$$

$$=-200\pm\sqrt{(200)^2-10^6}$$

$D_1=-200+j979.8$

$D_2=-200-j979.8$

Therefore the current

$i=e^{+K_1t}[C_1\cos K_2t+C_2\cos K_2t]$

$i=e^{-200t}[C_1\cos 979.8t+C_2\sin 979.8t]$ A

At t=0,the switch s is closed.

Since the inductor never allows sudden change in currents. At t= 0^- the current in the circuit is zero.Therefore at t=0^+, i =0

$i=0=(1)[C_1\cos 0+C_2\sin 0]$

$\Rightarrow C_1=0$ and $i=e^{-200t}[C_2\sin 979.8t]$ A

Differentiating w.r.t.t we get

$$\frac{di}{dt} = C_2\left[e^{-200t}979.8\cos 979.8t + e^{-200t}(-200)\sin 979.8t\right]$$

At t=0,the voltage across the inductor is 100V

$\Rightarrow L\frac{di}{dt}$=100or $\frac{di}{dt}$= 2000

Att=0,$\frac{di}{dt}$=2000= $C_2 979.8\cos 0$

$=\frac{2000}{979.8} \Rightarrow C_2$ 04

The current equation is
i= $e^{-200t}(2.04\sin 979.8t)\,A$

V. MAGNETIC CIRCUIT

The complete closed path followed by any group of magnetic flux lines is referred as magnetic circuit. The lines of magnetic flux never intersect, and each lineforms a closed path. Whenever a current is flowing through the coil there will be magnetic flux produced and the path followed by the magnetic flux is known as magnetic circuit.The operation of all the electrical devices like generators, motors, transformers etc. depend upon the magnetism produced by this magnetic circuit.Therefore,to obtain the required characteristics of these devices, their magnetic circuits have to be designed carefully.

Magneto Motive Force(MMF)

The magnetic pressure which sets up or tends to set up magnetic flux in a magnetic circuit is known as MMF.

1. Magneto motive force is the measure of the ability of a coil to produce flux.
2. The magnetic flux is due to the existence of the MMF caused by a current flowing through a coil having no. of turns.
3. ∴A coil with 'N' turns carrying a current of 'I' amperes represents a magnetic circuit producing an MMF of NI MMF=NI
4. Units of MMF = Ampere turns(AT)

MagneticFlux:

1. The amount of magnetic lines of force set-up in a magnetic circuit is called magnetic flux.
2. The magnetic flux, that is established in a magnetic circuit is proportional to the MMF and the proportional constant is the reluctance of the magnetic circuit.

Magneticflux α MMF

$$\text{Magneticflux} = \frac{MMF}{RELUCTANCE} = \frac{NI}{S}$$

3. The unit of magnetic flux is Weber.

Reluctance:

1. The opposition offered to the flow of magnetic flux in a magnetic circuit is called reluctance

2. Reluctance of a magnetic circuit is defined as the ratio of magneto motive force to the flux established.
3. Reluctance depends upon length (l), area of cross-section (a) and permeability of the material that makes up the magnetic circuit.(Sl,S a,S l/a)

$$S=\frac{l}{\mu a}$$

$$\text{RELUCTANCE}=\frac{MMF}{FLUX}$$

4. The unit of reluctance is AT/Wb

Magnetic field strength (H)

1. If the magnetic circuit is homogeneous, and of uniform cross-sectional area, the magnetic field strength is defined as the magneto motive force per unit length of magnetic circuit.

$$H=\frac{MMF}{LENGTH}=\frac{NI}{l}$$

2. The unit of magnetic field strength is AT/m

Magnetic flux density (B)

1. The magnetic flux density in any material is defined as the magnetic flux established per unit area of cross-section.

$$B = \frac{FLUX}{AREAOFCROSSSECTION} = \frac{\emptyset}{A}$$

2. The unit of magnetic flux density is wb/m^2 or TESLA

Relativepermeability

1. It is defined as the ratio of flux density established in magnetic material to the flux density established in air or vacuum for the same magnetic field strength.

SELF INDUCTANCE:

Inductance is the property of electrical circuits containing coils in which a change in the electrical current induces an electromotive force (emf). This value of induced emf opposes the change in current in electrical circuits and electric current 'I' produces a magnetic field which generates magnetic flux acting on the circuit containing coils. The ratio of the magnetic flux to the current is called the self-inductance.

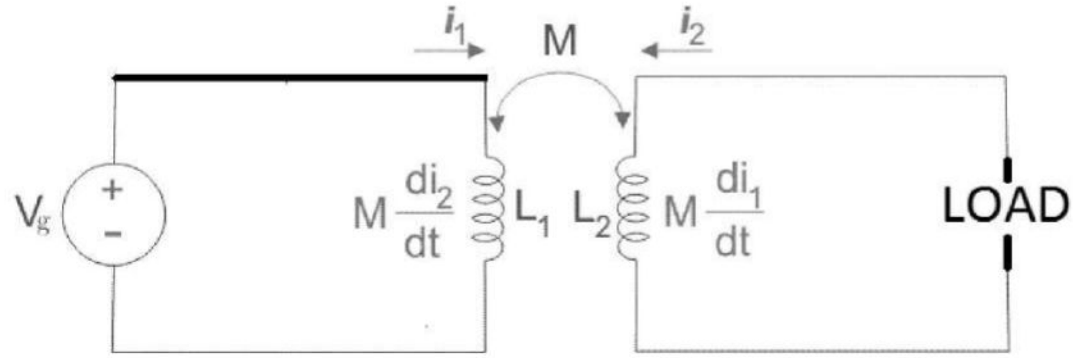
$$L = \frac{\psi = N\emptyset}{I}$$

The phenomenon of inducing an emf in a coil whenever a current linked with coil changes is called induction. Here units of L are Weber per ampere which is equivalent to Henry.

'Ø' denotes the magnetic flux through the area spanned by one loop, 'I' is the current flowing through the coil and N is the number of loops (turns) in the coil.

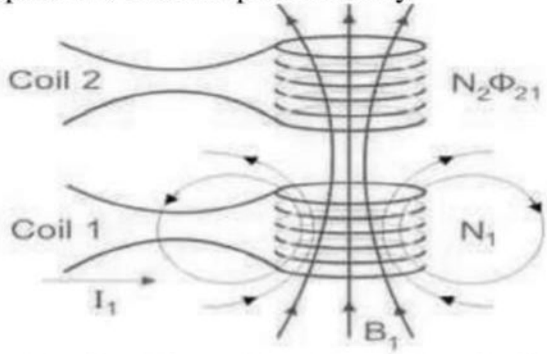
MUTUAL INDUCTANCE:

Mutual Inductance is the ratio between induced Electro Motive Force across a coil to the rate of change of current of another adjacent coil in such a way that two coils are in possibility of flux linkage. Mutual induction is a phenomenon when a coil gets induced in EMF across it due to rate of change current in adjacent coil in such a way that the flux of one coil current gets linkage of another coil. Mutual inductance is denoted as (M), it is called co-efficient of Mutual Induction between two coils



Mutual inductance for two coils gives the same value when they are in mutual induction with each other. Induction in one coil due to its own rate of change of current is called self inductance (L), but due to rate of change of current of adjacent coil it gives **mutual inductance**(M)

From the above figure, first coil carries current i1 and its self inductance is L1. Along with its self inductance it has to face mutual induction due to rate of change of current i2 in the second coil. Same case happens in the second coil also. Dot convention is used to mark the polarity of the mutual induction. Suppose two coils are placed nearby



Coil 1 carries I1 current having N1 number of turn. Now the flux density created by the coil 1 isB1. Coil 2 with N2 number of turn gets linked with this flux from coil 1. So flux linkage in coil 2 isN2.

φ21[φ21 is called leakage flux in coil 2 due to coil 1].

$$\varepsilon_2 = -N_2 . \frac{d\varphi_{21}}{dt} \ volt.$$

$$Again, \ \varepsilon_2 = -M_{21} . \frac{dl_1}{dt} \ volt.$$

Now it can be written from these equations,

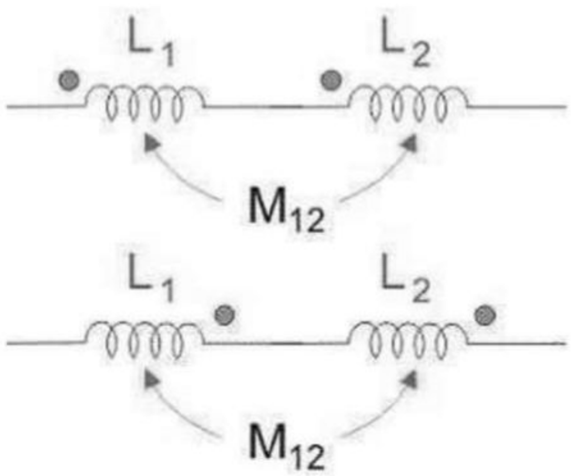
$$M_{21} = \frac{\varphi_{21} N_2}{I_1}$$

DOTCONVENTION:

- Dot convention is used to determine the polarity of a magnetic coil in respect of other magnetic coil.
- Dot convention is normally used to determine the total or equivalent inductance (**Leq**).

SERIES AIDING:

- Suppose two coils are in series with same place dot.
- When 2 dots are at the same place of both inductors (while at entering place or leaving place) as shown in below figure i.e. the total mutual inductance gets aided (added)

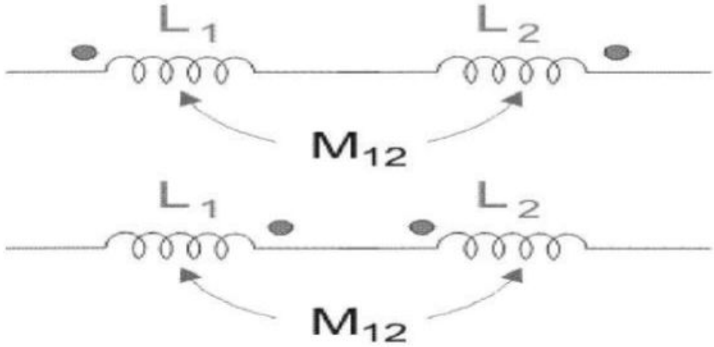


Mutual inductance between them is positive.

$$So, \;\; L_{eq} = L_1 + L_2 + 2M_{12}$$

SERIES OPPOSING:

- Suppose two coils are in series with opposite place dot.
- When 2 dots are at the opposite place of both inductors(while one at entering place and other at leaving place)as shown in below figure i.e. the total mutual inductance gets differed

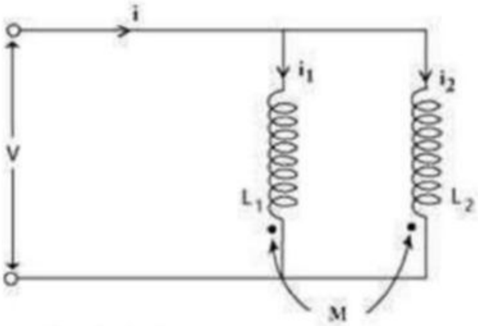


Mutual inductance between them is negative.

$$So, \;\; L_{eq} = L_1 + L_2 - 2M_{12}$$

PARALLEL AIDING:

- Suppose two coils are in parallel with same place dot.
- When 2 dots are at the same place of both inductors(while at entering place or leaving place)as shown in below figure i.e. the total mutual inductance gets aided(added)



$$V=L_1\frac{dI_1}{dt}+M\frac{dI_2}{dt}$$

$$V=L_2\frac{dI_2}{dt}+M\frac{dI_1}{dt}$$

$$\begin{bmatrix} L_1 & M \\ M & L_2 \end{bmatrix}\begin{bmatrix} \frac{di_1}{dt} \\ \frac{di_2}{dt} \end{bmatrix}=\begin{bmatrix} V \\ V \end{bmatrix}$$

$$\Delta=\begin{vmatrix} L_1 & M \\ M & L_2 \end{vmatrix}=L_1L_2-M^2$$

$$\frac{di_1}{dt}=\frac{\begin{vmatrix} V & M \\ V & L_2 \end{vmatrix}}{\Delta}=\frac{V(L_2-M)}{\Delta}\quad,\quad \frac{di_2}{dt}=\frac{\begin{vmatrix} L_1 & V \\ M & V \end{vmatrix}}{\Delta}=\frac{V(L_1-M)}{\Delta}$$

From the above figure,

$$i=i1+i2$$

$$\frac{di}{dt}=\frac{di_1}{dt}+\frac{di_2}{dt}=\frac{V(L_2-M)}{\Delta}+\frac{V(L_1-M)}{\Delta}=\frac{V(L_1+L_2-2M)}{\Delta}=\frac{V(L_1+L_2-2M)}{L_1L_2-M^2}$$

$$V=\frac{L_1L_2-M^2}{(L_1+L_2-2M)}\frac{di}{dt}$$

Therefore total inductance is given by,

$$L_{eq} = \frac{L_1L_2 - M^2}{(L_1 + L_2 - 2M)}$$

COEFFICIENTOF COUPLING:

The fraction of magnetic flux produced by the current in one coil that links with the other coil is called coefficient of coupling between the two coils. It is denoted by (k).
Two coils are taken coil A and coil B, when current flows through one coil it produces flux; the whole flux may not link with the other coil coupled, and this is because of leakage flux by a fraction (k) known as Coefficient of Coupling.

k=1 when the flux produced by one coil completely links with the other coil and is called magnetically tightly coupled.

k=0 when the flux produced by one coil does not link at all with the other coil and thus the coils are said to be magnetically isolated.

DERIVATION:

Consider two magnetic coils A and B. When current I1 flows through coil A.

$$L_1 = \frac{N_1\varphi_1}{I_1} \; and \; M = \frac{N_2\varphi_{12}}{I_1} \ldots\ldots(1) \; as \; (\varphi_{12} = k\varphi_1)$$

Considering coil B in which current I2 flows

$$L_2 = \frac{N_2\varphi_2}{I_2} \; and \; M = \frac{N_1\varphi_{21}}{I_2} = \frac{N_1 k\varphi_2}{I_2} \ldots\ldots(2) \quad as \; (\varphi_2 = k\varphi_2)$$

Multiplying equation (1) and (2)

$$MxM = \frac{N_2 k\varphi_1}{I_1} x \frac{N_1 k\varphi_2}{I_2}$$

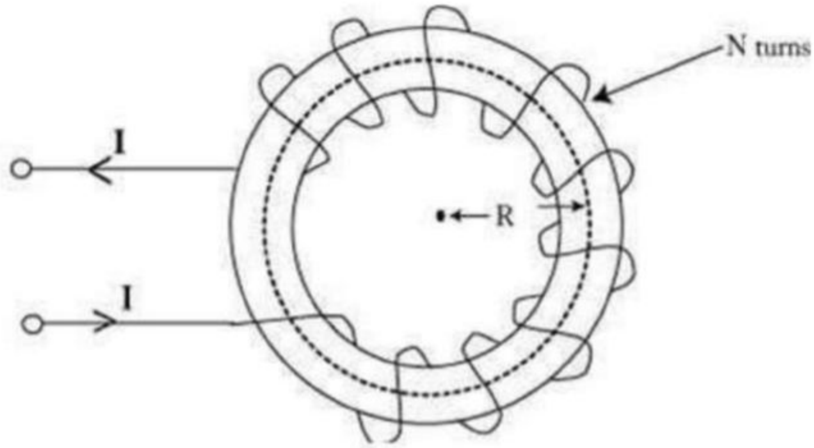
$$M^2 = k^2 \frac{N_1\varphi_1}{I_1} x \frac{N_2\varphi_2}{I_2} = k^2 L_1 L_2$$

$$\mathrm{M} = \sqrt{L_1 L_2} \quad \ldots\ldots\ldots(A)$$

The above equation (A) shows the relationship between mutual inductance and self inductance between two coils

SERIES MAGNETIC CIRCUIT:

- A series magnetic circuit is analogous to a series electric circuit. A magnetic circuit is said to be series, if the same flux is flowing through all the elements connected in a magnetic circuit. Consider a circular ring having a magnetic path of ‘l’ meters, area of cross section'a' m^2 with a mean radius of 'R' meters having a coil of 'N' turns carrying a current of 'I' amperes wound uniformly as shown in below fig



The flux produced by the circuit is given by

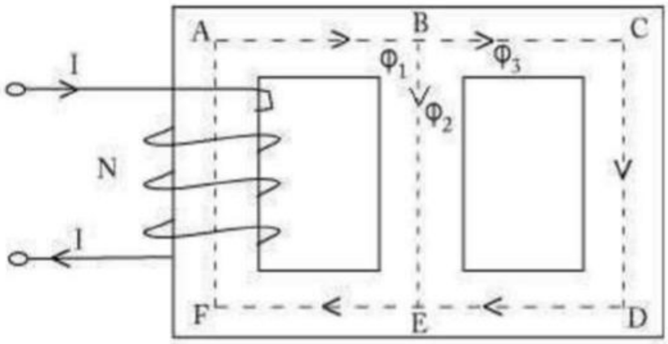
$$\text{Magneticflux} = \frac{MMF}{RELUCTANCE} = \frac{NI}{S} = \frac{NI}{l/\mu a}$$

In the above equation NI is the MMF of the magnetic circuit, which is analogous to EMF in the electrical circuit.

PARALLEL MAGNETIC CIRCUIT

- A magnetic circuit which has more than one path for magnetic flux is called a parallel magnetic circuit. It can be compared with a parallel electric circuit which has more than one path for electric current. The concept of parallel magnetic circuit is illustrated in fig.

2. Here a coil of 'N' turns wounded on limb 'AF' carries a current of 'I' amperes. The magnetic flux '$\varphi 1$' set up by the coil divides at 'B' into two paths namely
   Magnetic flux passes '$\varphi 2$' along the path'BE'
   Magnetic flux passes '$\varphi 3$'along the path 'BCDE'
   i.e $\varphi 1=\varphi 2+\varphi 3$



The magnetic paths 'BE'and'BCDE'are in parallel and form a parallel magnetic circuit. The AT required for this parallel circuit is equal to AT required for any one of the paths. Let S1=reluctance of path EFAB

Let, S1= reluctance ofpath EFAB

S2= reluctance of path BE

S3= reluctanceofpathBCDE

Total MMF = MMF for path EFAB+MMF for path BE or path BCD

NI=Φ1S1+Φ2S2=Φ1S1+Φ3S3

COMPOSITE MAGNETIC CIRCUIT:

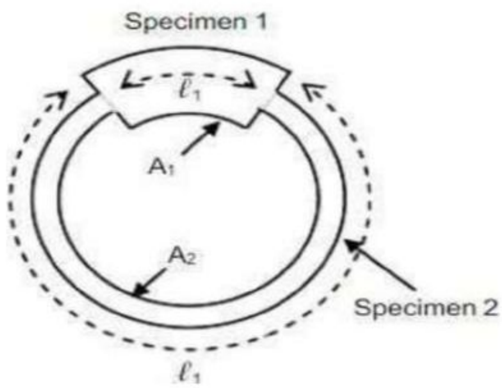
Consider a magnetic circuit which consists of two specimens of iron arranged as shown infigure. Let $\ell 1$ and $\ell 2$ be the mean lengths of specimen 1 and specimen2 in meters, A1 and A2 be their respective cross sectional areas in square meters, and 1 and 2 be their respective relative permeability's.

The reluctance of specimen1 is given as

$$S_1 = \frac{\ell_1}{\mu_0\mu_1 A_1} \quad (AT/Wb)$$

And that for specimen 2 is

$$S_2 = \frac{\ell_2}{\mu_0\mu_2 A_2} \quad (AT/Wb)$$



If a coil of N turns carrying a current I is wound on the specimen 1 and if the magnetic flux is assumed to be confined to iron core then the total reluctance is given by the sum of the individual reluctances S1and S2. This is equivalent to adding the resistances of a series circuit.Thus the total reluctance is given by

$$S = S_1 + S_2 = \frac{\ell_1}{\mu_0\mu_1 A_1} + \frac{\ell_2}{\mu_0\mu_2 A_2} \quad AT/Wb$$

And the total flux is given by

$$\Phi = \frac{\text{mmf}}{\text{S}} = \frac{\text{NI}}{\frac{\ell_1}{\mu_0\mu_1 A_1} + \frac{\ell_2}{\mu_0\mu_2 A_2}} \quad \left(\frac{\text{AT}}{(\text{AT}/\text{Wb})} \Rightarrow \text{Wb}\right)$$

IV.Alternating Current Theory

An alternating waveform is a periodic waveform which alternate between positive and negative values. Unlike direct waveforms, they cannot be characterised by one magnitude as their amplitude is continuously varying from instant to instant. Thus various forms of magnitudes are defined for such waveforms.

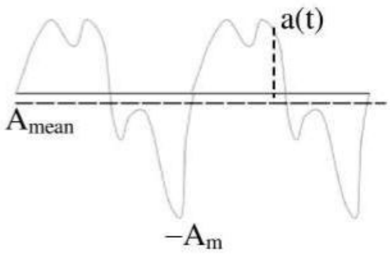
The advantage of the alternating waveform for electric power is that it can be stepped up or stepped down in potential easily for transmission and utilisation. Alternating waveforms can be of many shapes. The one that is used with electric power is the sinusoidal waveform. This has an equation of the form

$$v(t) = V_m \sin(\omega t + \phi)$$

If the period of the waveform is T, then its angular frequency ω corresponds to $\omega T = 2\pi$.

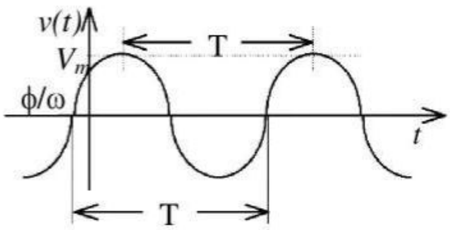
(a) **Instantaneous value:** The instantaneous value of a waveform is the value of the waveform at any given instant of time. It is a time variable *a(t)*.

For a sinusoid, Instantaneous value $a(t) = A_m \sin(\omega t + \phi)$



(b) **Peak value:** The peak value, or *maximum value*, of a waveform is the maximum instantaneous value of the waveform.

For a sinusoid, Peak value $= A_m$



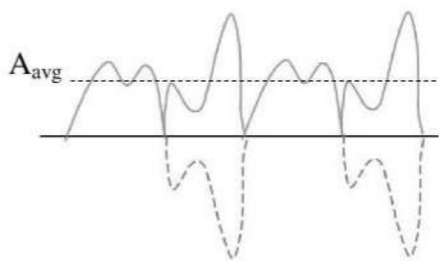
(c) **Mean value:** The mean value of a waveform is equal to the mean value over a complete cycle of the waveform. It also corresponds to the *direct component* of the waveform.

$$\text{Mean value } A_{mean} = \frac{1}{T}\int_{t_o}^{t_o+T} a(t).dt$$

The mean value of a waveform which has equal positive and negative half cycles must thus be always zero.

$$\text{For a sinusoid, Mean value} = \frac{1}{T}\int_{t_o}^{t_o+T} A_m \sin(\omega t + \phi).dt = 0$$

(d) **Average value (rectified):** The full-wave rectified average value or *average value* of a waveform is defined as the mean value of the rectified waveform over a complete cycle.

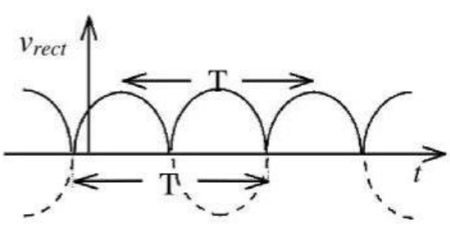


$$\text{Average value } A_{avg} = \frac{1}{T}\left[\int_{\substack{positive\\ hal\,fcycle}} a(t).dt - \int_{\substack{negative\\ hal\,fcycle}} a(t).dt\right]$$

For a sinusoid,

$$\text{Average value} = \frac{1}{T}\left[\int_0^{T/2} A_m \sin\omega t.dt - \int_{T/2}^{T} A_m \sin\omega t.dt\right]$$

$$= \frac{1}{\omega T}\bullet 2A_m = \frac{2}{\pi}A_m$$



The average value is defined in the above manner in electrical engineering .

(e) Effective value or r.m.s. value:

Neither the peak value, nor the mean value, nor the average value defines the useful value of the waveform with regard to the power or energy. Thus the effective value is defined based on the power equivalent of the quantity.

The *effective value* is thus defined as the constant value which would cause the same power dissipation as the original quantity over one complete period.

Thus considering current Power dissipation = $I_{eff}^{2} . R = \frac{1}{T}\int_{t_o}^{t_o+T} i^2(t).R.dt$

giving $\quad I_{eff} = \sqrt{\frac{1}{T}\int_{t_o}^{t_o+T} i^2(t).dt}$

or in general $\quad$ Effective value $A_{eff} = \sqrt{\frac{1}{T}\int_{t_o}^{t_o+T} a^2(t).dt}$

It is to be noted that the method of calculating the effective value involves the following processes. Taking the ***root*** of the ***mean*** of the ***square***d waveform over one complete cycle.

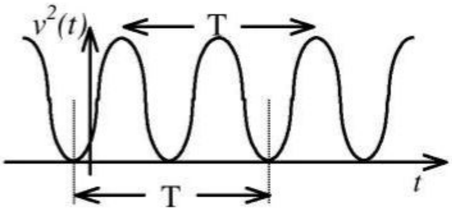
Thus it is also designated as the ***root mean square*** value, or the ***r.m.s.* value** of the waveform.

i.e. $\quad$ r.m.s. value $A_{rms} = \sqrt{\frac{1}{T}\int_{t_o}^{t_o+T} a^2(t).dt}$

This is defined for both current as well as voltage.

For a sinusoid,

r.m.s. value $= \sqrt{\frac{1}{T}\int_{t_o}^{t_o+T} A_m^2 \sin^2(\omega t+\phi).dt} = \frac{A_m}{\sqrt{2}}$



Unless otherwise specified, the rms value is the value that is always specified for ac waveforms, whether it be a voltage or a current. For example, **230 V** in the mains supply is an ***rms value*** of the voltage. Similarly when we talk about a **5 A**, **13 A** or **15A** socket outlet (plug point), we are again talking about the ***rms value*** of the rated current of the socket outlet.

For a given waveform, such as the *sinusoid*, the *peak value*, *average value* and the *rms value* are dependant on each other. The ***peak factor*** and the ***form factor*** are the two factors that are most commonly defined.

$$\text{Form Factor} = \frac{\textbf{\textit{rms value}}}{\textbf{\textit{average value}}}$$

and for a sinusoidal waveform, Form Factor = $\frac{V_m}{\sqrt{2}} \Big/ \frac{2V_m}{\pi} = 1.1107 \cong 1.111$

The form factor is useful such as when the average value has been measured using a rectifier type moving coil meter and the rms value is required to be found. [*Note:* You will be studying about these meters later]

Peak Factor = $\frac{\textbf{\textit{peak value}}}{\textbf{\textit{rms value}}}$ $\quad$ and for a sinusoidal waveform,

Peak Factor = $V_m \Big/ \frac{V_m}{\sqrt{2}} = \sqrt{2} = \mathbf{1.4142}$

The peak factor is useful when defining highly distorted waveforms such as the current waveform of compact fluorescent lamps.

Some advantages of the sinusoidal waveform for electrical power applications

a. Sinusoidally varying voltages are easily generated by rotating machines

b. Differentiation or integration of a sinusoidal waveform produces a sinusoidal waveform of the same frequency, differing only in magnitude and phase angle. Thus when a sinusoidal current is passed through (or a sinusoidal voltage applied across) a resistor, inductor or a capacitor a sinusoidal voltage waveform (or current waveform) of the same frequency, differing only in magnitude and phase angle, is obtained.

   If $i(t) = I_m \sin(\omega t+\phi)$,

   for a resistor, $\boldsymbol{v(t) = R.i(t) = R.I_m \sin(\omega t+\phi) = V_m \sin(\omega t+\phi)}$

   – magnitude changed by R but no phase shift

   for an inductor,

   $$v(t) = L.\frac{di}{dt} = L.\frac{d}{dt}(I_m \sin(\omega t+\phi)) = L.\omega.I_m \cos(\omega t+\phi) = L.\omega.I_m \sin(\omega t+\phi+\pi/2)$$

   – magnitude changed by $L\omega$ and phase angle changed by $\pi/2$

   for a capacitor,

   $$v(t) = \frac{1}{C}\cdot\int i\cdot dt = \frac{1}{C}\int I_m \sin(\omega t+\phi)\cdot dt = \frac{-1}{C\cdot\omega}.I_m \cos(\omega t+\phi) = \frac{1}{C\cdot\omega}.I_m \sin(\omega t+\phi-\pi/2)$$

   – magnitude changed by $1/C\omega$ and phase angle changed by $-\pi/2$

c. Sinusoidal waveforms have the property of remaining unaltered in shape and frequency when other sinusoids having the same frequency but different in magnitude and phase are added to them.

   $A \sin(\omega t+\alpha) + B \sin(\omega t+\beta)$

   $$= A \sin \omega t.\cos\alpha + A\cos\omega t.\sin\alpha + B\sin\omega t.\cos\beta + B\cos\omega t.\sin\beta$$
   $$= (A.\cos\alpha + B.\cos\beta)\sin\omega t + (A.\sin\alpha + B.\sin\beta)\cos\omega t$$
   $$= C\sin(\omega t+\theta),$$

   where $C = \sqrt{A^2 + B^2 + 2AB\cos(\alpha-\beta)}$, and $\theta = \tan^{-1}\dfrac{A\cos\theta + B\sin\theta}{A\sin\theta + B\cos\theta}$ are constants.

d. Periodic, but non-sinusoidal waveforms can be broken up to direct terms, its fundamental and harmonics.

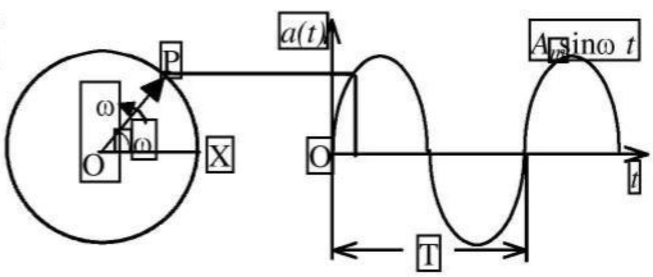
   $f(t) = F_o + F_1\sin(\omega t+\theta_1) + F_2\sin(2\omega t+\theta_2) + F_3\sin(3\omega t+\theta_3) + F_4\sin(4\omega t+\theta_4) + \ldots\ldots$

   where F_n and θ_n are constants dependant on the function f(t).

e. Sinusoidal waveforms can be represented by the projections of a rotating phasor.

Phasor Representation of Sinusoids

You may be aware that sin θ can be written in terms of exponentials and complex numbers.



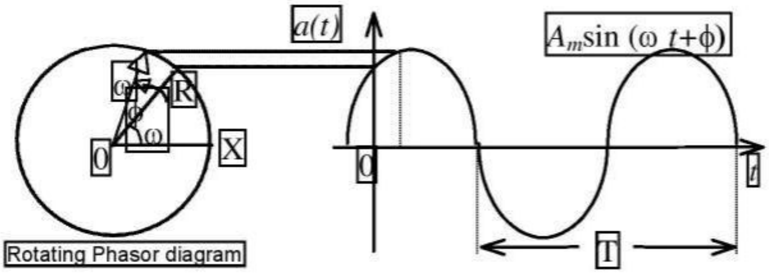
i.e. $\boldsymbol{e^{j\theta} = \cos\theta + j\sin\theta}$

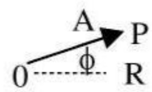
or $\boldsymbol{e^{j\omega t} = \cos\omega t + j\sin\omega t}$

Consider a line OP of length A_m which is in the horizontal direction OX at time t=0.

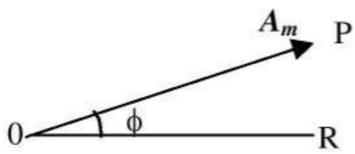
If OP rotates at an angular velocity ω , then in time t its position would correspond to an angle of ω ***t*****.**

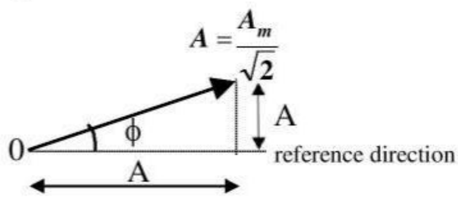
The projection of this rotating phasor OP (a phasor is somewhat similar to a vector, except that it does not have a physical direction in space but a *phase* angle) on the y-axis would correspond to OP sin ωt or $A_m \sin \omega t$ and on the x-axis would correspond to $A_m \cos \omega t$. Thus the sinusoidal waveform can be thought of being the projection on a particular direction of the complex exponential $e^{j\omega t}$.





If we consider more than one phasor, and each phasor rotates at the same angular frequency, then there is no relative motion between the phasors. Thus if we fix the reference phasor OR in a particular reference direction (without showing its rotation), then all others phasors moving at the same angular frequency would also be fixed at a relative position. Usually this reference direction is chosen as horizontal on the diagram for convenience.





Phasor diagram

It is also usual to draw the ***Phasor diagram*** using the ***rms value A*** of the sinusoidal waveform, rather than with the ***peak value*** A_m. This is shown on an enlarged diagram. Thus unless otherwise specified it is the rms value that is drawn on a phasor diagram.

It should be noted that the values on the phasor diagram are no longer time variables. The phasor **A** is characterised by its magnitude $|\mathbf{A}|$ and its phase angle ϕ. These are also the polar co-ordinates of the phasor and is commonly written as $|\mathbf{A}|\underline{/\phi}$. The phasor **A** can also be characterised by its cartesian co-ordinates $\mathbf{A_x}$ and $\mathbf{A_y}$ and usually written using complex numbers as $\mathbf{A = A_x + j\,A_y}$.

Note: In electrical engineering, the letter ***j*** is always used for the complex operator $\sqrt{-1}$ because the letter ***i*** is regularly used for electric current.

It is worth noting that $|A| = \sqrt{A_x^2 + A_y^2}$ and that $\tan\phi = \dfrac{A_y}{A_x}$ or $\phi = \tan^{-1}\left(\dfrac{A_y}{A_x}\right)$

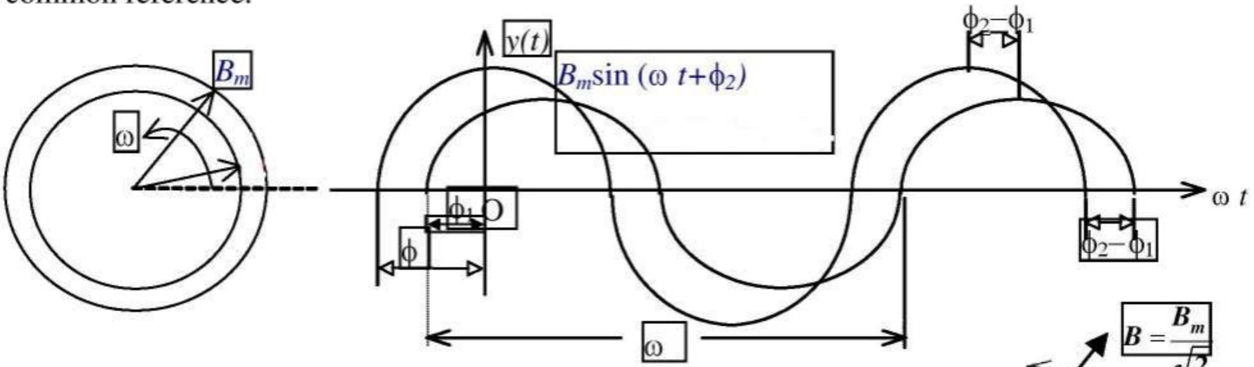
Also, $\mathbf{A_x} = |\mathbf{A}| \cos\phi$, $\mathbf{A_y} = |\mathbf{A}| \sin\phi$ and $|\mathbf{A}|\, e^{j\phi} = |\mathbf{A}| \cos\phi + \mathbf{j}|\mathbf{A}| \sin\phi = \mathbf{A_x} + \mathbf{j}\,\mathbf{A_y}$

Note: If the period of a sinusoidal waveform is **T**, then the corresponding angle would be ω **T.** Also, the period of a waveform corresponds to 1 complete cycle or 2 π radians or 360^0.

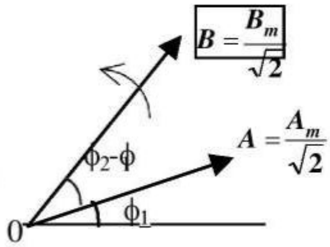
$\therefore\ \omega\,\mathbf{T} = 2\pi$

Phase difference

Consider the two waveforms $A_m \sin(\omega t+\phi_1)$ and $B_m \sin(\omega t+\phi_2)$ as shown in the figure. It can be seen that they have different amplitudes and different phase angles with respect to a common reference.



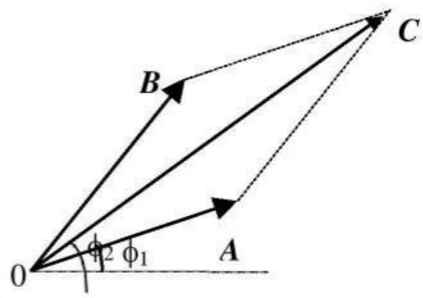
These two waveforms can also be represented by either rotating phasors $A_m\, e^{j(\omega t+\phi_1)}$ and $B_m\, e^{j(\omega t+\phi_2)}$ with peak amplitudes A_m and B_m, or by a normal phasor diagram with complex values A and B with polar co-ordinates $|\mathbf{A}| \angle \phi_1$ and $|\mathbf{B}| \angle \phi_2$ as shown .



Any particular value (such as positive peak, or zero) of a(t) is seen to occur at a time T after the corresponding value of b(t). i.e. the positive peak A_m occurs after an angle $(\phi_2 - \phi_1)$ after the positive peak B_m. Similarly the zero of a(t) occurs after an angle $(\phi_2 - \phi_1)$ after the corresponding zero of b(t). In such a case we say that the waveform **b(t)** ***leads*** the waveform **a(t)** by a phase angle of $(\phi_2 - \phi_1)$. Similarly we could say that the waveform **a(t)** ***lags*** the waveform **b(t)** by a phase angle of $(\phi_2 - \phi_1)$. [*Note:* Only the angle less than 180° is used to specify whether a waveform leads or lags another waveform].

We could also define, **lead** and **lag** by simply referring to the phasor diagram. Since angles are always measured anticlockwise (convention), we can see from the phasor diagram, that ***B*** **leads** ***A*** by an angle of $(\phi_2 - \phi_1)$ anticlockwise or that ***A*** **lags** ***B*** by an angle $(\phi_2 - \phi_1)$.

Addition and subtraction of phasors can be done using the same parallelogram and triangle laws as for vectors, generally using complex numbers. Thus the addition of phasor ***A*** and phasor ***B*** would be



$$\boldsymbol{A + B = (A \cos\phi_1 + j A \sin\phi_1) + (B \cos\phi_2 + j B \sin\phi_2)}$$

$$\boldsymbol{= (A \cos\phi_1 + B \cos\phi_2) + j (A \sin\phi_1 + B \sin\phi_2)}$$

$$\boldsymbol{= C_x + jC_y = |C| \angle \phi_c = C}$$

where $|C| = \sqrt{C_x^2 + C_y^2} = \sqrt{(A\cos\phi_1 + B\cos\phi_2)^2 + (A\sin\phi_1 + B\sin\phi_2)^2}$

and $\phi_c = \tan^{-1}\left(C_y / C_x\right) = \tan^{-1}\left((A\sin\phi_1 + B\sin\phi_2) / (A\cos\phi_1 + B\cos\phi_2)\right)$

Example 1

Find the addition and subtraction of the 2 complex numbers given by $10\angle 30^\circ$ and $25\angle 48^\circ$.

Addition $= 10 \angle 30^\circ + 25 \angle 48^\circ = 10(0.8660 + j\,0.5000) + 25(0.6691 + j\,0.7431)$

$= (8.660 + 16.728) + j\,(5.000 + 18.577) = 25.388 + j\,23.577 = 34.647 \angle 42.9^\circ$

Subtraction $= 10\angle 30^\circ - 25\angle 48^\circ = (8.660 - 16.728) + j\,(5.000 - 18.577)$

$= -8.068 - j\,13.577 = 15.793\angle 239.3^\circ$

Multiplication and division of phasors is most easily done using the polar form of complex numbers.

Thus the multiplication of phasor ***A*** and phasor ***B*** would be

$\boldsymbol{A} * \boldsymbol{B} = |\boldsymbol{A}| \angle\phi_1 * |\boldsymbol{B}| \angle\phi_2 = |\boldsymbol{A}|\ e^{j\phi_1} * |\boldsymbol{B}|\ e^{j\phi_2} = |\boldsymbol{A}| * |\boldsymbol{B}|\ e^{j(\phi_1+\phi_2)} = |\boldsymbol{A}| * |\boldsymbol{B}|$ $\angle(\phi_1 + \phi_2) = |\boldsymbol{C}| \angle\phi_c$

where $|\boldsymbol{C}| = |\boldsymbol{A}| * |\boldsymbol{B}|$ and $\phi_c = \phi_1 + \phi_2$

In a similar way, it can be easily seen that for division

$|\boldsymbol{C}| = |\boldsymbol{A}| / |\boldsymbol{B}|$ and $\phi_c = \phi_1 - \phi_2$

Thus, whenever we need to do addition and subtraction, we use the cartesian form of complex numbers, whereas for multiplication or division we use the polar form.

Example 2

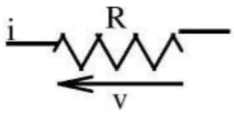
Find the multiplication and the division of the two complex numbers given by $10\angle 30^o$ and $25\ \angle 48^o$.

Multiplication = $10\ \angle 30^o * 25\angle 48^o = 250\ \angle 78^o$

Division = $10\ \angle 30^o \div 25\angle 48^o = 0.4\ \angle{-18^o}$

Currents and voltages in simple circuit elements

(1) Resistor



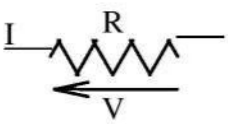
for a sinusoid, consider i(t) =*Real part of* [$I_m\, e^{(j\omega t+\theta)}$] or $I_m \cos (\omega t+\theta)$

$\therefore v(t) = Real\ [\boldsymbol{R}.\ \boldsymbol{I_m}\ \mathbf{e}^{(j\omega t+\theta)}] = Real\ [\boldsymbol{V_m}.\ \mathbf{e}^{(j\omega t+\theta)}]$

$\boldsymbol{v(t) = R.i(t)}$

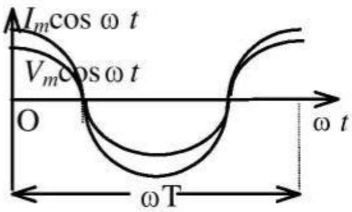
or $v(t) = R.I_m \cos(\omega t+\theta) = V_m \cos(\omega t+\theta)$

$\therefore \boldsymbol{V_m = R.I_m}$ and $V_m/\sqrt{2} = R.I_m/\sqrt{2}$



i.e. $\boldsymbol{V = R\ .\ I}$

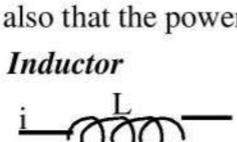
Note: ***V*** and ***I*** are rms values of the voltage and current and no additional phase angle change has occurred in the resistor .



Phasor diagram

Note also that the power dissipated in the resistor ***R*** is equal to $\boldsymbol{R\ .\ I^2 = V\ .\ I}$

(2) Inductor



for a sinusoid, consider i(t) =*Real part of* [$I_m\, e^{(j\omega t+\theta)}$] or $I_m \cos (\omega t+\theta)$

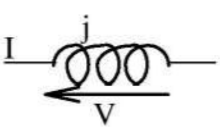
$\therefore v(t) = Real\ [\boldsymbol{L}.\frac{d}{dt}\ \boldsymbol{I_m}\ \mathbf{e}^{(j\omega t+\theta)}] = Real\ [\boldsymbol{L}.\ j\omega.\ \boldsymbol{I_m}\ \mathbf{e}^{j(\omega t+\theta)}] = Real\ [\boldsymbol{j\ V_m}\ \mathbf{e}^{j(\omega t+\theta)}]$

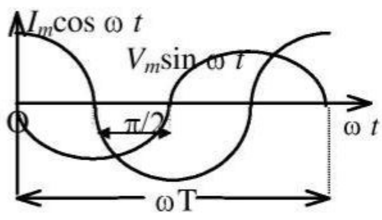
$v(t) = L\dfrac{d\,i(t)}{dt}$

or $v(t) = \boldsymbol{L}.\frac{d}{dt}\boldsymbol{I_m}\ \boldsymbol{\cos}\ (\omega \boldsymbol{t}+\theta) = -L.\omega.I_m \sin(\omega t+\theta) = L.\ \omega.I_m \cos(\omega t+\theta+\pi/2)$

$= V_m \cos(\omega t+\theta+\pi/2)$

$\therefore \boldsymbol{V_m = \omega L.I_m}$ and $V_m/\sqrt{2} = \omega \boldsymbol{L}.I_m/\sqrt{2}$





It can be seen that the ***rms*** magnitude of voltage is related to the ***rms*** magnitude of current by the multiplying factor ωL. It also seen that the voltage waveform leads the current waveform by 90^o or $\pi/2$ radians or that the current waveform lags the voltage waveform by 90^o for an inductor.

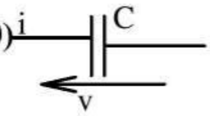
Phasor diagram

Thus it is usual to write the relationship as $\boldsymbol{V} = j\omega L.\boldsymbol{I}$ or $\underline{V} = \omega\, L.\boldsymbol{I}\ 90^o$

The impedance $\boldsymbol{Z}$ of the inductance may thus be defined as $j\omega L$, and $\boldsymbol{V} = \boldsymbol{Z}\,.\,\boldsymbol{I}$ corresponds to the generalised form of Ohm's Law.

Remember also that the power dissipation in a pure inductor is zero, as energy is only stored and as there is no resistive part in it, but that the product $\boldsymbol{V}\,.\,\boldsymbol{I}$ is not zero.

(3) Capacitor



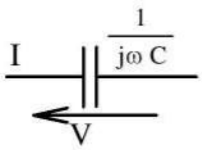
for a sinusoid, consider i(t) = *Real part of* [$I_m\, e^{j(\omega t+\theta)}$] or $I_m \cos(\omega t+\theta)$

$v(t) = Real\left[\frac{1}{C}\int I_m\, e^{j(\omega t+\theta)}.dt\right] = Real\left[\frac{1}{C.j\omega}.I_m\, e^{(j\omega t+\theta)}\right] = Real\left[\frac{1}{j}V_m\, e^{(j\omega t+\theta)}\right]$

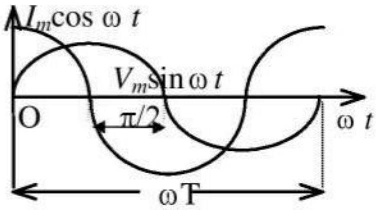
or $v(t) = \frac{1}{C}\int I_m \cos(\omega t+\theta).dt = \frac{1}{C\omega}.I_m \sin(\omega t+\theta) = \frac{1}{C\omega}.I_m \cos(\omega t+\theta-\pi/2)$

$= V_m \cos(\omega t+\theta-\pi/2)$

$v(t) = \frac{1}{C}\int i(t).dt$



$\therefore V_m = \frac{1}{\omega C}.I_m$ and $V_m/\sqrt{2} = \frac{1}{\omega C}.I_m/\sqrt{2}$



It can be seen that the ***rms*** magnitude of voltage is related to the ***rms*** magnitude of current by the multiplying factor $\frac{1}{\omega C}$.

It also seen that the voltage waveform lags the current waveform by 90^o or $\pi/2$ radians or that the current waveform leads the voltage waveform by 90^o for a capacitor.

Phasor diagram

Thus it is usual to write the relationship as $\boldsymbol{V} = \frac{1}{j\omega C}.\boldsymbol{I}$ or $\underline{V} = \frac{1}{\omega C}.\boldsymbol{I}\ 90^o$

The impedance $\boldsymbol{Z}$ of the inductance may thus be defined as $\frac{1}{j\omega C}$, and $\boldsymbol{V} = \boldsymbol{Z}\,.\,\boldsymbol{I}$ corresponds to the generalised form of Ohm's Law.

Remember also that the power dissipation in a pure capacitor is zero, as energy is only stored and as there is no resistive part in it, but that the product $\boldsymbol{V}\,.\,\boldsymbol{I}$ is not zero.

Impedance and Admittance in an a.c. circuit

The *impedance* $\boldsymbol{Z}$ of an a.c. circuit is a complex quantity. It defines the relation between the complex rms voltage and the complex rms current. *Admittance* $\boldsymbol{Y}$ is the inverse of the impedance $\boldsymbol{Z}$.

$$\boldsymbol{V} = \boldsymbol{Z}.\boldsymbol{I}, \qquad \boldsymbol{I} = \boldsymbol{Y}.\boldsymbol{V} \qquad \text{where} \quad \boldsymbol{Z} = \boldsymbol{R} + j\,\boldsymbol{X}, \quad \text{and} \quad \boldsymbol{Y} = \boldsymbol{G} + j\,\boldsymbol{B}$$

It is usual to express $\boldsymbol{Z}$ in cartesian form in terms of $\boldsymbol{R}$ and $\boldsymbol{X}$, and $\boldsymbol{Y}$ in terms of $\boldsymbol{G}$ and $\boldsymbol{B}$.

The real part of the impedance $\boldsymbol{Z}$ is resistive and is usually denoted by a resistance $\boldsymbol{R}$, while the imaginary part of the impedance $\boldsymbol{Z}$ is called a *reactance* and is usually denoted by a reactance $\boldsymbol{X}$.

It can be seen that the pure inductor and the pure capacitor has a reactance only and not a resistive part, while a pure resistor has only resistance and not a reactive part.

Thus $\mathbf{Z = R + j\,0}$ for a resistor, $\mathbf{Z = 0 + j\omega L}$ for an inductor, and $\mathbf{Z} = \dfrac{1}{j\omega C} = 0 - j\dfrac{1}{\omega C}$ for a capacitor.

The real part of the admittance ***Y*** is a *conductance* and is usually denoted by **G**, while the imaginary part of the admittance **Y** is called a *susceptance* and is denoted by **B**.

Relationships exist between the components of ***Z*** and the components of ***Y*** as follows.

$$G + jB = Y = \frac{1}{Z} = \frac{1}{R + jX} = \frac{R - jX}{R^2 + X^2} \quad \text{so that } G = \frac{R}{R^2 + X^2}, \quad \text{and} \quad B = \frac{-X}{R^2 + X^2}$$

The reverse process can also be similarly done if necessary.

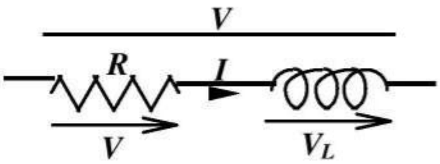
However, it must be remembered that in a complex circuit, ***G*** does not correspond to the inverse of the resistance **R** but its effective value is influenced by **X** as well as seen above.

Simple Series Circuits

In the case of single elements ***R, L*** and ***C*** we found that the angle difference between the voltage and the current was either zero, or $\pm 90^o$. This situation changes when there are more than one component in a circuit.

R-L series circuit

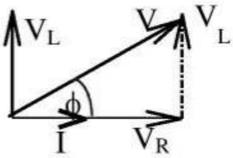
In the series R-L circuit, considering current **I** as reference



$V_R = R.I$, $V_L = j\omega L.I$, and $V = V_R + V_L$

$\therefore \quad V = (R + j\omega L).I \quad$ so that

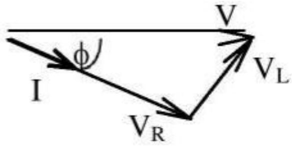
the total series impedance is $Z = R + j\omega L$



Phasor diagram

The above phasor diagram has been drawn with ***I*** as reference. [i.e. ***I*** is drawn along the x-axis direction]. The current was selected as reference in this example, because it is common to both the resistance and the inductance and makes the drawing of the circuit diagram easier. In this diagram, the voltage across the resistor $\mathbf{V_R}$ is in phase with the current, where as the voltage across the inductor $\mathbf{V_L}$ is 90^o leading the current. The total voltage **V** is then obtained by the phasor addition (similar to vector addition) of V_R and V_L.

If the total voltage was taken as the reference, the diagram would just rotate as shown. In this diagram, the current is seen to be lagging the voltage by the same angle that in the earlier diagram the voltage was seen to be leading the current. V_L has been drawn from the end of V_R rather than from the origin for ease of obtaining the resultant V from the triangular law.

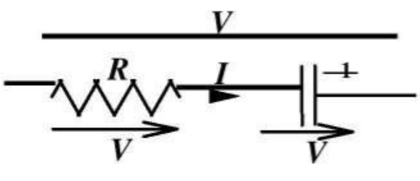


Phasor diagram

In an R-L circuit, the current lags the voltage by an angle less than 90^o and the circuit is said to be inductive.

Note that the power dissipation can only occur in the resistance in the circuit and is equal to $\boldsymbol{R\,.\,I^2}$ and that this is not equal to product $\boldsymbol{V\,.\,I}$ for the circuit.

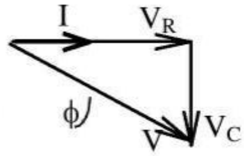
R-C series circuit



In the series R-C circuit,

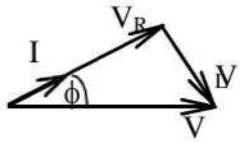
$V_R = R.I$, $V_C = \dfrac{1}{j\omega C}\,.\,I = -j\dfrac{1}{\omega C}\,.\,I$

and $V = V_R + V_C$



Phasor diagram

$\therefore \quad V = (R + \frac{1}{j\omega C}).I$ so that the total series impedance is $Z = R + \frac{1}{j\omega C}$

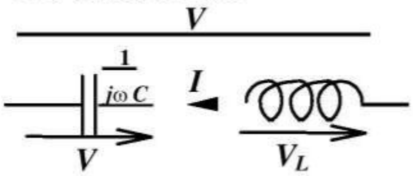


The phasor diagrams has been drawn first with current **I** as reference and then with voltage **V** as reference.

In an R-C circuit, the current leads the voltage by an angle less than 90^o and the circuit is said to be capacitive.

Note that the power dissipation can only occur in the resistance in the circuit and is equal to $\boldsymbol{R\,.\,I^2}$ and that this is not equal to product $\boldsymbol{V\,.\,I}$ for the circuit.

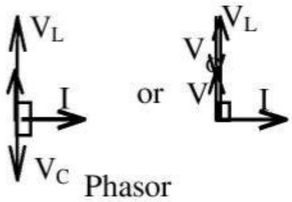
L-C series circuit



In the series L-C circuit,

$V_L = j\omega L.I$, $V_C = \frac{1}{j\omega C}.I = -j\frac{1}{\omega C}$

and $V = V_L + V_C$



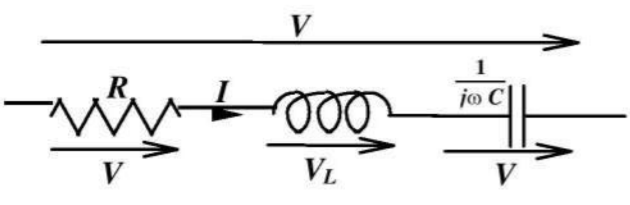
$\therefore \quad V = (j\omega L + \frac{1}{j\omega C}).I$ so that the total series impedance is $Z = j\omega L + \frac{1}{j\omega C} = j\omega L - j\frac{1}{\omega C}$

It is seen that the total impedance is purely reactive, and that all the voltages in the circuit are inphase but perpendicular to the current. The resultant voltage corresponds to the algebraic difference of the two voltages V_L and V_C and the direction could be either up or down depending on which voltage is more.

When $\omega L = \frac{1}{\omega C}$ the total impedance of the circuit becomes zero, so that the circuit current for a given supply voltage would become very large (only limited by the internal impedance of the source). This condition is known as series resonance. In an L-C circuit, the current either lags or leads the voltage by an angle equal to 90^o and the resultant circuit is either purely inductive or capacitive.

Note that no power dissipation can occur in the circuit and but that the product $\boldsymbol{V\,.\,I}$ for the circuit is non zero.

R-L-C series circuit



In the series R-L-C circuit,

$V_R = R.I$, $V_L = j\omega L.I$,

$V_C = \frac{1}{j\omega C}.I = -j\frac{1}{\omega C}.I$

and $V = V_R + V_L + V_C$

$\therefore \quad V = (R + j\omega L + \frac{1}{j\omega C}).I$ so that the total series impedance is $Z = R + j\omega L + \frac{1}{j\omega C}$

$$Z = R + j(\omega L - \quad)$$

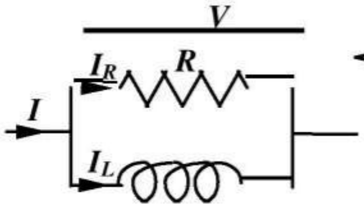
$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$ has a minimum value at $\omega L = \frac{1}{\omega C}$. This is the series resonance condition.

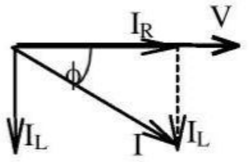
In an R-L-C circuit, the current can either lag or lead the voltage, and the phase angle difference between the current and the voltage can vary between -90^o and 90^o.

Note that the power dissipation can only occur in the resistance in the circuit and is equal to $\boldsymbol{R \,.\, I^2}$ and that this is not equal to product $\boldsymbol{V \,.\, I}$ for the circuit.

Simple Parallel Circuits

R-L parallel Circuit





Phasor

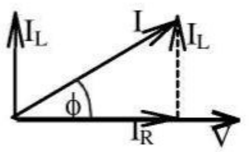
In the parallel R-L circuit,

considering **V** as reference

$V = R.I_R$, $V = j\omega L.I_L$, and $I = I_R + I_L$

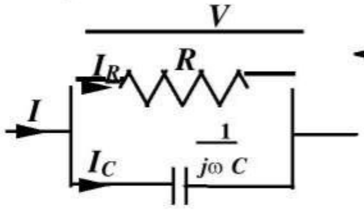
$$\therefore\ I = \frac{V}{R} + \frac{V}{j\omega L}$$

$\therefore$ total shunt admittance $= \dfrac{1}{R} + \dfrac{1}{j\omega L}$



Phasor

R-C parallel Circuit



In the parallel R-C circuit,

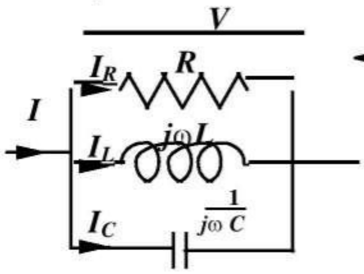
considering **V** as reference

$V = R.I_R$, $I_C = j\omega C.V$, and $I = I_R + I_L$

$$\therefore\ I = \frac{V}{R} + V \,.\, j\omega C$$

$\therefore$ total shunt admittance $= \dfrac{1}{R} + j\omega C$

R-L-C parallel Circuit



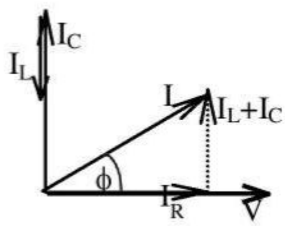
In the parallel R-L-C circuit ,

considering **V** as reference

$V = R.I_R$, $V = j\omega L.I_L$, $I_C = j\omega C.V$

and $I = I_R + I_L + I_C$

$$\therefore\ I = \frac{V}{R} + \frac{V}{j\omega L} + V.j\omega C$$



Phasor

$\therefore$ total shunt admittance $= \dfrac{1}{R} + \dfrac{1}{j\omega L} + j\omega C$

As in the case of the series circuit, shunt resonance will occur when $\dfrac{1}{\omega L} = \omega C$ giving a minimum value of shunt admittance.

Note that even in the case of a parallel circuit, power loss can only occur in the resistive elements and that the product $\boldsymbol{V.\, I}$ is not usually equal to the power loss.

Power and Power Factor

It was noted that in an a.c. circuit, power loss occurs only in resistive parts of the circuit and in general the power loss is not equal to the product $\boldsymbol{V \,.\, I}$ and that purely inductive parts and purely capacitive parts of a circuit did not have any power loss. To account for this apparent discrepancy, we define the product $\boldsymbol{V \,.\, I}$ as the ***apparent power S*** of the circuit.

Apparent power has the unit volt-ampere (VA) and not the watt (W), and watt (W) is used only for the ***active power P*** of the circuit (which we earlier called the ***power***)

apparent power S = V . I

Since a difference exists between the ***apparent power*** and the ***active power*** we define a new term called the ***reactive power Q*** for the reactance ***X.***

The instantaneous value of power ***p(t)*** is the product of the instantaneous value of voltage ***v(t)*** and the instantaneous value of current ***i(t).***

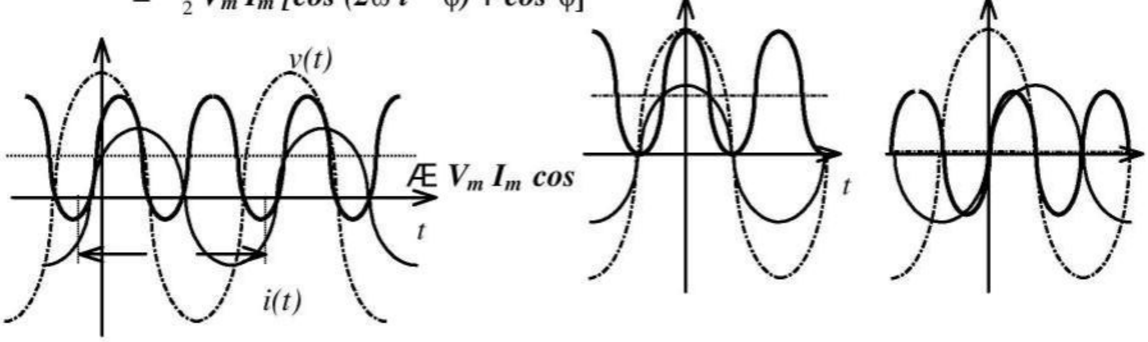
i.e. $\quad p(t) = v(t) \, . \, i(t)$

If $v(t) = V_m \cos \omega t$ and $i(t) = I_m \cos (\omega t - \phi)$,

where the voltage has been taken as reference and the current lags the voltage by a phase angle ϕ

then $\quad p(t) = V_m \cos \omega t \, . \, I_m \cos (\omega t - \phi) = V_m I_m \, . \, \frac{1}{2} \, . \, 2 \cos \omega t \, . \, \cos (\omega t - \phi)$

$$= \tfrac{1}{2} V_m I_m [\cos (2\omega t - \phi) + \cos \phi]$$



current lagging voltage by angle ϕ inphase

It can be seen that the waveform of power p(t) has a sinusoidally varying component and a constant component.

Thus the average value of power (active power) ***P*** would be given by the constant value $\frac{1}{2} V_m I_m \cos \phi$.

active power $P = \frac{1}{2} V_m I_m \cos \phi = \frac{V_m}{\sqrt{2}} \, . \, \frac{I_m}{\sqrt{2}} \cos \phi = V \, . \, I \cos \phi$

The term ***cos*** ϕ is defined as the ***power factor***, and is the ***ratio*** of the ***active power*** to the ***apparent power***.

Note that for a ***resistor***, $\phi = 0^o$ so that $P = V \, . \, I$

and that for an ***inductor***, $\phi = 90^o$ lagging (i.e. current is lagging the voltage by 90^o) so that $P = 0$

and that for an ***capacitor***, $\phi = 90^o$ leading (i.e. current is leading the voltage by 90^o) so that $P = 0$

For combinations of ***resistor***, ***inductor*** and ***capacitor***, ***P*** lies between ***V. I*** and ***0***

For an inductor or capacitor, ***V.*** I exists although $P = 0$. For these elements the product ***V. I*** is defined as the ***reactive power Q***. This occurs when the ***voltage*** and the ***current*** are ***quadrature*** (90^o out of phase). Thus reactive power is defined as the product of voltage and current components which are quadrature.

This gives

reactive power $Q = V. \, I \sin \phi$

RESONANCE

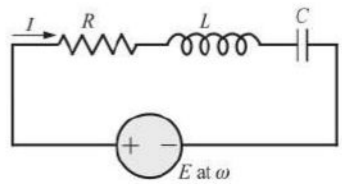
A.C Circuits made up of resistors, inductors and capacitors are said to be resonant circuits when the current drawn from the supply is in phase with the impressed sinusoidal voltage. Then

1. the resultant reactance or susceptance is zero.
2. the circuit behaves as a resistive circuit.
3. the power factor is unity.

Series Resonance

Fig. represents a series resonant circuit.
Resonance can be achieved by

1. varying frequency ω
2. varying the inductance L
3. varying the capacitance C



Series Resonant Circuit

The current in the circuit is

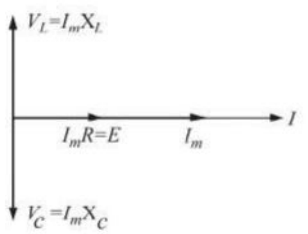
$$I = \frac{E}{R + j(X_L - X_C)} = \frac{E}{R + jX}$$

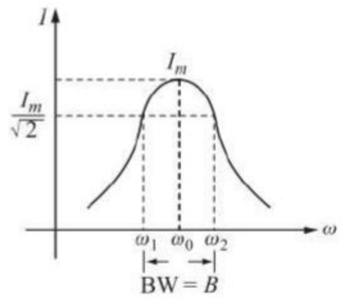
At resonance, X is zero. If ω_0 is the frequency at which resonance occurs, then $\omega_0 L = \frac{1}{\omega_0 C}$ or $\omega_0 = \frac{1}{\sqrt{LC}}$ = resonant frequency.

The current at resonance is $I_m = \frac{V}{R}$ = maximum current.

The phasor diagram for this condition is shown in Fig.

The variation of current with frequency is shown in Fig

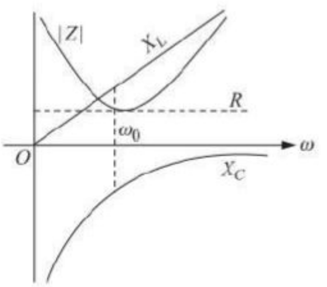


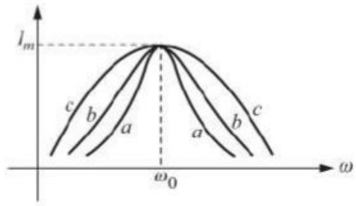


It is observed that there are two frequencies, one above and the other below the resonant frequency, ω_0 at which current is same.

Fig. represents the variations of $X_L = \omega L; X_C = \dfrac{1}{\omega C}$ and $|Z|$ with ω.

From the equation $\omega_0 = \dfrac{1}{\sqrt{LC}}$ we see that any constant product of L and C give a particular resonant frequency even if the ratio $\dfrac{L}{C}$ is different. The frequency of a constant frequency source can also be a resonant frequency for a number of L and C combinations. Fig. shows how the sharpness of tuning is affected by different $\dfrac{L}{C}$ ratios, but the product LC remaining constant.





For larger $\dfrac{L}{C}$ ratio, current varies more abruptly in the region of ω_0. Many applications call for narrow band that pass the signal at one frequency and tend to reject signals at other frequencies.

Bandwidth & Quality Factor

At resonance $I = I_m$ and the power dissipated is

$$P_m = I_m^2 R \text{ watts.}$$

When the current is $I = \dfrac{I_m}{\sqrt{2}}$ power dissipated is

$$\frac{P_m}{2} = \frac{I_m^2 R}{2} \text{ watts.}$$

From $\omega - I$ characteristic shown in Fig. 6.3, it is observed that there are two frequencies ω_1 and ω_2 at which the current is $I = \dfrac{I_m}{\sqrt{2}}$. As at these frequencies the power is only one half of that at ω_0, these are called half power frequencies or cut off frequencies.

The ratio, $$\frac{\text{current at half power frequencies}}{\text{Maximum current}} = \frac{I_m}{\sqrt{2} I_m} = \frac{1}{\sqrt{2}}$$

When expressed in dB it is $20 \log \dfrac{1}{\sqrt{2}} = -3\text{dB}$.

Therefore ω_1 and ω_2 are also called -3 dB frequencies.

As $\frac{I_m}{\sqrt{2}} = \frac{E}{\sqrt{2}R}$, the magnitude of the impedance at half-power frequencies is $\sqrt{2}R = |R + j(X_L - X_C)|$

Therefore, the resultant reactance, $X = X_L - X_C = R$.

The frequency range between half - power frequencies is $\omega_2 - \omega_1$, and it is referred to as passband or band width.

$$\text{BW} = \omega_2 - \omega_1 = B.$$

The sharpness of tuning depends on the ratio $\frac{R}{L}$, a small ratio indicating a high degree of selectivity. The quality factor of a circuit can be expressed in terms of R and L of the inductor.

$$\text{Quality factor} = Q = \frac{\omega_0 L}{R}$$

Writing $\omega_0 = 2\pi f_0$ and multiplying numerator and denominator by $\frac{1}{2}I_m^2$, we get,

$$Q = 2\pi f_0 \frac{\frac{1}{2}L{I_m}^2}{\frac{1}{2}{I_m}^2 R} = 2\pi \times \frac{\frac{1}{2}L{I_m}^2}{\frac{1}{2}{I_m}^2 RT}$$
$$= 2\pi \times \frac{\text{Maximum energy stored}}{\text{total energy lost in a period}}$$

Selectivity is the reciprocal of Q.

As $$Q = \frac{\omega_0 L}{R} \text{ and } \omega_0 L = \frac{1}{\omega_0 C},$$
$$Q = \frac{1}{\omega_0 CR}$$

and since $\omega_0 = \frac{1}{\sqrt{LC}}$, we have

$$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Half Power Frequencies

At half power frequencies ω_1 and ω_2,

$$I = \frac{E}{\sqrt{2}R} = \frac{E}{\{R^2 + (X_L - X_C)^2\}^{\frac{1}{2}}}$$

$$\therefore \qquad |X_L - X_C| = R \quad \text{i.e.,} \quad \left|\omega L - \frac{1}{\omega C}\right| = R$$

At $\omega = \omega_2$, $$R = \omega_2 L - \frac{1}{\omega_2 C}$$

Simplifying, $\omega_2^2 LC - \omega_2 CR - 1 = 0$

Solving, we get

$$\omega_2 = \frac{RC + \sqrt{R^2C^2 + 4LC}}{2LC} = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \qquad \text{......................} \qquad (1)$$

Note that only + sign is taken before the square root. This is done to ensure that ω_2 is always positive.

At $\omega = \omega_1$,

$$R = \frac{1}{\omega_1 C} - \omega_1 L$$

$$\Rightarrow \quad \omega_1^2 LC + \omega_1 CR - 1 = 0$$

Solving,

$$\omega_1 = \frac{-RC + \sqrt{R^2C^2 + 4LC}}{2LC}$$

$$= \frac{-R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}} \qquad \text{......................} \qquad (2)$$

While determining ω_1, only positive value is considered.

Subtracting equation (2) from equation (1), we get

$$\omega_2 - \omega_1 = \frac{R}{L} = \text{Band width.}$$

Since $Q = \dfrac{\omega_0 L}{R}$, Band width is expressed as

$$B = \omega_2 - \omega_1 = \frac{R}{L} = \frac{\omega_0}{Q}.$$

and therefore

$$Q = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{\omega_0}{B}$$

Multiplying equations (1) and (2), we get

$$\omega_1\omega_2 = \frac{R^2}{4L^2} + \frac{1}{LC} - \frac{R^2}{4L^2} = \frac{1}{LC} = \omega_0^2$$

or

$$\omega_0 = \sqrt{\omega_1\omega_2}$$

The resonance frequency is the geometric mean of half power frequencies.

Normally $\dfrac{R}{2L} << \dfrac{1}{\sqrt{LC}}$, in which case $Q \geq 5$

Then,

$$\omega_1 \simeq -\frac{R}{2L} + \sqrt{\frac{1}{LC}} \quad \text{and} \quad \omega_2 \simeq \frac{R}{2L} + \frac{1}{\sqrt{LC}}$$

$$= \frac{R}{2L} + \omega_0 \quad \text{and} \quad \omega_2 = \frac{R}{2L} + \omega_0$$

$\therefore$

$$\omega_0 = \frac{\omega_1 + \omega_2}{2} = \text{Arithmetic mean of } \omega_1 \text{ and } \omega_2$$

Since $\dfrac{R}{L} = \dfrac{\omega_0}{Q}$, Equations for ω_1 and ω_2 as given by equations (1) and (2) can be expressed in terms of Q as

$$\omega_2 = \frac{\omega_0}{2Q} + \sqrt{\left(\frac{\omega_0}{2Q}\right)^2 + \omega_0^2}$$

$$= \omega_0 \left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q} \right)^2} \right]$$

Similarly
$$\omega_1 = \omega_0 \left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q} \right)^2} \right]$$

Normally, $\frac{R}{2L} << \frac{1}{\sqrt{LC}}$ and then $Q > 5$.

Consequently ω_1 and ω_2 can be approximated as

$$\omega_1 \simeq -\frac{R}{2L} + \sqrt{\frac{1}{LC}} = -\frac{R}{2L} + \omega_0 = -\frac{B}{2} + \omega_0$$

$$\omega_2 \simeq \frac{R}{2L} + \sqrt{\frac{1}{LC}} = +\frac{R}{2L} + \omega_0 = \frac{B}{2} + \omega_0$$

so that
$$\omega_0 = \frac{\omega_1 + \omega_2}{2}.$$

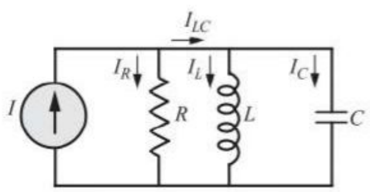
Parallel Resonance

The dual of a series resonant circuit is often considered as a parallel resonant circuit and it is as shown in Fig.

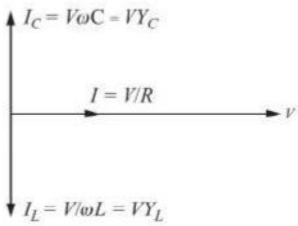
The phasor diagram for resonance is shown in Fig

The admittance as seen by the current source is

$$Y(j\omega) = Y_R + Y_L + Y_C$$
$$= \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right) = G + jB$$



Parallel Resonance Circuit



Phasor Diagram

If the resonance occurs at ω_0, then the susceptance B is zero. That is,

$$\omega_0 C = \frac{1}{\omega_0 L}$$

or

$$\omega_0 = \frac{1}{\sqrt{LC}} \text{ rad/ sec}.$$

At resonance,

$$I_{C0} = -I_{L0} = j\omega_0 CRI$$

and

$$I_{LC} = I_{C0} + I_{L0} = 0$$

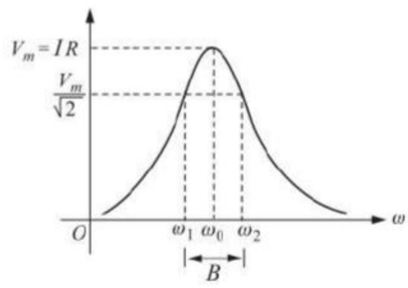
The quality factor, as in the case of series resonant circuit is defined as

$$Q = 2\pi \frac{\text{Maximum energy stored}}{\text{Energy dissipated in a period}}$$
$$= 2\pi \frac{\frac{1}{2}CV_m^2}{\frac{1}{2}\frac{V_m^2}{R}T}$$
$$= 2\pi f_0 CR = \omega_0 CR.$$

Since $$\omega_0 C = \frac{1}{\omega_0 L},$$

$$Q = \frac{R}{\omega_0 L}.$$

On either side of ω_0 there are two frequencies at which the voltage is same. At resonance, the voltage is maximum and is given by $V_m = IR$ and is evident from the response curve as shown in Fig. At this frequency, $p = p_m = \frac{V_m^2}{R}$ watts. The frequencies at which the voltage is $\frac{1}{\sqrt{2}}$ times the maximum voltage are called half power frequencies or cut off frequencies, since at these frequencies,



$$p = \frac{\left(\frac{V_m}{\sqrt{2}}\right)^2}{R} = \frac{V_m^2}{2R} = \text{half of the maximum power.}$$

At any ω,

$$Y = \frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)$$

At ω_1 and ω_2,

$$|Y| = \frac{1}{\sqrt{2}R} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}$$

Squaring,

$$\frac{1}{2R^2} = \frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2$$

Therefore, $$\left(\omega C - \frac{1}{\omega L}\right) = \frac{1}{R}$$

At $\omega = \omega_2$,

$$\omega_2 C - \frac{1}{\omega_2 L} = \frac{1}{R}$$

$$\omega_2^2 LC - 1 = \frac{\omega_2 L}{R}$$

$$\omega_2^2 LCR - R - \omega_2 L = 0$$

Hence, $$\omega_2 = \frac{L + \sqrt{L^2 + 4LCR^2}}{2LCR}$$

Note that only positive sign is used before the square root to ensure that ω_2 is positive.

Thus, $$\omega_2 = \frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

Similarly, $$\omega_1 = -\frac{1}{2RC} + \sqrt{\left(\frac{1}{2RC}\right)^2 + \frac{1}{LC}}$$

So that, bandwidth

$$B = \omega_2 - \omega_1 = \frac{1}{RC}$$

and
$$\omega_1\omega_2 = \left(\frac{1}{2RC}\right)^2 + \frac{1}{LC} - \left(\frac{1}{2RC}\right)^2$$
$$= \frac{1}{LC} = \omega_0^2$$

Thus,
$$\omega_0 = \sqrt{\omega_1\omega_2}$$

As $\omega_0 = \dfrac{1}{\sqrt{LC}}$ and $Q = \omega_0 RC = \dfrac{R}{\omega_0 L}$
$$Q = \frac{R}{L}\sqrt{LC} = R\sqrt{\frac{C}{L}}$$

Since $\dfrac{1}{2RC} = \dfrac{B}{2}$
$$\omega_2 = \frac{B}{2} + \sqrt{\left(\frac{B}{2}\right)^2 + \omega_0^2}$$

and
$$\omega_1 = -\frac{B}{2} + \sqrt{\left(\frac{B}{2}\right)^2 + \omega_0^2}$$

Using $B = \dfrac{\omega_0}{Q}$,
$$\omega_2 = \omega_0\left[\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

and
$$\omega_1 = \omega_0\left[-\frac{1}{2Q} + \sqrt{1 + \left(\frac{1}{2Q}\right)^2}\right]$$

VI. THREE PHASE CIRCUITS

INTRODUCTION

There are two types of system available in electric circuit, single phase and **three phase system**. In single phase circuit, there will be only one phase, i.e. the current will flow through only one wire and there will be one return path called neutral line to complete the circuit. So in the single phase minimum amount of power can be transported. Here the generating station and load station will also be single phase

In poly phase system, more than one phase canbe used for generating, transmitting and for load system.**Three phase circuit**is the polyphase system where three phases are sent together from the generator to the load. Each phase is having a phase difference of 120°, i.e. 120° angle electrically. So from the total of 360°, three phases are equally divided into 120° each. The power in **three phase system** is continuous as allthe three phases are involved in generating the total power. The sinusoidal waves for 3 phase system is shown below

The three phases can be used as single phase each. So if the load is single phase, then onephase can be taken from the **three phase circuit** and the neutral can be used as ground tocompletethecircuit.

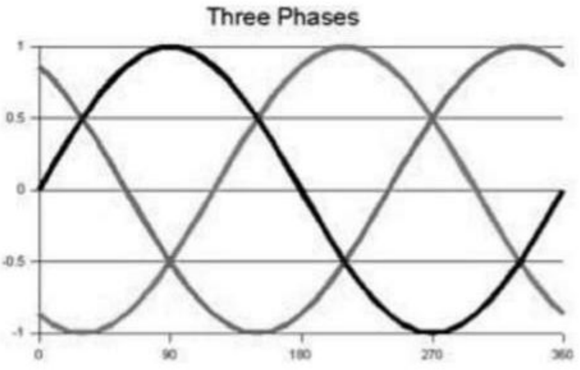


Fig.1

IMPORTANCEOFTHREEPHASEOVERSINGLEPHASE

There are various reasons for this because there are a number of advantages over single phase circuit.The three phase system can be used as three single phase lines o it can act as three single phase system. The three phase generation and single phase generation are the same in the generator except the arrangement of coil in the generator to get 120° phase difference. The conductor needed in three phase circuit is 75% that of conductors needed in a single phase circuit. And also the instantaneous power in a single phase system falls down to zero as in singlephase, we can see from the sinusoidal curve, but in three phase systemthe net power from all the phases gives a continuous power to theload.

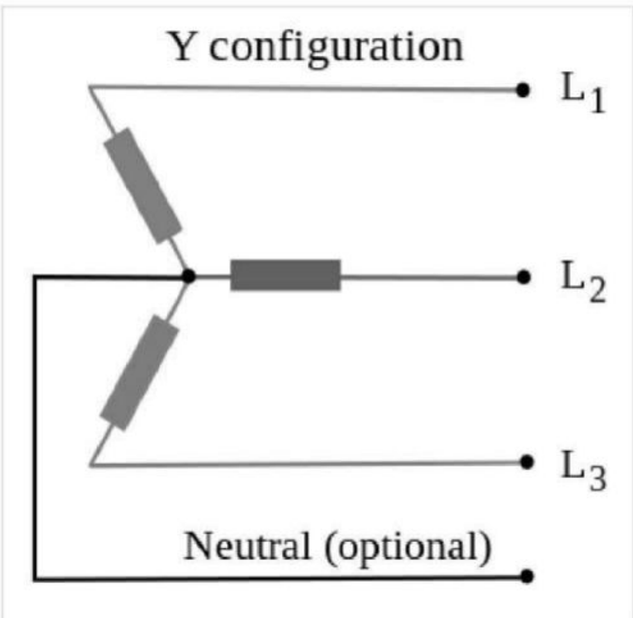
In athreephase circuit,connectionscanbegivenintwotypes:

1. Star connection
2. Delta connection

STAR CONNECTION

In star connection, there are four wires, three wires are phase wire and fourth is neutral which is taken from the star point. Star connection is preferred for long distance power transmission because it is having the neutral point. In this we need to come to the concept of balanced and unbalanced current in power system.

When equal current will flow through all the three phases, then it is called as balanced current. And when the current will not be equal in any of the phases, then it is unbalanced current. In this case, during balanced condition, there will be no current flowing through the



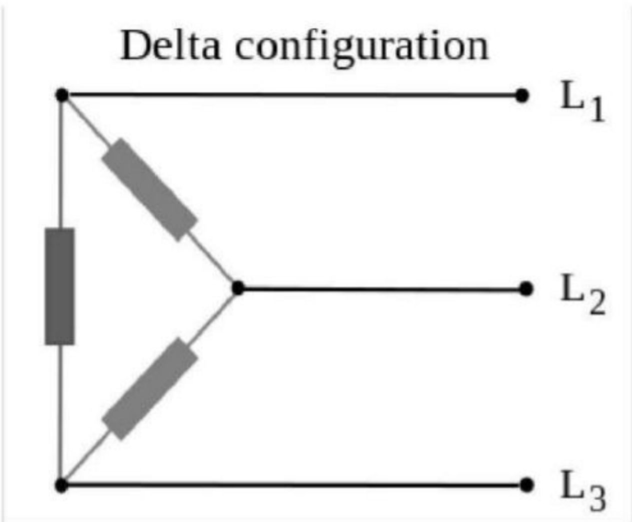
Neutral line and hence there is no use of the neutral terminal. But when there will be an unbalanced current flowing in the three phase circuit, neutral is having a vital role. It will take the unbalanced current through to the ground and protect the transformer.

Unbalanced current affects transformer and it may also cause damage to the transformer and for this star connection is preferred for long distance transmission.

In star connection, the line voltage is √3 times of phase voltage. Line voltage is the voltage between two phases in three phase circuit and phase voltage is the voltage between one phase to the neutral line. And the current is same for both line and phase. It is shown as expression below

$$E_{line} = \sqrt{3}E_{ph} \quad ad \quad I_{Line} = I_{phase}$$

DELTA CONNECTION



In delta connection, there are three wires alone and no neutral terminal is taken. Normally delta connection is preferred for short distance due to the problem of unbalanced current in the circuit. The figure is shown below for delta connection. In the load station, ground can be used as the neutral path if required.

In delta connection, the line voltage is same with that of phase voltage. And the line current is √3 times of the phase current. It is shown as expressed below,

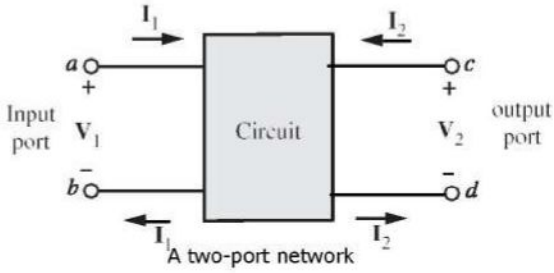
$$E_{line} = E_{phase} \quad and \quad I_{Lin} = \sqrt{3} I_{phase}$$

VII. TWO PORT NETWORKS

Terminals and Ports

A pair of terminals through which a current may enter or leave a network is known as a port. A port is an access to the network and consists of a pair of terminals. The parameters of a two-port network completely describes its behaviour in terms of the voltage and current at each port.. Two-port networks are also important in modeling electronic devices and system components. For example, in electronics, two-port networks are employed to model transistors and Op-amps. Other examples of electrical components modeled by two-ports are transformers and transmission lines.

Four popular types of two-ports param- eters are examined here: impedance, admit- tance, hybrid, and transmission.



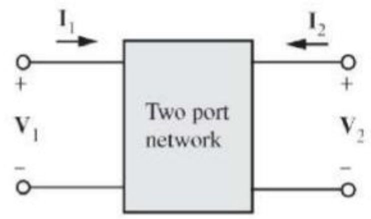
A two-port network

Fig represents a two-port network. A four terminal network is called a two-port network when the current entering one ter- minal of a pair exits the other terminal in the pair. For example, I_1 enters terminal and exits terminal of the input terminal pair - .

We assume that there are no independent sources or nonzero initial conditions within the lineartwo-port network.

Impedance parameters

Let us assume the two port network shown in Fig. is a linear network that contains no independent sources. Then using superposition theorem, we can write the input and output voltages as the sum of two components, one due to I_1 and other due to I_2:



$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

Putting the above equations in matrix from, we get

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

The **z** parameters are defined as follows:

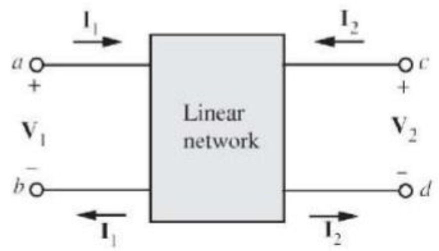
$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \quad z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \quad z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \quad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

In the preceeding equations, letting I_1 or $I_2 = 0$ is equivalent to open-circuiting the input or output port. Hence, the **z** parameters are called *open-circuit impedance parameters.* z_{11} is defined as the *open-circuit input impedance,* z_{22} is called the *open-circuit output impedance,* and z_{12} and z_{21} are called the *open-circuit transfer impedances.*

If $z_{12} = z_{21}$, the network is said to be **reciprocal network**. Also, if all the **z**-parameter are identical, then it is called a **symmetrical network**.

Admittance parameters

The network shown in Figis assumed to be linearand contains no independent sources. Hence, princi- ple of superposition can be applied to determine the current I_1, which can be written as the sum of two components, one due to V_1 and the other due to V_2. Using this principle, we can write



A linear two-port network

$$I_1 = y_{11}V_1 + y_{12}V_2$$

where y_{11} and y_{12} are the constants of proportionality with units of Siemens.

In a similar way, we can write

$$I_2 = y_{21}V_1 + y_{22}V_2$$

Hence, the two equations that describe the two-port network are

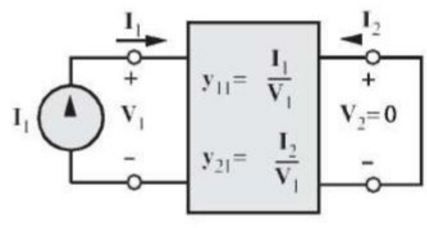
$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

Putting the above equations in matrix form, we get

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Here the constants of proportionality y_{11} y_{12} y_{21} and y_{22} are called **y** parameters for a network. If these parameters y_{11}, y_{12}, y_{21} and y_{22} are known, then the input/output operation of the two-port is completely defined.

From equations , we can determine**y** parameters. We obtain y_{11} and y_{21} by connecting a current source I_1 to port 1 and short-circuiting port 2 as shown in Fig. 7.3, finding V_1 and I_2, and then calculating,



Determination of y_{11} and y_{12}

$$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=0} \qquad y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=0}$$

Since y_{11} is the admittance at the input measured in siemens with the output short-circuited, it is called short-circuit input admittance. Similarly, y_{21} is called the short-circuit transfer admittance

Similarly, we obtain y_{12} and y_{22} by connecting a current source I_2 to port 2 and short-circuiting port 1 as in Fig., finding I_1 and V_2, and then calculating,

Hybrid parameters

The **z** and **y** parameters of a two-port network do not always exist. Hence, we define a third set of parameters known as hybrid parameters. In the pair of equations that define these parameters, V_1 and I_2 are the dependent variables. Hence, the two-port equations in terms of the hybrid parameters are

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

or in matrix form,

$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

These parameters are particularly important in transistor circuit analysis. These parameters are obtained via the following equations:

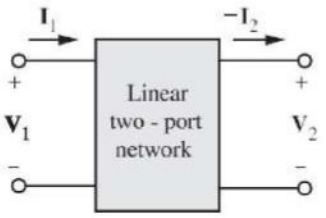
$$h_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=0} \quad h_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=0} \quad h_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0} \quad h_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=0}$$

The parameters h_{11} h_{12} h_{21} and h_{22} represent the *short-circuit input impedance*, the *open-circuit reverse voltage gain,* the *short-circuit forward current gain,* and the open-circuit *output admittance* respectively. Because of this mix of parameters, they are called **hybrid parameters.**

Transmission parameters

The transmission parameters are defined by the equations:

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$



Terminal variables used to define the ABCD Parameters

Putting the above equations in matrix form we get

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

Please note that in computing the transmission parameters, I_2 is used rather than I_2, because the current is considered to be leaving the network as shown in Fig.

These parameters are very useful in the analysis of circuits in cascade like transmission lines and cables. For this reason they are called Transmission Parameters. They are also known as **ABCD** parameters. The parameters are determined via the following equations:

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} \qquad B = \left.\frac{V_1}{I_2}\right|_{V_2=0} \qquad C = \left.\frac{I_1}{V_2}\right|_{I_2=0} \qquad D = \left.\frac{I_1}{I_2}\right|_{V_2=0}$$

A, **B**, **C** and **D** represent the *open-circuit voltage ratio*, the *negative short-circuit transfer impedance*, the *open-circuit transfer admittance*, and the *negative short-circuit current ratio*, respectively. When the two-port network does not contain dependent sources, the following relation holds good.

$$AD - BC = 1$$

Relations between two-port parameters

If all the two-port parameters for a network exist, it is possible to relate one set of parameters to another, since these parameters interrelate the variables V_1 I_1 V_2 and I_2 To begin with let us first derive the relation between the **z** parameters and **y** parameters.

The matrix equation for the **z** parameters is

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V = zI$$

Similarly, the equation for **y** parameters is

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$I = yV$$

Substituting equation, we get

$$V = zyV$$

Hence $$z = y^{-1} = \frac{adj(y)}{\Delta y}$$

where $$\Delta y = y_{11}y_{22} - y_{21}y_{12}$$

This means that we can obtain **z** matrix by inverting **y** matrix. It is quite possible that a two-port network has a **y** matrix or a **z** matrix, but not both.

Next let us proceed to find **z** parameters in terms of **ABCD** parameters.

The **ABCD** parameters of a two-port network are defined by

$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

$$V_2 = \frac{1}{C}(I_1 + DI_2)$$

$$V_2 = \frac{1}{C}I_1 + \frac{D}{C}I_2$$

$$V_1 = A\left(\frac{I_1}{C} + \frac{DI_2}{C}\right) - BI_2$$

$$= \frac{AI_1}{C} + \frac{AD-BC}{C} I_2$$

Comparing above equations with

$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

respectively, we find that

$$z_{11} = \frac{A}{C} \qquad z_{12} = \frac{AD - BC}{C} \qquad z_{21} = \frac{1}{C} \qquad z_{22} = \frac{D}{C}$$

Next, let us derive the relation between hybrid parameters and **z** parameters.

$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_2 = z_{21}I_1 + z_{22}I_2$$

From equation we can write

$$I_2 = -\frac{z_{21}}{z_{22}}I_1 + \frac{V_2}{z_{22}}$$

Substituting this value of I_2 in equation, we get

$$V_1 = z_{11}I_1 + z_{12}\ \frac{z_{21}I_1}{z_{22}} + \frac{V_2}{z_{22}}$$

$$= \frac{z_{11}z_{22} - z_{12}z_{21}}{z_{22}}\ I_1 + \frac{z_{12}V_2}{z_{22}}$$

Comparing above equations with

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

we get,

$$h_{11} = \frac{z_{11}z_{22} - z_{12}z_{21}}{z_{22}} = \frac{\Delta z}{Z_{22}}$$

$$h_{12} = \frac{z_{12}}{z_{22}} \qquad h_{21} = \frac{-z_{21}}{z_{22}} \qquad h_{22} = \frac{1}{z_{22}}$$

where $\Delta z = z_{11}z_{22} - z_{12}z_{21}$

Finally, let us derive the relationship between **y** parameters and **ABCD** parameters.

$$I_1 = y_{11}V_1 + y_{12}V_2$$
$$I_2 = y_{21}V_1 + y_{22}V_2$$

From equation we can write

$$V_1 = \frac{I_2}{y_{21}} + \frac{y_{22}}{y_{21}}V_2$$
$$= \frac{-y_{22}}{y_{21}}V_2 + \frac{1}{y_{21}}I_2$$

Substituting equation we get

$$I_1 = \frac{y_{11}y_{22}}{y_{21}}V_2 + y_{12}V_2 + \frac{y_{11}}{y_{21}}I_2$$
$$= \frac{\Delta y}{y_{21}}V_2 + \frac{y_{11}}{y_{21}}I_2$$

Comparin above g equations with the following equations,

$$V_1 = AV_2 - BI_2$$
$$I_1 = CV_2 - DI_2$$

we get $\quad A = \frac{y_{22}}{y_{21}} \qquad B = \frac{-1}{y_{21}} \qquad C = \frac{\Delta y}{y_{21}} \qquad D = \frac{-y_{11}}{y_{21}}$

where $\Delta y = y_{11}y_{22} - y_{12}y_{21}$

VI. FILTERS

A filter is a reactive network that freely passes the desired band of frequencies while almost totally suppressing all other bands. A filter is constructed from purely reactive elements

Low Pass Filter

By definition a low pass (LP) filter is one which passes without attenuation all frequencies up to the cut-off frequency f_c, and attenuates all other frequencies greater than f_c. The attenuation characteristic of an ideal LP filter is shown in fig..1.This transmits currents of all frequencies from zero up to the cut-off frequency. The band is called pass band or transmission band. Thus, the pass band for the LP filter is the frequency range 0 to f_c .The frequency range over which transmission does not take place is called the stop band or attenuation band. The stop band for a LP filter is the frequency range above f_c.

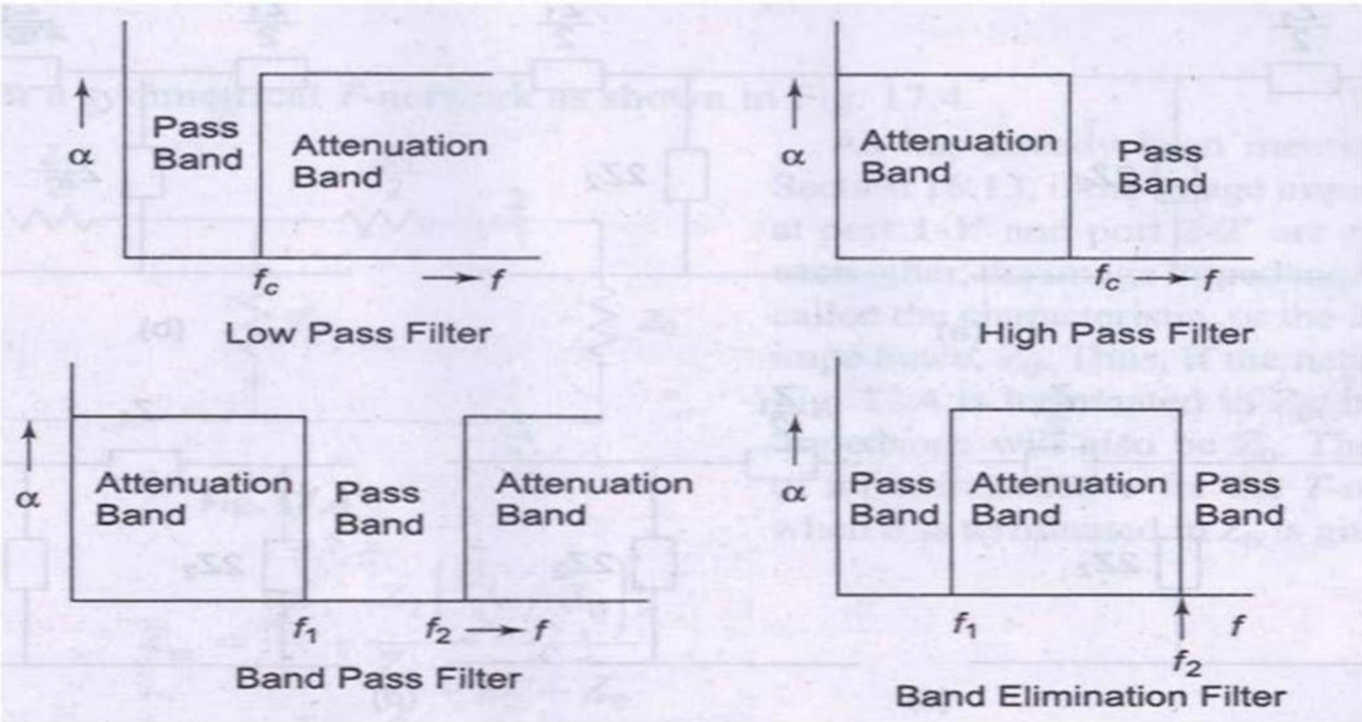


Fig.1

High Pass Filter

A high pass (HP) filter attenuates all frequencies below adesignated cut-off frequency, f_c, and passes all frequencies above f_c. Thus the pass band of this filter is the frequency range above f_c, and the stop band is the frequency range below f_c. The attenuation characteristic of a HP filter is showninfig..1.

Band Pass Filter

A band pass filter passes frequencies between two designated cut-off frequencies and attenuates all other frequencies. It is abbreviated as BP filter. As shown in fig. 1, a BP filter has two cut-off frequencies and will have the pass band $f_2 - f_1$; f_1 is called the lower cut – off frequency, while f_2 is called the upper cut-of ffrequency.

Band Elimination filter

A band elimination filter passes all frequencies lying outside a certain range, while it attenuates all frequencies between the two designated frequencies. It is also referred as band stop filter. The characteristic of an ideal band elimination filter is shown in fig..1.All frequencies between f_1 and f_2 will be attenuated while frequencies below f_1 and above f_2 will be passed.

CONSTANT–K LOW PASS FILTER

A network, either T or π, is said to be of the constant – k type if Z_1 and Z_2 of the network satisfy the relation

$$Z_1Z_2 = k^2$$

Where Z_1 and Z_2 are impedances in the T and π sections as shown in Fig.below The above Equation states that Z_1 and Z_2 are inverse if their product is a constant, independent of frequency. K is a real constant that is the resistance. k is often termed as design impedance or nominal impedance of the constant k –filter.

The constant k, T or π type filter is also known as the *prototype* because other more complex network can be derived from it. A prototype T and π– section areshown in

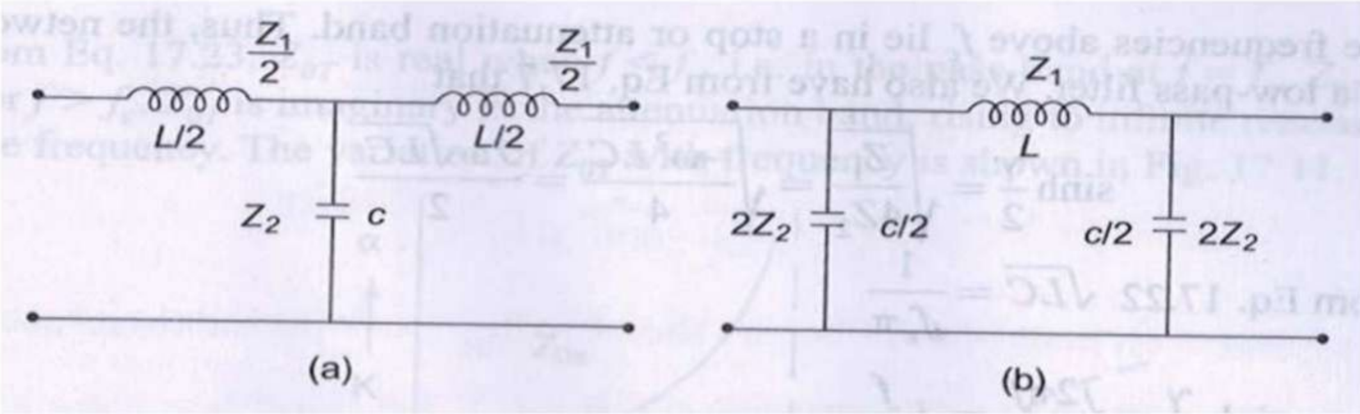


Fig. (a) and (b), where $Z_1 = j\omega$ Land $Z_2 = 1 / j\omega C$. Hence $Z_1Z_2 = L/C = k^2$ which is independent of frequency.

$$Z_1Z_2 = k^2 = \frac{L}{C} \quad \text{or} \quad k = \sqrt{\frac{L}{C}}$$

Since the product Z_1 and Z_2 is constant, the filter is a constant – k type. The cut-off frequencies are given by

$$Z_1 / 4Z_2 = 0,$$

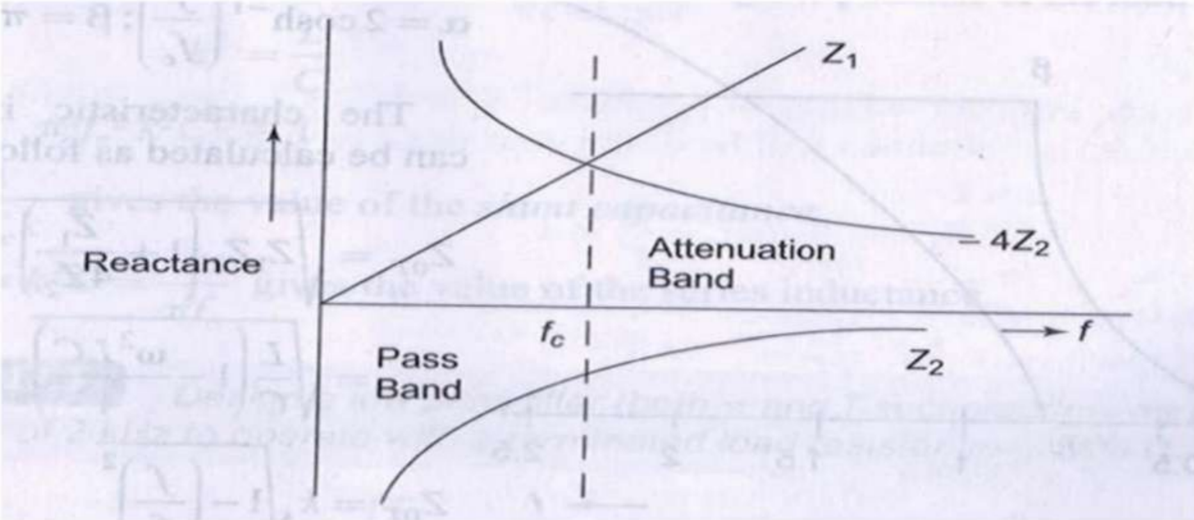
i.e. $$\frac{-\omega^2 LC}{4} = 0$$

i.e. $$f = 0 \text{ and } \frac{Z_1}{4Z_2} = -1$$

$$\frac{-\omega^2 LC}{4} = -1$$

or $$f_c = \frac{1}{\pi\sqrt{LC}}$$

The pass band can be determined graphically. The reactances of Z_1 and $4Z_2$ will vary with frequency as drawn in Fig.below The cut-off frequency at the intersection of the curves Z_1 and $-4z_2$ is indicated as f_C . On the X – axis as $Z_1 = -4Z_2$ at cut-off frequency, the pass band lies between the frequencies at which $Z_1 = 0$, and $Z_1 = -4Z_2$.



All the frequencies above f_C lie in a stop or attenuation band , thus, the network is called a low-passfilter. We also have

$$\sinh\frac{\gamma}{2}=\sqrt{\frac{Z_1}{4Z_2}}=\sqrt{\frac{-\omega^2 LC}{4}}=\frac{J\omega\sqrt{LC}}{2}$$

$$\sqrt{LC}=\frac{1}{f_c\pi}$$

$$\therefore \quad \sinh\frac{\gamma}{2}=\frac{j2\pi f}{2\pi f_c}=j\frac{f}{f_c}$$

We also know that in the pass band

$$-1<\frac{Z_1}{4Z_2}<0$$

$$-1<\frac{-\omega^2 LC}{4}<0$$

$$-1<-\left(\frac{f}{f_c}\right)^2<0$$

or $$\frac{f}{f_c}<1$$

and $$\beta=2\sin^{-1}\left(\frac{f}{f_c}\right);\ \alpha=0$$

In the attenuation band,

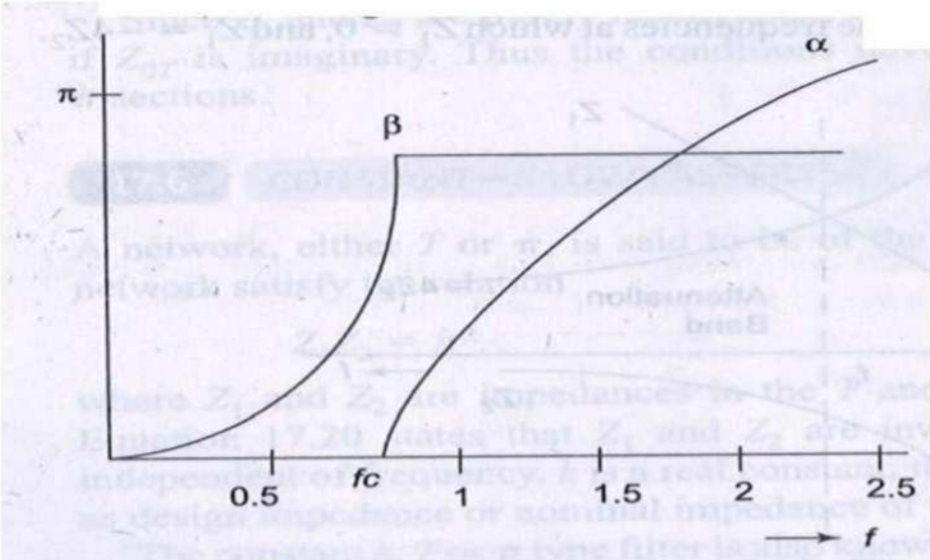
$$\frac{Z_1}{4Z_2}<-1,\ \text{i.e.}\frac{f}{f_c}<1$$

$$\alpha=2\cosh^{-1}\left|\frac{Z_1}{4Z_2}\right|=2\cosh^{-1}\left(\frac{f}{f_c}\right);\ \beta=\pi$$

The plots of α and β for pass and stop bands are shown in Fig below

Thus, from Fig., $\alpha = 0$, $\beta = 2\sinh^{-1}(f/f_C)$ for $f < f_C$

And $\alpha = 2\cosh^{-1}(f/f_C)$; $\beta = \pi$ for $f > f_C$



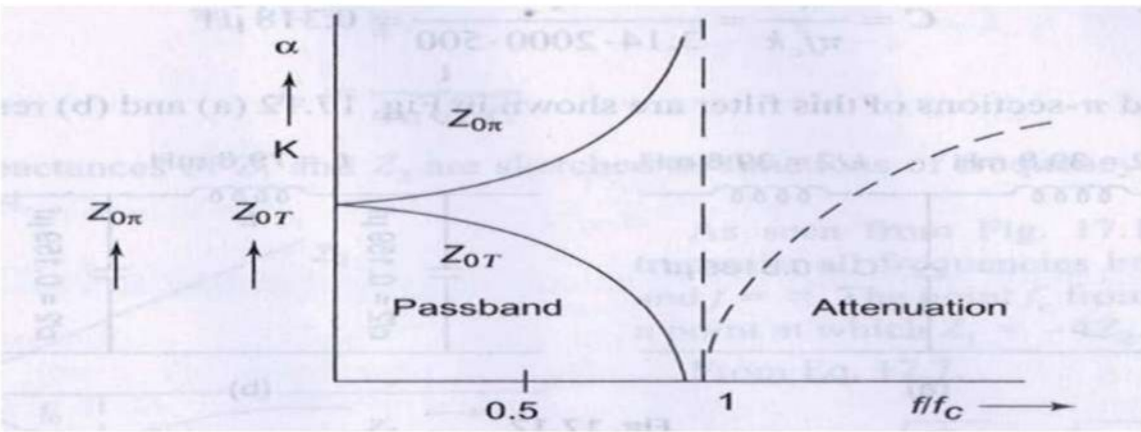
The characteristics impedance can be calculated as follows

$$Z_{OT} = \sqrt{Z_1 Z_2\left(1+\frac{Z_1}{4Z_2}\right)}$$

$$= \sqrt{\frac{L}{C}\left(1-\frac{\omega^2 LC}{4}\right)}$$

$$Z_{OT} = k\sqrt{1-\left(\frac{f}{f_c}\right)^2}$$

Thus , Z_{OT} is real when $f < f_C$, i.e.in the pass band at $f = f_C$, Z_{OT} is zero; and for $f > f_C$, Z_{OT} is imaginary in the attenuation band , rising to infinite reactance at infinite frequency . The variation of Z_{OT} with frequency is shown in Fig.below



Similarly, the characteristics impedance of a π–network is given by

$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} = \frac{k}{\sqrt{1-\left(\frac{f}{f_c}\right)^2}}$$

The variation of $Z_{O\pi}$ with frequency is shown in Fig.above. For $f < f_C$, $Z_{O\pi}$ is real ; at $f = f_C$, Z_{OT} is infinite , and for $f > f_C$, $Z_{O\pi}$ is imaginary . A low pass filter can be designed from the specifications of cut-off frequency and load resistance.

At cut-off frequency, $Z_1 = -4Z_2$

$$j\omega_c L = \frac{-4}{j\omega_c C}$$

$$\pi^2 f_c^2 LC = 1$$

Also we know that $k = \sqrt{L/C}$ is called the design impedance or the load resistance

$$\therefore \quad k^2 = \frac{L}{C}$$

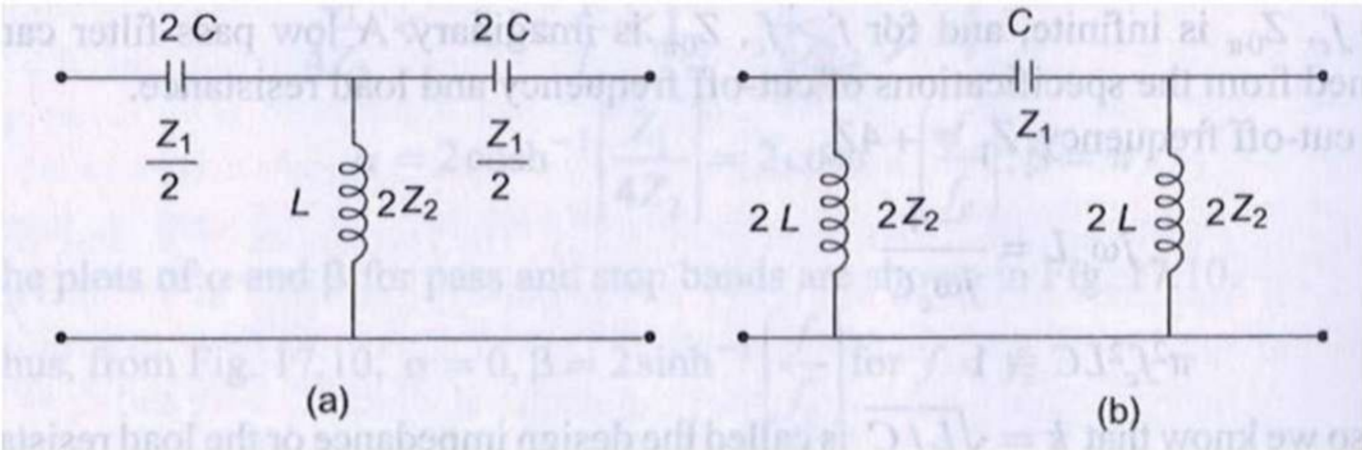
$$\pi^2 f_c^2 k^2 C^2 = 1$$

$C = \dfrac{1}{\pi f_c k}$ gives the value of the *shunt capacitance*

and $L = k^2 C = \dfrac{k}{\pi f_c}$ gives the value of the series inductance.

CONSTANT K – HIGH PASS FILTER

The proto type high pass filters are shown in Fig.below, where Z_1 = -j/ωC and Z_2 = jωL .



Again, it can be observed that the product of Z_1 and Z_2 is independent of frequency, and the filter design obtained will be of the constant k type. Thus, Z_1Z_2 are given by

$$Z_1Z_2 = \frac{-j}{\omega C} j\omega L = \frac{L}{C} = k^2$$

$$k = \sqrt{\frac{L}{C}}$$

The cut-off frequencies are given by Z_1=0 and Z_2= -4Z_2.

Z_1=0 indicates j/ωC=0, or ω→∞

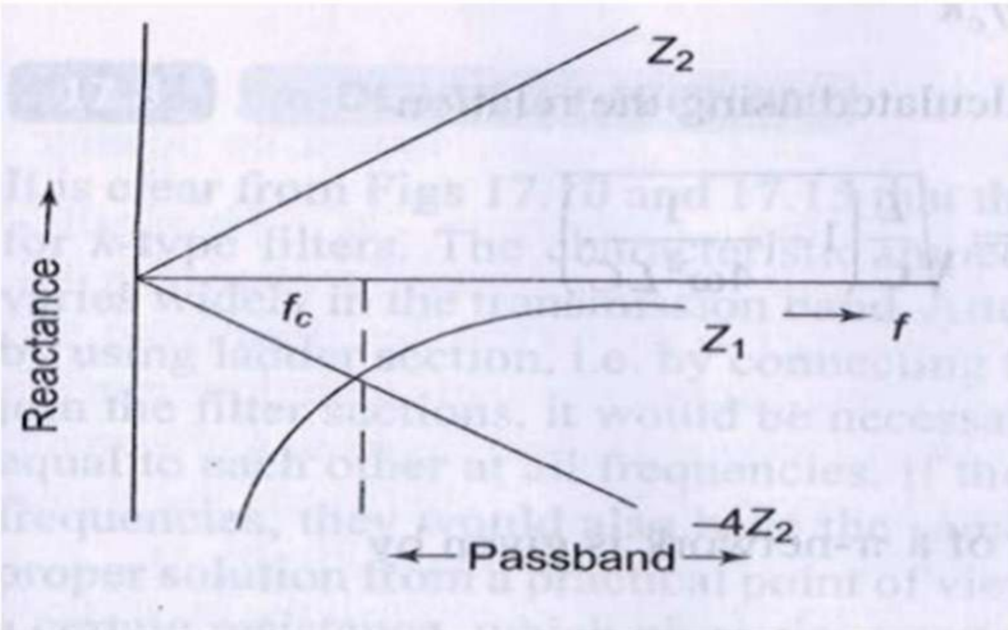
From $Z_1 = -4Z_2$

$$-j/\omega C = -4\, j\omega L$$

$$\omega^2 LC = 1/4$$

or $$f_c = \frac{1}{4\pi\sqrt{LC}}$$

The reactances of Z_1 and Z_2 are sketched as functions of frequency as shown in Fig.below



As seen from Fig. the filter transmits all frequencies between $f = f_C$ and $f = \infty$.The point f_C from the graph is a point at which $Z_1 = -$

$4Z_2$. Again

$$\sinh\frac{\gamma}{2} = \sqrt{\frac{Z_1}{4Z_2}} = \sqrt{\frac{-1}{4\omega^2 LC}}$$

$$f_c = \frac{1}{4\pi\sqrt{LC}}$$

$$\therefore \quad \sqrt{LC} = \frac{1}{4\pi f_c}$$

$$\therefore \quad \sinh\frac{\gamma}{2} = \sqrt{\frac{-(4\pi)^2 (f_c)^2}{4\omega^2}} = j\frac{f_c}{f}$$

In the pass band, $-1 < Z_1/4Z_2 < 0$, $\alpha = 0$ or the region in which $f_C/f < 1$ is a pass band

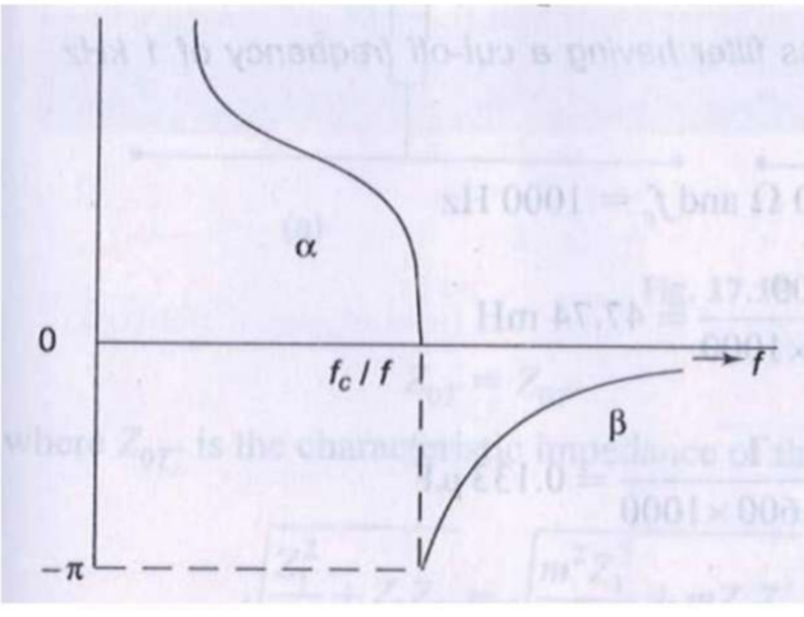
$\beta = 2\sin^{-1}(f_C/f)$

In the attenuation band $Z_1/4Z_2 < -1$, i.e. $f_C/f > 1$

$$\alpha = 2\cosh^{-1}[Z_1/4Z_2]$$

$$= 2\cos^{-1}(f_C/f);$$

$$\beta = -\pi$$



The plots of α and β for pass and stop bands of a high pass filter network are shown in Fig.above

A high pass filter may be designed similar to the low pass filter by choosing a resistive load R equal to the constant k, such that $R = k = \sqrt{L/C}$

$$f_c = \frac{1}{4\pi\sqrt{L/C}}$$

$$f_c = \frac{k}{4\pi L} = \frac{1}{4\pi Ck}$$

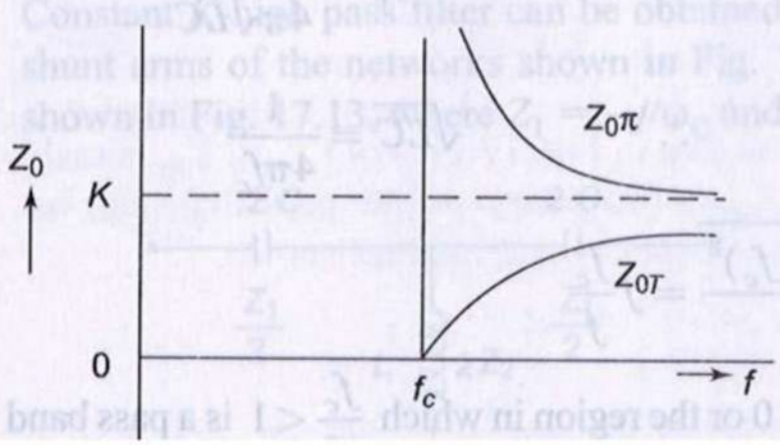
Since $\sqrt{C} = \frac{L}{k}$,

$$L = \frac{k}{4\pi f_c} \text{ and } C = \frac{1}{4\pi f_c k}$$

The characteristic impedance can be calculated using the relation

$$Z_{0T} = \sqrt{Z_1 Z_2\left(1 + \frac{Z_1}{4Z_2}\right)} = \sqrt{\frac{L}{C}\left(1 - \frac{1}{4\omega^2 LC}\right)}$$

$$Z_{0T} = k\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$



$$Z_{0\pi} = \frac{Z_1 Z_2}{Z_{0T}} = \frac{k^2}{Z_{0T}} = \frac{k}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}$$

The plot of characteristic impedances with respect to frequency is shown in Fig.above