

ENGINEERING MECHANICS

2nd Sem.

STUDY UNIT 1: Engg. Mechanics

Fundamentals of Engg. Mechanics

Definition of mechanics :— It is that branch of applied science, which deals with the laws & principles of mechanics, along with their applications to engineering problems.

It is very essential for an engineer in planning, designing & construction of his various types of structures & machines.

Statics — Statics deals with the forces & their effects while acting upon the bodies at rest.

Dynamics — Dynamics deals with the forces & their effects, while acting upon the bodies in motion.

Types — Kinetics & Kinematics

Kinetics — Which deals with the bodies in motion due to the application of forces.

Kinematics — Which deals with the bodies in motion without any references to the forces which are responsible for the motion.

Fundamental units — Every quantity is measured in terms of some arbitrary but internationally accepted units called "fundamental units".

ex — length, mass, time

Derived units — The units which are derived from fundamental units known as derived units.

ex — units of area, velocity, pressure etc.

Rigid bodies — It is a collection of particles with the property that the distance between particles remains unchanged during the course of motions of the body.

Mass — The amount of matter in any solid object or in any volume of liquid or gas.

Weight — The weight of an object is the force acting in the object due to gravity.

Length — The measurement or extent of something from end to end. The greater of two or the greatest of three dimensions of an object.

Time — The measured or measurable period during which an action, process or condition exists or continues.

Scalar & Vector — Scalar quantities are those quantities which have magnitude only. But vector quantities have both magnitude & directions.

ex — Scalar quantities — length, mass, distance etc.

Vector quantities — force, displacement, velocity etc.

S.I. units — The globally-agreed system of measurement units is named as International System of units or S.I. units.

Force

Definition of force — It is defined as an agent which produces or tends to produce, destroys or tends to destroy motion in called as force.

units in S.I. system — Newton.

Characteristics & effects of a force

Effects — i) It may change the motion of a body
ii) It may retard the motion of a body
iii) It may retard the forces already acting on a body thus bringing it to rest or in equilibrium.
iv) It may give rise to the internal stresses in the body on which it acts.

Characteristics — i) Magnitude of the force (10N, 100N etc)
ii) The direction of the line along which the force acts.
i.e. along OX, OY, at 30° N etc.
iii) Nature of the force i.e. whether the force is push or pull.
iv) The point at which the force acts on the body.

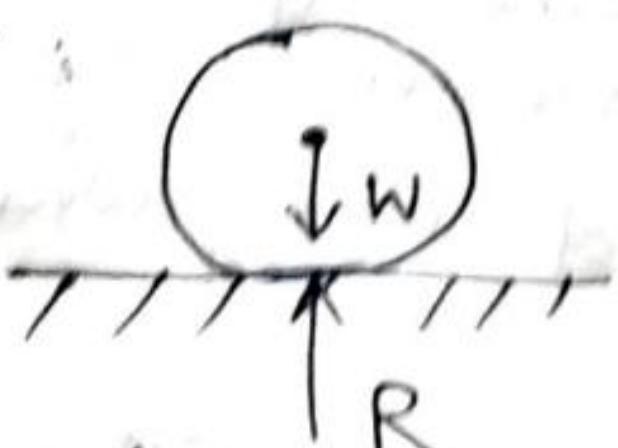
Action & Reaction force — When two bodies interact then action force & reaction force comes in picture. Even though their magnitude are same & their direction are opposite.



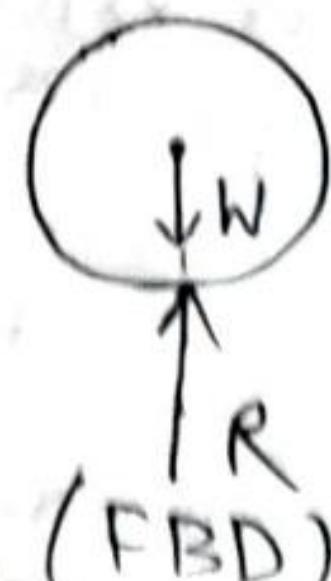
(action & reaction force)

Free body diagram (FBD) — These are diagrams used to show the relative magnitude & direction of all forces acting upon an object in a given situation.

example —



(Space diagram)



(FBD)

Principle of Transmissibility

It states that "if a force acts at any point on a rigid body, it may also be considered to act at any other point on its line of action, provided this point is rigidly connected with the body."

Resolution of a Force

The process of splitting up the given force into a number of components, without changing its effects on the body is called as "resolution of a force."

A force is generally resolved in two mutually perpendicular directions.

Method of Resolution

- 1) Resolve all the forces horizontally & find the algebraic sum of all the horizontal components (i.e. $\sum H$).
- 2) Resolve all the forces vertically & find the algebraic sum of all the vertical components (i.e. $\sum V$).
- 3) The resultant force (R) is given by —

$$R = \sqrt{\sum H^2 + \sum V^2}$$

- 4) The resultant force will be inclined at an angle θ , with the horizontal, such that,

$$\tan \theta = \frac{\sum V}{\sum H}$$

When $\sum V$ is +ve, R makes an angle between 0° & 180° .
When $\sum V$ is -ve, R makes an angle between 180° & 360° .

But $\sum V$ is -ve then R makes an angle between 0° to 90° .
Similarly, when $\sum H$ is +ve, R makes an angle between 0° to 90° or 270° to 360° .
But when $\sum H$ is -ve, then R makes an angle between 90° to 270° .

Force system

When two or more forces act on a body, they are called as force system or to form a system of force.

- a) Coplanar forces — whose lines of action lie on the same plane.
- b) Collinear forces — whose lines of action lie on the same line.
- c) Concurrent forces — Forces which meet at one point, but they may or may not be collinear.
- d) Coplanar concurrent forces — Forces which meet at one point & their lines of action also lie on the same plane.
- e) Coplanar non-concurrent forces — Forces which don't meet at one point but their lines of action lie on the same plane.

f) Non-coplaner concurrent forces — Forces which meet at one point but their ~~lines~~ lines of action don't lie on the same plane.

g) Non-coplaner non-concurrent forces — Forces which don't meet at one point & their lines of action don't lie on the same plane.

Composition of forces

Resultant force — If a number of forces P, Q... etc are acting simultaneously on a particle, then it is possible to find a single force which could replace them i.e. which would produce the same effect as produced by the all given forces. This single force is called as resultant force & is denoted by "R". And the given forces P, Q are called as component forces.

Composition of forces — The process of finding out the resultant force of a number of given forces is called "composition of forces".

Method of composition of forces are

a) Analytical method (Law of Parallelogram of forces).

b) Method of resolution.

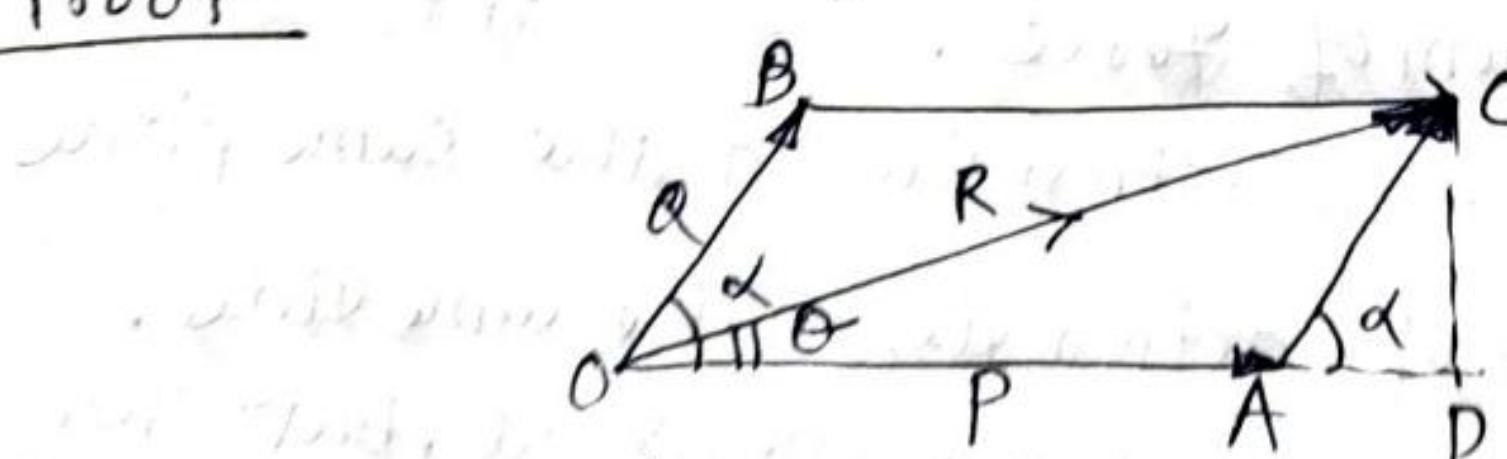
Law of Parallelogram of forces

It states that "if two forces, acting simultaneously on a particle be represented in magnitude & direction by the two sides (adjacent) of a parallelogram ; their resultant may be represented in magnitude & direction by the diagonal of the parallelogram, which passes through their point of intersection."

Mathematically,

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos\alpha}$$

Proof



Let P & Q forces acting at pt.'O' represented in magnitude & direction by the sides OA & OB respectively.

Now make it to a parallelogram OACB as shown above. Then according to the law, the resultant 'R' of P & Q represented in magnitude & direction by the diagonal "OC" of the parallelogram OACB.

Now from 'C' draw perpendicular CD by producing OA.

As AC is parallel to OB,

$$\therefore \angle CAD = \angle AOB = \alpha$$

(where α = angle between OA & OB)

Now in the right angled triangle OCD,

$$OC^2 = OD^2 + CD^2 = (OA + AD)^2 + CD^2$$

$$\text{or } OC^2 = (OA + Ac \cos \alpha)^2 + (Ac \sin \alpha)^2$$

$$\text{or } R^2 = (P + Q \cos \alpha)^2 + (Q \sin \alpha)^2$$

$$\text{or } R^2 = P^2 + 2PQ \cos \alpha + Q^2 \cos^2 \alpha + Q^2 \sin^2 \alpha$$

$$(\because \cos^2 \alpha + \sin^2 \alpha = 1)$$

$$\text{or } R = \sqrt{P^2 + 2PQ \cos \alpha + Q^2} \quad \text{--- (1)}$$

Let θ = angle made by R with P.

so, in the right angle triangle OCD,

$$\tan \theta = \frac{CD}{OD} = \frac{Ac \sin \alpha}{OA + AD} = \frac{Ac \sin \alpha}{OA + Ac \cos \alpha} = \frac{Q \sin \alpha}{P + Q \cos \alpha} \quad \text{--- (2)}$$

$$\therefore R = \sqrt{P^2 + 2PQ \cos \alpha + Q^2}$$

$$\& \tan \theta = \frac{Q \sin \alpha}{P + Q \cos \alpha}$$

Proved

Problems

No1. Two forces of 100N & 150N are acting simultaneously at a point. What is the resultant of these two forces if the angle between them is 45° ?

Ans — Given $P = 100\text{N}$, $Q = 150\text{N}$ & $\alpha = 45^\circ$.

We know that,

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \alpha}$$

$$= \sqrt{100^2 + 150^2 + 2 \times 100 \times 150 \cos 45^\circ} \text{ N}$$

$$= \sqrt{10000 + 22500 + 30000 \times 0.707} \text{ N}$$

$$= 232 \text{ N.}$$

Ans

No2 Find the magnitude of the two forces, such that if they act at 90° , their resultant is $\sqrt{10}$ N. But if they act at 60° , their resultant is $\sqrt{13}$ N.

Ans — Let the two forces be F_1 & F_2 .

When the angle is $90^\circ \rightarrow R = \sqrt{F_1^2 + F_2^2} = \sqrt{10}$

$$R = \sqrt{F_1^2 + F_2^2} \quad (\text{Resultant})$$

$$\sqrt{10} = \sqrt{F_1^2 + F_2^2} \quad (\text{Given})$$

$$\text{or } 10 = F_1^2 + F_2^2 \quad (\text{squaring both sides})$$

When the angle is $60^\circ \rightarrow$

$$\sqrt{13} = \sqrt{F_1^2 + F_2^2 + 2F_1 F_2 \cos 60^\circ}$$

$$\text{or } 13 = F_1^2 + F_2^2 + 2F_1 F_2 (0.5)$$

$$\text{or } F_1 F_2 = 13 - 10 = 3 \quad (\because F_1^2 + F_2^2 = 10)$$

$$\text{We know that, } (F_1 + F_2)^2 = F_1^2 + F_2^2 + 2F_1 F_2 = 10 + 6 = 16$$

$$\text{or } F_1 + F_2 = \sqrt{16} = 4 \quad \text{--- (1)}$$

$$\text{Similarly, } (F_1 - F_2)^2 = F_1^2 + F_2^2 - 2F_1 F_2 = 10 - 6 = 4$$

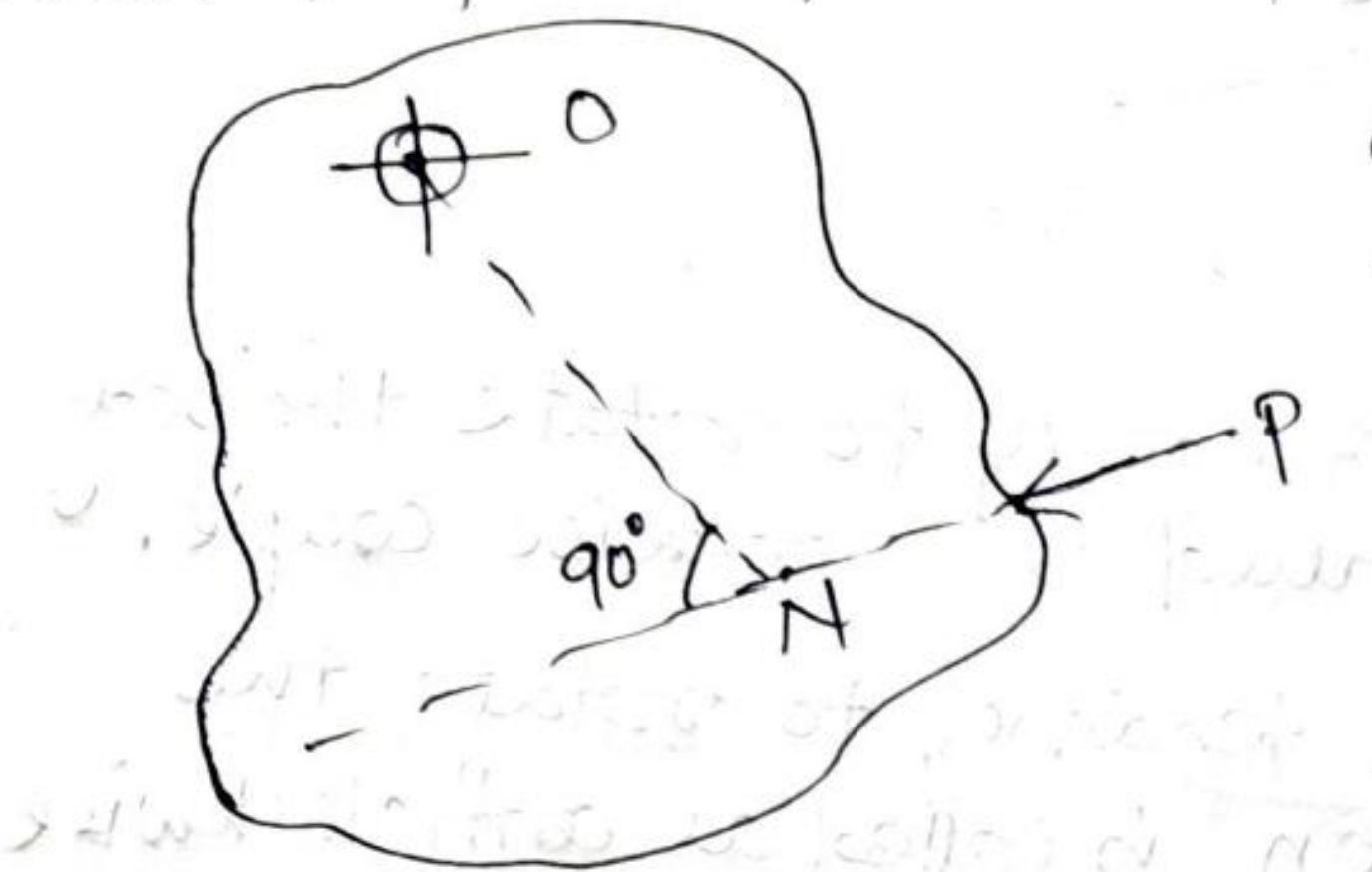
$$\text{or } F_1 - F_2 = \sqrt{4} = 2 \quad \text{--- (2)}$$

Now, solving equations (1) & (2) \rightarrow

$$F_1 = 3 \text{ N} \quad \& \quad F_2 = 1 \text{ N.} \quad \underline{\text{Ans}}$$

Moment of force

Moment of a force about a point is the product of the force & the perpendicular distance of the point from the line of action of the force.



Let a force P act on a body which is fixed at 'O'. Then moment of ' P ' about the point 'O' is given by —

$$= P \times ON$$

where, $ON = \text{perpendicular distance of } 'O' \text{ from the line of action of the force } 'P'$.

Unit

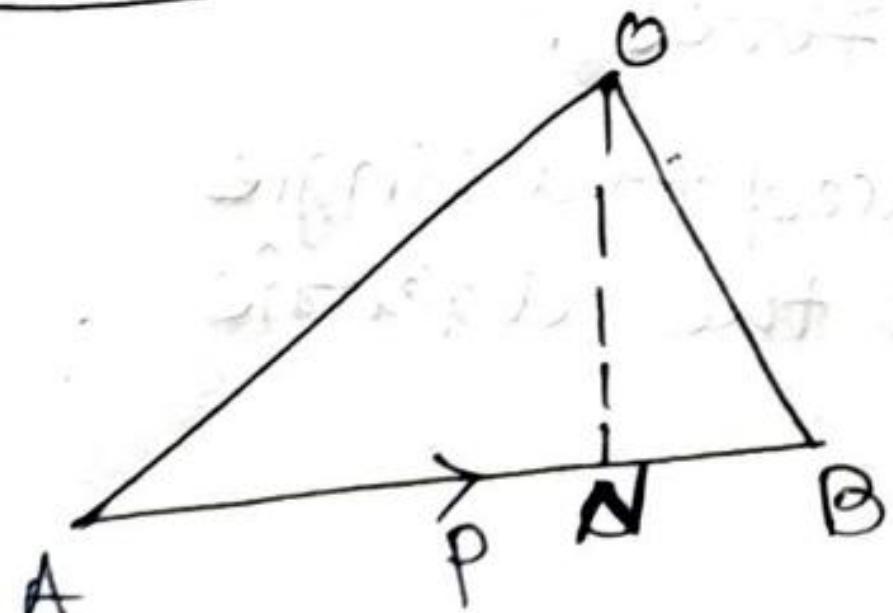
In SI system, unit of moment is kNm , Nmm etc.

Types of moment

Two types are there as —

- a) clockwise moment — in which, the moment has the tendency to rotate the body in clockwise direction.
- & b) anticlockwise moment — in which, the moment has the tendency to rotate the body in anticlockwise direction.

Geometrical representation of moment of a force



Let a force ' P ' represented in magnitude & direction by AB be acting on a body & let ' O ' be any point in the plane of the force P .

From ' O ', draw perpendicular ON to AB . Then moment of ' P ' about ' O ' is —

$$= P \times ON = 2 \times \frac{1}{2} P \times ON$$

$$= 2 \times \frac{1}{2} AB \times ON = 2 \times \text{Area of } \triangle AOB.$$

SO, moment of a force about any point is equal to twice the area of the triangle, whose base is the line to represent the force & whose vertex is the point about which the moment is taken.

Couple

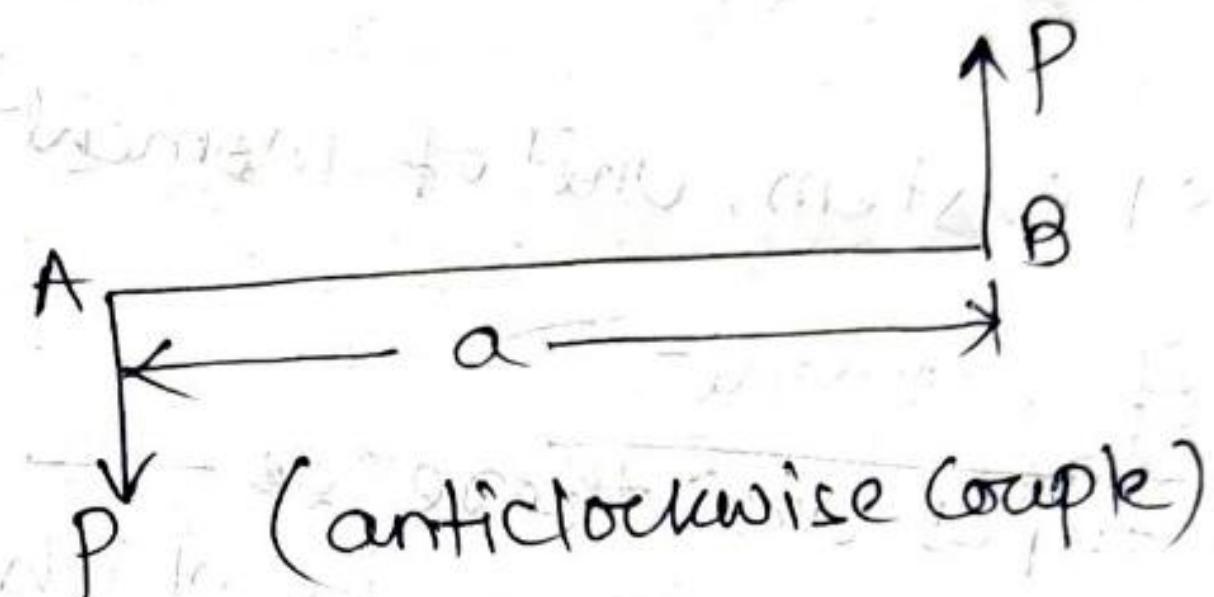
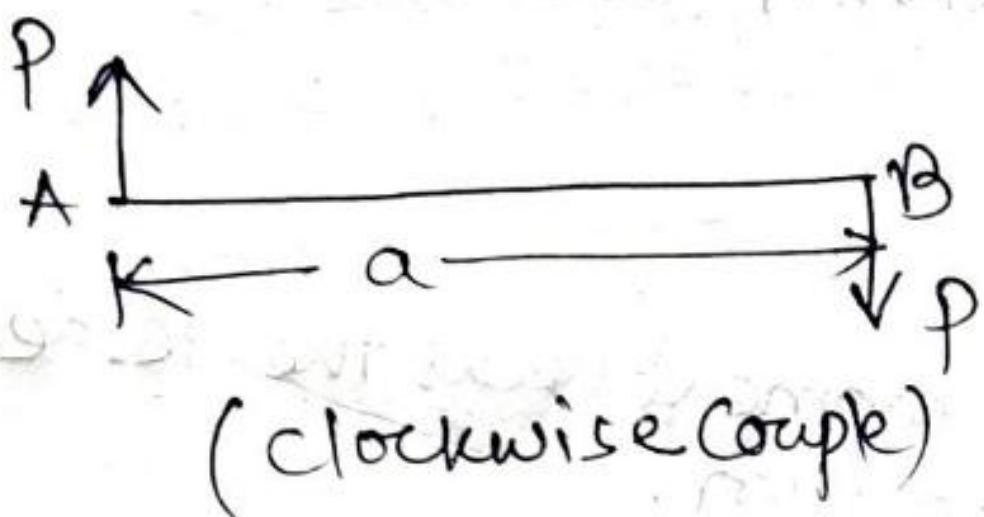
Definition— A pair of two equal & unlike parallel forces is known as a couple.

S.I. unit — Nm, KNm etc.

Classification of a Couple

It is classified as

- clockwise couple — whose tendency to rotate the body in a clockwise direction is called as clockwise couple.
- anticlockwise couple — whose tendency to rotate the body in anticlockwise direction is called as anticlockwise couple.



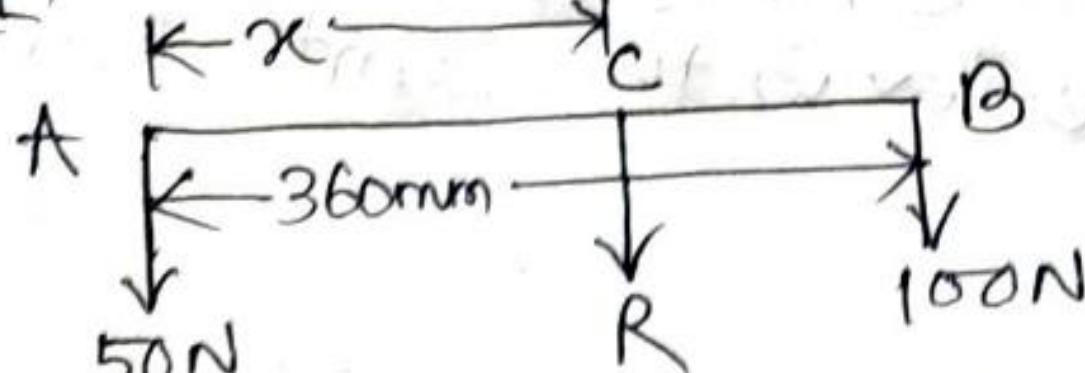
Characteristics of a Couple

- The algebraic sum of the forces constituting the couple is zero.
- The algebraic sum of the moments of the forces forming the couple about any pt. is same & equal to the moment of the couple itself.
- A couple can't be balanced by a single force.
- Any no. of coplaner couples can be reduced to a single couple whose magnitude will be equal to the algebraic sum of the moments of all the couples.

Problem

No1. Two like parallel forces of 50N & 100N act at the ends of a rod 360mm long. Find the value of resultant force & the point where it acts.

Soln



As the forces are like & parallel so, the value of R is —

$$R = 50 + 100 = 150 \text{ N} \quad \underline{\text{Ans.}}$$

Let x = the pt. where the resultant will act i.e. the distance between R & pt. A.

Now taking the clockwise & anticlockwise moments of the forces about pt. 'C' & equating the same —

$$50x = 100(360 - x) = 36000 - 100x$$

$$\text{or } 150x = 36000$$

$$\text{or, } x = \frac{36000}{150} = 240 \text{ mm.} \quad \underline{\text{Ans}}$$

Equilibrium

A little consideration will show that if the resultant of a number of forces acting on a particle is zero, then the particle will be in equilibrium. Set of forces whose resultant is zero are called equilibrium forces.

Methods for the equilibrium of coplanar forces

- 1) Analytical method
- 2) Graphical method

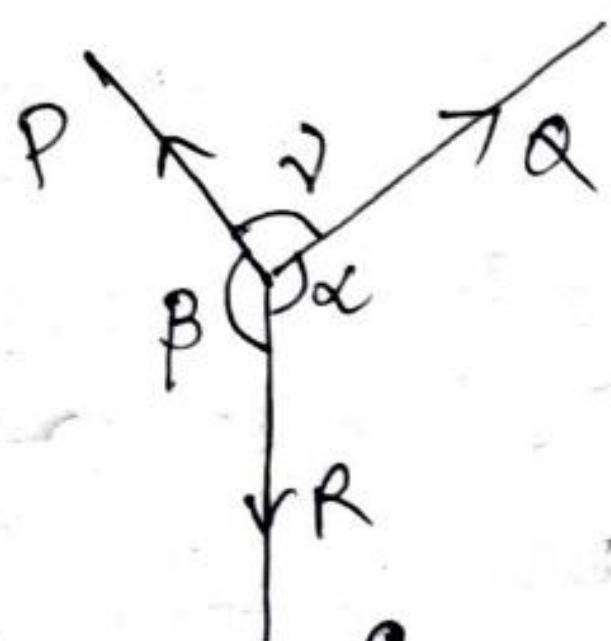
Lami's theorem

Statement — "If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two."

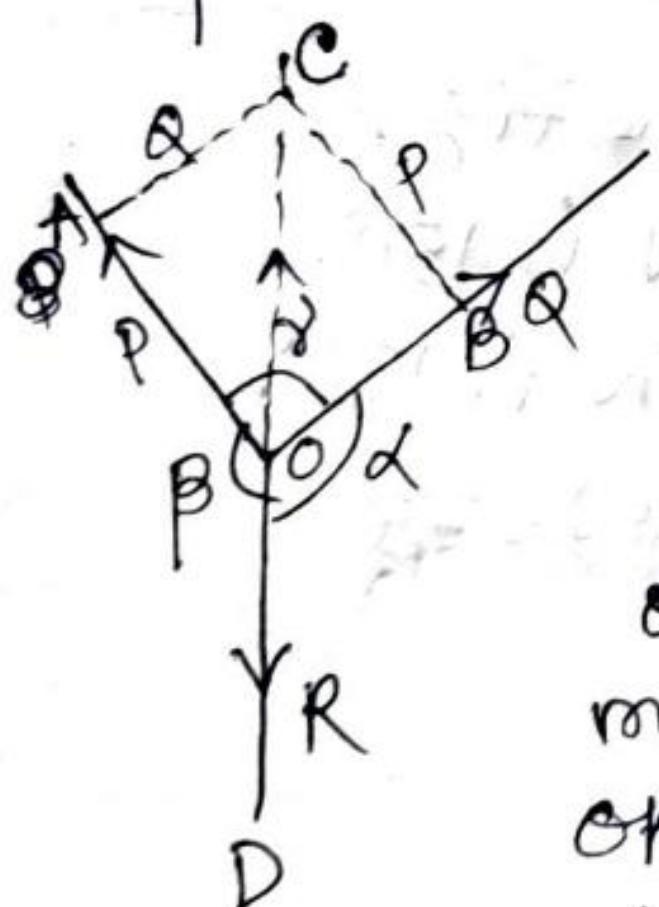
Mathematically,

$$\frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma}$$

where P, Q, R are three forces & $\alpha, \beta \& \gamma$ are the angles.



Proof



Take P, Q, R three coplanar forces acting at 'O'. Let the opposite angles to the three forces be $\alpha, \beta \& \gamma$ as shown.

Now, let complete the parallelogram OACB with OA & OB as adjacent sides. The resultant of $P \& Q$ will be the diagonal OC both in magnitude & direction of the parallelogram OACB.

As the forces are in equilibrium, so the resultant of $P \& Q$ must be in line with OD & equal to R but in opposite direction.

∴ From the geometry of the fig. →

$$BC = P \& AE = Q$$

$$\therefore \angle AOC = (180^\circ - \beta)$$

$$\text{or } \angle ACO = \angle BOC = (180^\circ - \alpha)$$

$$\therefore \angle CAO = 180^\circ - (\angle AOC + \angle ACO) = 180^\circ - [(180^\circ - \beta) + (180^\circ - \alpha)] \\ = 180^\circ - 180^\circ + \beta - 180^\circ + \alpha \\ = \alpha + \beta - 180^\circ$$

$$\text{But } \alpha + \beta + \gamma = 360^\circ \quad \text{--- (1)}$$

Now subtracting 180° from both sides of eqn (1) →

$$(\alpha + \beta - 180^\circ) + \gamma = 360^\circ - 180^\circ = 180^\circ$$

$$\text{or } \angle CAO = 180^\circ - \gamma$$

We know that in $\triangle AOC$,

$$\frac{OA}{\sin \angle ACO} = \frac{AC}{\sin \angle AOC} = \frac{OC}{\sin \angle CAO}$$

$$\text{or, } \frac{OA}{\sin(180^\circ - \alpha)} = \frac{AC}{\sin(180^\circ - \beta)} = \frac{OC}{\sin(180^\circ - \gamma)}$$

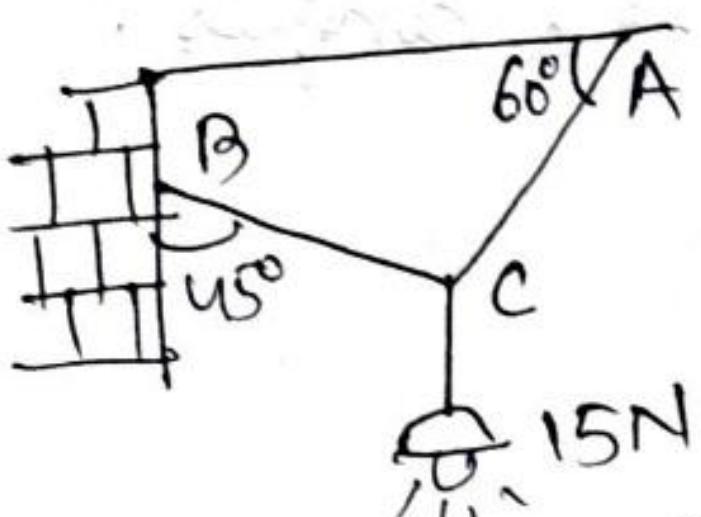
$$\text{or, } \frac{p}{\sin \alpha} = \frac{q}{\sin \beta} = \frac{R}{\sin \gamma} \quad (\because \sin(180^\circ - \theta) = \sin \theta)$$

Proved

Problem

problem

No1. An electric light fixture weighing 15N hangs from a point 'C' by two strings AC & BC. The strings AC is inclined at 60° to the horizontal & BC at 45° to the vertical shown in fig. Using Lami's theorem find the forces in the strings AC & BC.



soln

Let weight at C = 15 N.

~~Weight~~ at C = 15 N
~~Weight~~ = Force in the string AC

T_{AC} = Force in the strong BC.
 $\& T_{BC}$ = Force in the strong BC.

From the geometry of the fig.

angle between TAE & 15N is 150° .
angle between TA_T & 15N is 135°

angle between the & " T_{BC} & 15N is 135°.

$$8^{\circ} 11' " \quad BC \quad 120^{\circ} \quad (150^{\circ} - 60^{\circ}) = 75^{\circ}$$

By Lami's theorem —

$$\frac{15}{\sin 75^\circ} = \frac{T_{AE}}{\sin 135^\circ} = \frac{T_{BC}}{\sin 150^\circ}$$

$$\text{or, } \frac{15}{\sin 75^\circ} = \frac{T_{Ae}}{\sin 45^\circ} = \frac{T_{BC}}{\sin 60^\circ}$$

$$\therefore T_{AC} = \frac{15 \sin 45^\circ}{\sin 75^\circ} = \frac{15 \times 0.707}{0.9659} = 10.98 \text{ N.}$$

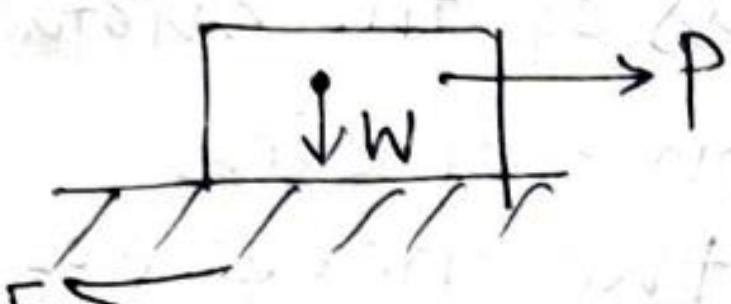
$$U_{TBC} = \frac{1.5 \sin 30^\circ}{\sin 75^\circ} = \frac{1.5 \times 0.5}{0.9659}$$

or, $T_{BC} = 7.76 \text{ N.}$ Ans

Ans

Friction

Definition — When an external force is applied to a body to move it over the surface of another body, an opposing force comes into play along the common surface of contact of two bodies. This opposing force is called "friction" or "frictional force". It is denoted by "F".



Types — static friction & dynamic friction.

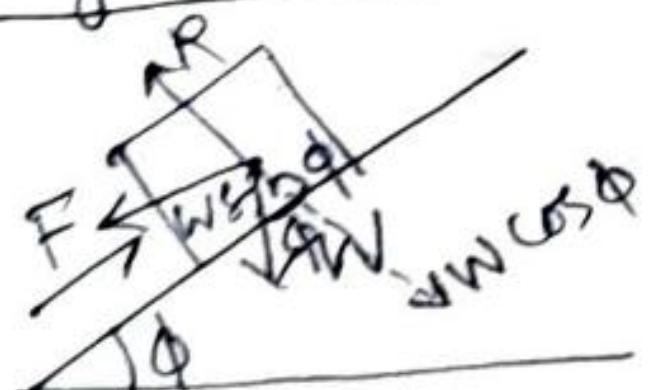
Limiting frictional force — There is a limit beyond which the force of friction cannot increase. If the force applied exceeds this limit, the frictional force cannot balance it & the body begins to move, in the direction of applied force. This maximum frictional force which comes into play when a body just begins to slide over the surface of other body is called "limiting friction".

Angle of friction



The angle of inclined plane (α) at which a body just begins to slide down the plane is called the "angle of friction".

Angle of repose



action of its own weight.

It is the maximum angle of inclination of an inclined plane with the horizontal for which a body lying on the inclined plane will be on the point of sliding down the inclined plane under the

Coefficient of friction (C.O.F.)

It is the ratio of limiting friction to normal reaction between the two bodies & is denoted by " μ ".

Mathematically,

$$C.O.F. (\mu) = \frac{F}{R} = \tan \phi$$

or $\mu = \frac{F}{R}$ where ϕ = angle of friction
 R = normal reaction betn the two bodies.

$$\text{or } F = \mu R \quad \mu = C.O.F.$$

Laws of friction

- 1) The force of friction always act along the common surface of contact bet'n two bodies.
- 2) The direction of friction is always opposite to the direction of motion of one body over the surface of another.
- 3) The friction is independent of the area of contact between the two surfaces.
- 4) The friction depends upon the roughness of the surface.
- 5) For moderate speeds, the force of friction remains constant. But it decrease slightly with the increase of speed.

Advantages & disadvantages of friction

Advantages — a) Friction is responsible for many types of motion.

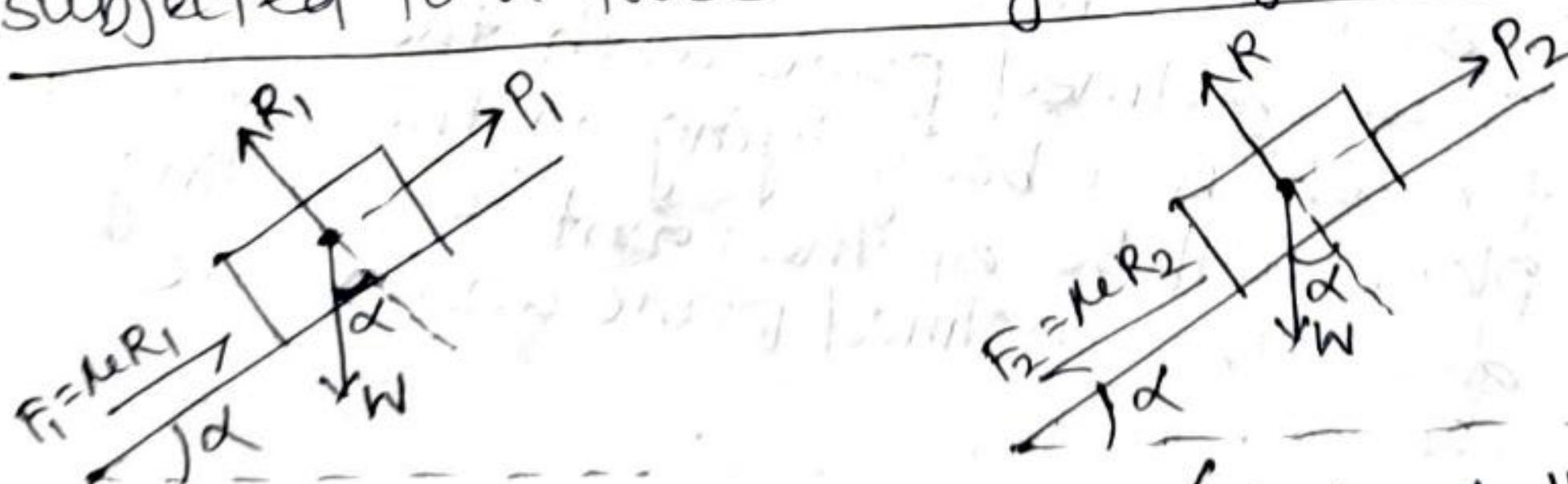
- b) It helps us walk on the ground
- c) Brakes in a car makes use of friction to stop the car.
- d) It helps in the generation of heat when we rub hands.

Disadvantages — a) It produces unnecessary heats.

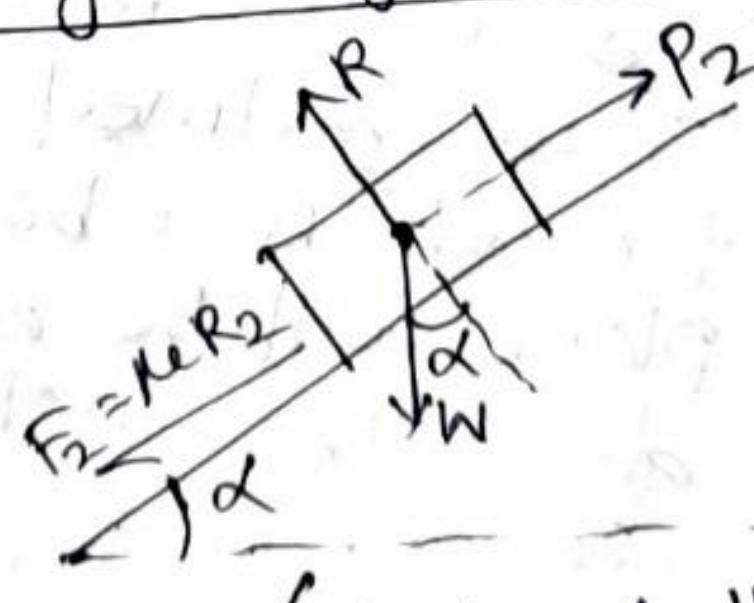
- leading to wastage of energy.
- b) The friction acts in the opposite direction of motion, so it slows down the motion of moving objects.
 - c) A lot of money goes into preventing friction.

Equilibrium of a body on a rough inclined plane

Subjected to a force acting along the inclined plane



(Body at the pt. of
sliding down)
(a)



(Body at the pt. of
sliding up)
(b)

Take a body lying on a rough inclined plane subjected to a force acting along the inclined plane, which keeps it in equilibrium as shown in fig a & b.

Let, W = Weight of the body

Angle, α = inclined plane makes with the horizontal

R = normal reaction

f = C.O.F. between the body & the inclined plane

ϕ = angle of friction such that $f_e = \tan \phi$

A little consideration will show that if the force is not there, the body will slide down the plane.

Now we take the two cases:

a) Min. force (P_1) which will keep the body in equilibrium when it is at the point of sliding downwards →

In this case, $F_1 = \mu R_1$ will act upwards, as the body is at the pt. of sliding downwards as shown in fig(a).

Now resolving the forces along the plane —

$$P_1 = W \sin \alpha - \mu R_1 \quad \text{--- (1)}$$

& now resolving the forces perpendicular to the plane —

$$R_1 = W \cos \alpha \quad \text{--- (2)}$$

Putting the value of R_1 in eqn (1) →

$$P_1 = W \sin \alpha - \mu W \cos \alpha = W(\sin \alpha - \mu \cos \alpha)$$

Now put $\mu = \tan \phi$ then we get,

$$P_1 = W \sin \alpha - \tan \phi W \cos \alpha \quad (\because \mu = \tan \phi)$$

Multiplying both sides by $\cos \phi$, →

$$P_1 \cos \phi = W(\sin \alpha \cos \phi - \sin \phi \cos \alpha) = W \sin(\alpha - \phi)$$

$$\therefore P_1 = W \times \frac{\sin(\alpha - \phi)}{\cos \phi}$$

b) Max. force (P_2) which will keep the body in equilibrium, when it is at the point of sliding upwards →

In this case, $F_2 = \mu R_2$ will act downwards as the body is at the point of sliding upward as shown in fig(b).

Now resolving the forces along the plane —

$$P_2 = W \sin \alpha + \mu R_2 \quad \text{--- (1)}$$

& now, resolving the forces perpendicular to the plane —

$$R_2 = W \cos \alpha \quad \text{--- (2)}$$

Put the value of R_2 in eqn (1) →

$$P_2 = W \sin \alpha + \mu W \cos \alpha = W(\sin \alpha + \mu \cos \alpha)$$

& now put the value of $\mu = \tan \phi$, we get —

$$P_2 = W(\sin \alpha + \tan \phi \cos \alpha)$$

Multiplying both sides by $\cos \phi$, we get —

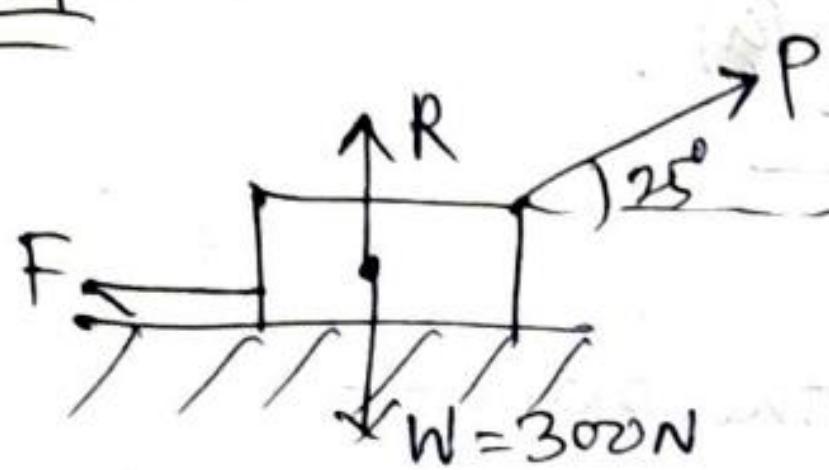
$$P_2 \cos \phi = W(\sin \alpha \cos \phi + \sin \phi \cos \alpha) = W[\sin(\alpha + \phi)]$$

$$\text{or, } P_2 = W \times \frac{\sin(\alpha + \phi)}{\cos \phi}$$

Problem

No1 A body of weight- 300N is lying on a rough horizontal plane having C.O.F. of 0.3. Find the magnitude of the force, which can move the body, while acting at an angle of 25° with the horizontal.

Soln



Given

$$W = 300\text{N}$$

$$\mu = 0.3$$

$$\& \text{angle}(\alpha) = 25^\circ$$

Let P = magnitude of the force which can move the body
 & F = force of friction

Now resolving the forces horizontally,

$$F = P \cos \alpha = P \cos 25^\circ = P \times 0.9063$$

& now resolving the forces vertically,

$$R = W - P \sin \alpha = 300 - P \sin 25^\circ$$

$$= 300 - P \times 0.4226$$

We know that force of friction (F) \rightarrow

$$0.9063 P = \mu R = 0.3 \times (300 - P \times 0.4226) = 90 - 0.1268 P$$

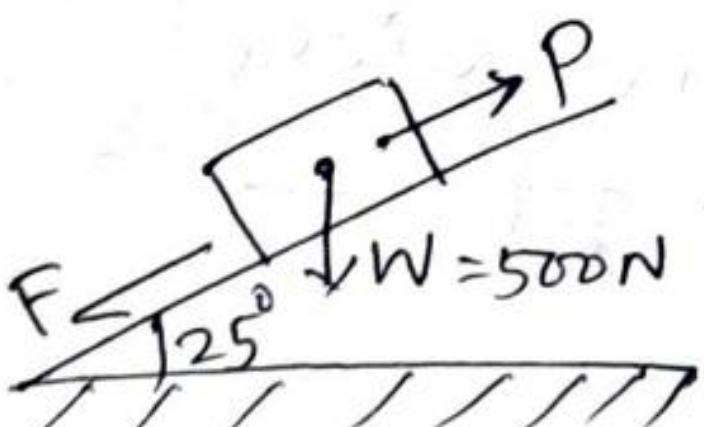
$$\text{or } 90 = 0.9063 P + 0.1268 P = 1.0331 P$$

$$\text{or, } P = \frac{90}{1.0331} = 87.1\text{N.} \quad \underline{\text{Ans}}$$

No2

A body of weight- 500N is lying on a rough plane inclined at an angle of 25° with the horizontal. It is supported by an effort (P) parallel to the plane as shown.

Find the minimum & maximum values of P , for which the equilibrium can exist, if the angle of friction is 20° .



Soln

$$\text{Given, } W = 500\text{N,}$$

$$\alpha = 25^\circ$$

$$\phi = \text{angle of friction} = 20^\circ$$

Minimum value of P

For $P_{\min.}(P_1)$, the body is at the pt. of sliding down. Also we know that, when the body is at the point of sliding downwards, then the force

$$P_1 = W \times \frac{\sin(\alpha - \phi)}{\cos \phi} = 500 \times \frac{\sin(25^\circ - 20^\circ)}{\cos 20^\circ}$$

$$\text{or, } P = 500 \times \frac{\sin 5^\circ}{\cos 20^\circ} = 500 \times \frac{0.0872}{0.9397} = 46.4 \text{ N. } \underline{\text{Ans.}}$$

Maximum of P

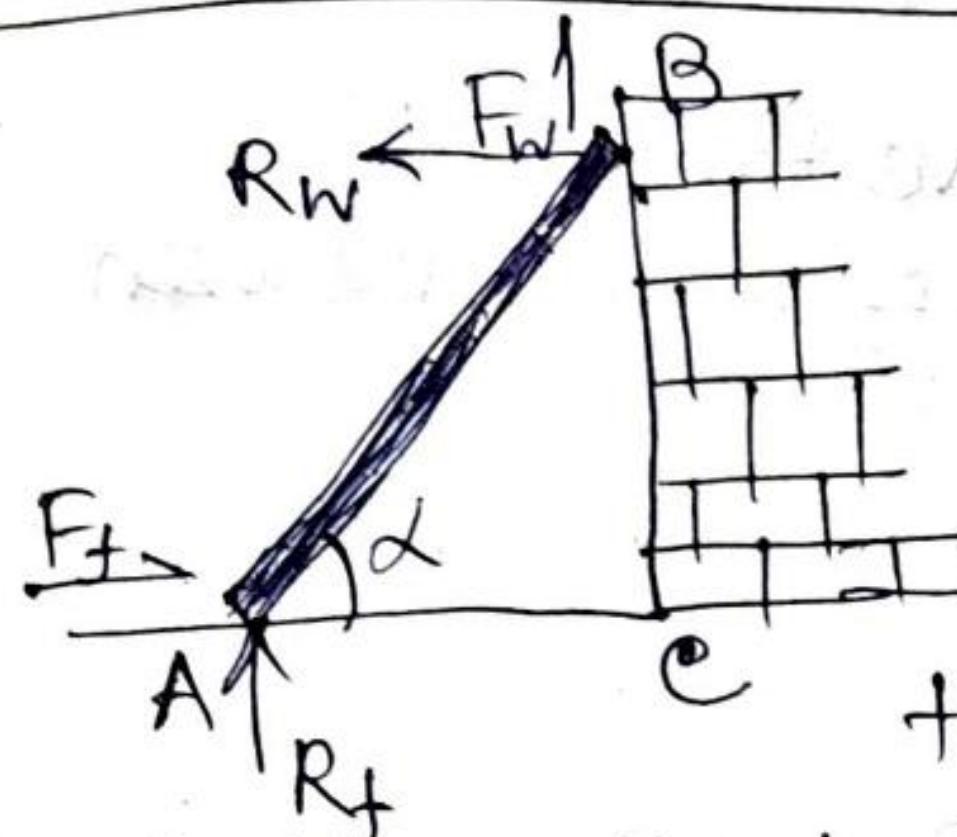
We know that, the $P_{\max}(P_2)$ value, the body is at the point of sliding upwards. We also know that, the body is at the point of sliding upwards, then the force is

$$P_2 = W \times \frac{\sin(\alpha + \phi)}{\cos \phi} = 500 \times \frac{\sin(25^\circ + 20^\circ)}{\cos 20^\circ}$$

$$\text{or, } P_2 = 500 \times \frac{0.7071}{0.9397} = 376.2 \text{ N.}$$

Ans

Ladder friction



Take a ladder AB resting on the rough ground & leaning against a wall as shown.

As the upper end tends to slip downwards so, the direction of the friction between the ladder & the wall (F_w) will be upwards

Similarly, as the lower end of the ladder tends to slip away from the wall, so the direction of the force of friction between the ladder & the floor (R_f) (F_f) will be towards the wall as shown.

As the system is in equilibrium, so the algebraic sum of the horizontal & vertical components of the forces must be equals to zero.

* The normal reaction at the floor (R_f) will act perpendicular to the floor. Similarly, normal reaction of the wall (R_w) will also act perpendicular to the wall.

Problem

No1 A uniform ladder of length 3.25m & weight 250N is placed against a smooth vertical wall with its lower end 1.25m from the wall. The C.O.F. betw the ladder & floor is 0.3.

What is the frictional force acting on the ladder at the point of contact between the ladder & the floor? Show that the ladder will remain in equilibrium in this position.

Soln Given, l = length of the ladder = 3.25m

$$W = 250N$$

Distance between the ladder's lower end & the wall = 1.25m

$$\mu_f = 0.3$$

Frictional force acting on the ladder

The forces acting on the ladder are shown in fig. given.

let F_f = force of friction acting on the ladder at the point of contact between the ladder & floor.

R_f = normal reaction at the floor.

As the ladder is placed against a smooth vertical wall, so there will be no friction at the pt. of contact between the ladder & the wall.

Resolving the forces vertically,

$$R_f = 250N$$

From the fig, we get —

$$BC = \sqrt{3.25^2 - 1.25^2} = 3.0m$$

Taking moments about B & equating the same —

$$F_f \times 3 = (R_f \times 1.25) - (250 \times 0.625)$$

$$= (250 \times 1.25) - 156.3 = 156.2N$$

$$\therefore F_f = \frac{156.2}{3} = 52.1N. \text{ Ans}$$

Equilibrium of ladder

\therefore Maximum friction available at the pt. of contact between the ladder & the floor is —

$$= \mu_f R_f = 0.3 \times 250 = 75N$$

\therefore At the point of contact, the value of friction is 75N which is more than the friction required for equilibrium (52.1N). So, the ladder will remain in equilibrium position.

Ans

Engg. Mechanics (2nd Sem.) (Diploma)

(~~class~~) Simple Lifting Machine

Machine — A machine is a device which receives energy in some available form & utilise it for doing a particular useful work.

So, machine help us in doing a specific work much easily in a convenient manner.

ex — A motor converts elect. energy into mech. energy to do some useful work.

Simple lift^g m/c — It is a device which enables us to lift a heavy load by applying comparatively lesser effort.

Compound lifting m/c — It is a device consist^g of a number of simple lift^g m/c which can work at a faster speed or with a much less effort as compared to a simple lifting m/c.

Thus a compound lift^g m/c is a combination of no. of simple lift^g m/c.

Mechanical Advantage (M. A.)

The mechanical advantage (M. A.) is the ratio of the wt. lifted (W) to the effort applied (P) & is always expressed in pure number.

Mathematically,

$$\boxed{M. A. = \frac{W}{P}}$$

Input of a machine

The input of a m/c is the work done on the m/c. In a lifting m/c, it is measured by the product of effort & the distance through which it has moved.

Output of a machine

The output of a m/c is the actual work done by the machine. In a lift'g m/c, it is measured by the product of the weight lifted & the distance through which it has been lifted.

Efficiency of a machine

It is the ratio of output to the input of a machine & is generally expressed as a percentage.

Mathematically,

$$\text{Efficiency}(\eta) = \frac{\text{Output}}{\text{Input}} \times 100$$

Ideal machine

If the efficiency of a m/c is 100%, i.e. if the output is equal to input, the m/c is called as a perfect or an ideal m/c.

Velocity Ratio (V.R.) - It is the ratio of the dist. moved by effort (y) to the distance moved by the load (x). & is always expressed in pure number.

Mathematically,

$$V.R. = \frac{y}{x}$$

Relation between efficiency, Mech. advantage

Velocity ratio of a lifting m/c

It is an important relation of a lifting m/c.

Consider a lifting m/c.

Let W = Load lifted by the m/c

P = Effort reqd. to lift the load

y = Distance moved by the effort in lifting the load

x = Distance moved by the load.

We know that, $M.A. = \frac{W}{P}$ & $V.R. = \frac{y}{x}$

Also we know that,

Input of a machine = Effort applied \times Dist.

through which the effort has moved

$$= P \times y \quad \text{--- (1)}$$

and output of a machine = Load lifted \times Dist.

through which load has been lifted

$$= W \times x \quad \text{--- (2)}$$

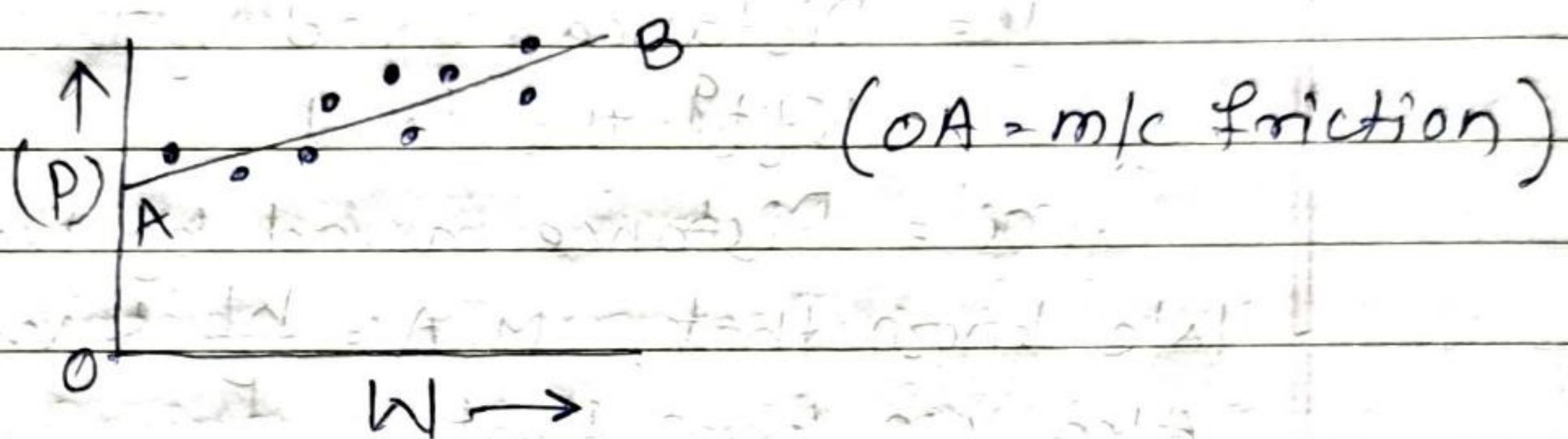
$$\therefore \text{Efficiency}(\eta) = \frac{\text{Output}}{\text{Input}} = \frac{W \times x}{P \times y}$$

$$= \frac{W/P}{y/x} = \frac{M.A.}{V.R.}$$

It is seen from the above relation that the values of MA & VR are equal only in case of machines whose η is 100%. But in actual practice it is not possible. Here the m/c is treated as ideal machine.

Law of a Machine

It is defined as the relationship between the load lifted & the effort applied. Thus for any m/c, if we record the various efforts required to raise the corresponding loads & plot a graph between effort & load, we get a straight line AB as shown in fig.



We know that the intercept OA represents the amount of friction offered by the m/c. Or in other words, this is the effort reqd. by the m/c to overcome the friction, before it can lift any load.

Mathematically, the law of a lifting machine is given by the relation :—

$$P = mw + c$$

where, P = Effort applied to lift the load
 m = A constant called Coefficient of friction which equals the slope of the line AB

w = Load lifted

c = Another constant, which represents the m/c friction (i.e. OA).

(Diploma) ~~Ch~~ (Simple Lifting Machine)

Reversibility of Machine → In some cases, there are some machines which are capable of doing work in the reversed direction after the effort is removed. Such ~~machines~~ machines are called reversible machine and the action of such machine is called as reversibility of the machine.

Conditions for reversibility of a machine

Take a reversible machine.

Let W = Load to be lifted.

P = Effort reqd. to lift W .

y = distance moved by the effort.

x = distance moved by the load.

∴ Input of the m/c = Pxy — ①

& Output of the m/c = Wxx — ②

We know that machine friction is →

$$= \text{Input} - \text{Output} = (Pxy) - (Wxx) - ③$$

A little consideration will show that, for a reversible machine, the output is more than the machine friction when effort (P) = 0, i.e.

$$Wxx > Pxy - Wxx$$

$$\text{or, } 2Wxx > Pxy$$

$$\text{or, } \frac{Wxx}{Pxy} > \frac{1}{2}$$

$$\text{or, } \frac{W}{P} / \frac{y}{x} > \frac{1}{2} \text{ or, } \frac{MA}{VR} > \frac{1}{2} \left(\because \frac{W}{P} = MA \right. \\ \left. \text{and } \frac{y}{x} = VR \right)$$

$$\therefore \text{Efficiency}(\eta) > \frac{1}{2} = 50\%$$

So, condition for a reversible machine is that, its efficiency should be more than 50%.

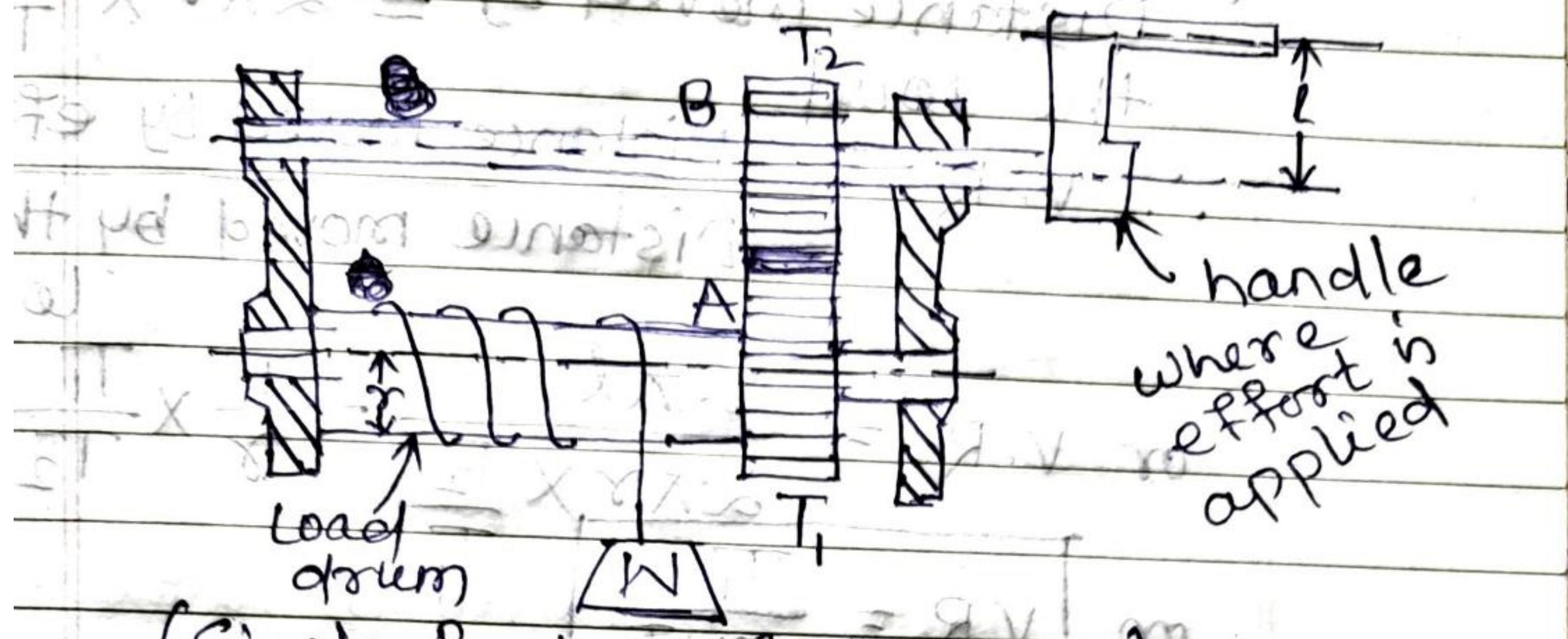
Self-Locking machine —

Sometimes, a machine is not able to do any work in the reversed way, when the effort is removed.

Such a machine is called a non-reversible or self-locking machine.

The condition for self-locking machine is that the efficiency(η) shouldn't be more than 50%.

Single Purchase Crab Winch



(Single Purchase Crab Winch)

In this a rope is fixed to the drum & is wrapped by few turns on the drum. The free end of the rope carries the load 'W' as shown in fig. A toothed wheel B called pinion is geared with toothed wheel A as shown in fig.

Let T_1 = No. of teeth on A
 T_2 = No. of teeth on B
 l = length of the handle
 r = radius of the load drum
 W = Load to be lifted
 & P = effort applied to lift the load at the end of the handle.

\therefore Distance moved by the effort, for one revolution of the handle = $2\pi l$
 Number of revolution made by pinion $B = \frac{l}{r}$
 & number of revolution made by wheel A = T_2/T_1

Distance moved by the load

$$\therefore \text{Distance moved by effort} = 2\pi r \times \frac{T_2}{T_1}$$

$$\therefore V.R. = \frac{\text{Distance moved by effort}}{\text{Distance moved by the load}}$$

$$\text{or } V.R. = \frac{2\pi l}{2\pi r \times T_2} \times \frac{T_1}{T_2}$$

$$\therefore V.R. = \frac{T_1/W}{\pi r \times T_2}$$

$$M.A. = \frac{W}{P}$$

$$\therefore \text{efficiency } (\eta) = \frac{M.A.}{V.R.}$$

Problem

Ques. In a single purchase crab winch, the no. of teeth on pinion is 25 and that on the spur wheel 250. Radius of the drum is 150 mm & length of handle is 300mm. Find the η of the m/c if an effort of 20N can lift a load of 300N.

Solution

Given: $T_2 = 25$, $T_1 = 250$, $r = 150 \text{ mm}$, $l = 300 \text{ mm}$

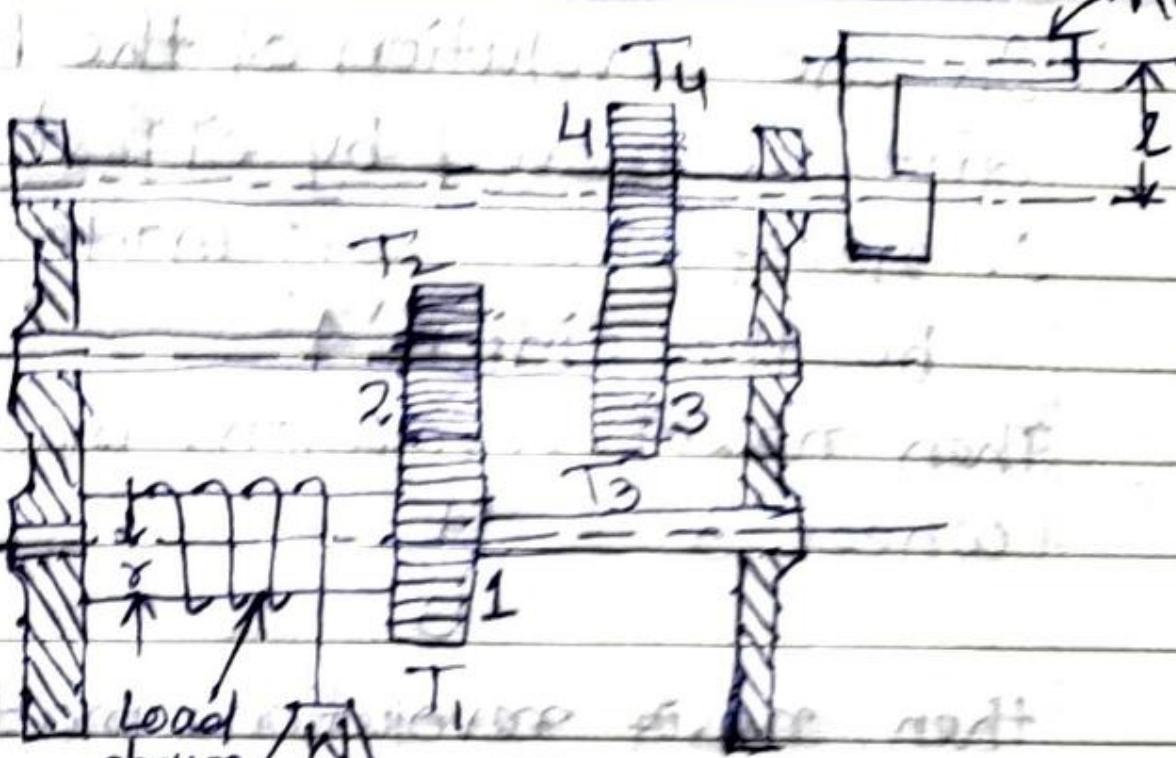
$$P = 20 \text{ N} \quad W = 300 \text{ N}$$

$$\therefore M.A. = \frac{W}{P} = \frac{300}{20} = 15, V.R. = \frac{T_1}{2\pi r \times T_2} = \frac{250}{150 \times 25} = \frac{2}{3}$$

$$\therefore V.R. = 20, \text{ Then, } \eta = \frac{M.A.}{V.R.} = \frac{15}{20} = 0.75 = 75\%$$

Double Purchase Crab Winch

(2)



(Double purchase crab winch)

It is a modified design of single purchase crab winch, to get a high V.R. In this there are two spur wheels of teeth T_2 & T_3 with two pinions of teeth T_1 & T_4 . The arrangement of wheels & pinions are in such a way that, T_1 mesh with pinion of teeth T_2 & wheel of teeth T_3 mesh with the pinion of teeth T_4 . The effort is applied to the handle as shown in fig. above.

Let T_1 & T_3 = No. of teeth of spur wheels.

T_2 & T_4 = No. of teeth of the pinions

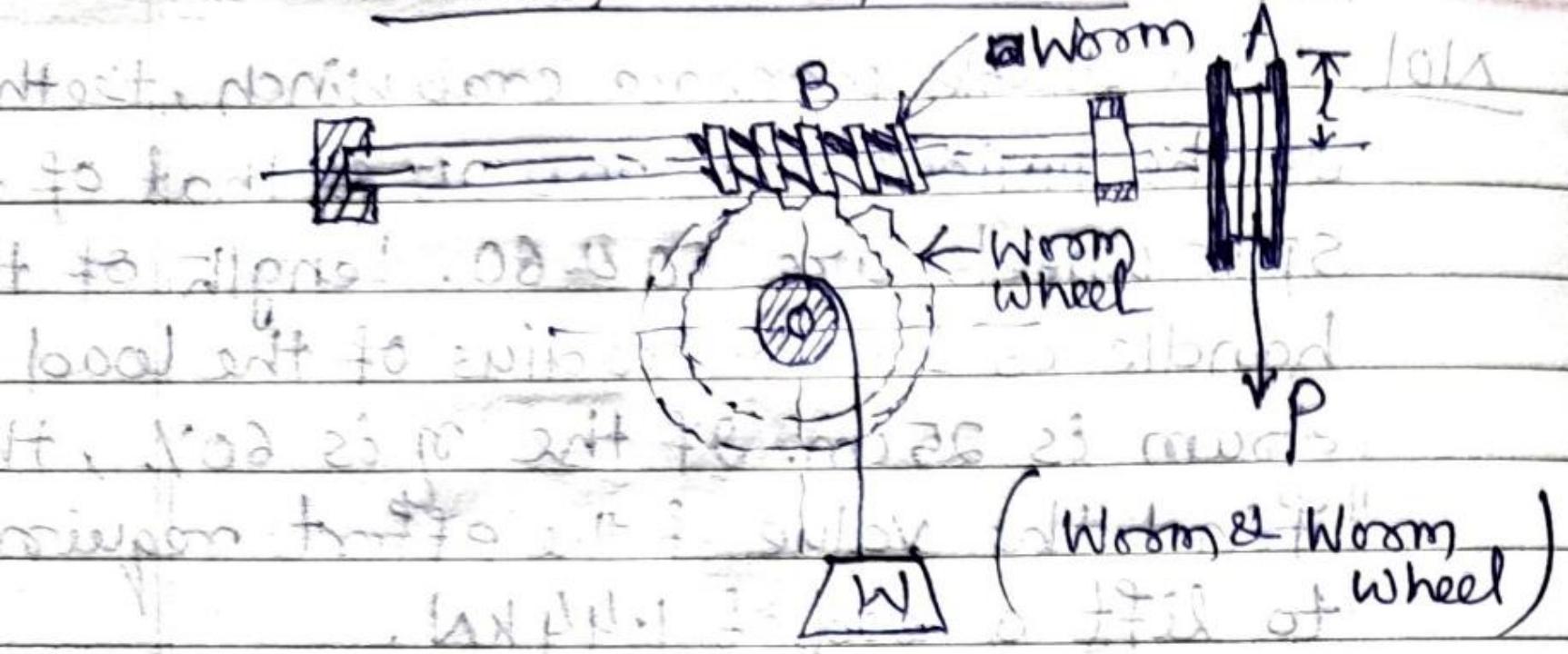
l = length of the handle

r = radius of the load drum

W = Load Lifted

P = Effort applied at the end of the handle.

Worm & Worm Wheel



It consists of a square threaded screw with 'B' known as worm and a toothed wheel called as worm wheel as shown in fig. A wheel 'A' is attached to the worm and a rope is wrapped round 'A' to give the effort. A load drum is securely mounted on the worm wheel. Both worm & worm wheel are geared with each other as shown in fig. above.

Let l = radius of effort wheel

r = radius of load drum

W = Load to be lifted

P = effort applied to lift the

Q T = No. of teeth on load worm wheel

\therefore The distance moved by the effort, for one revolution of wheel = $2\pi l$

If the worm is of single threaded, then the 'B' pushes the worm wheel through one teeth, then the distance moved by the load drum = $\frac{1}{T}$ revolution

& the distance moved by 'W' = $\frac{2\pi r}{T}$

$$\therefore VR = \frac{\text{Dist. moved by the effort (P)}}{\text{Dist. moved by the load (W)}}$$

$$= \frac{2\pi l}{2\pi r} = \frac{l \times T}{r}$$

$$\text{Now } MA = \frac{W}{P}, \therefore \eta = \frac{MA}{VR} = \frac{W/P}{l \times T} = \frac{W}{l \times T}$$

Problem — In a worm & worm wheel, the number of teeth on the worm wheel is 50. The dia. of the effort wheel is 200mm & that of load drum is 100mm. Find VR? If the η of the machine is 30%, then find the effort required to lift a load of 300N.

Solution — Given,

$$T = 50, l = \frac{200}{2} = 100\text{mm}, r = \frac{100}{2} = 50\text{mm}$$

$$\therefore \eta = 30\% = 0.3 \text{ & } W = 300\text{N}. \text{ Then } P = ?$$

$$\text{Given } VR = \frac{l \times T}{r} = \frac{100 \times 50}{50} = 100 \text{ (Ans)}$$

$$\text{Then } M.A. = \frac{W}{P} = \frac{300}{P}$$

$$\text{But, } \eta = 0.3 \therefore \eta = \frac{MA}{VR}$$

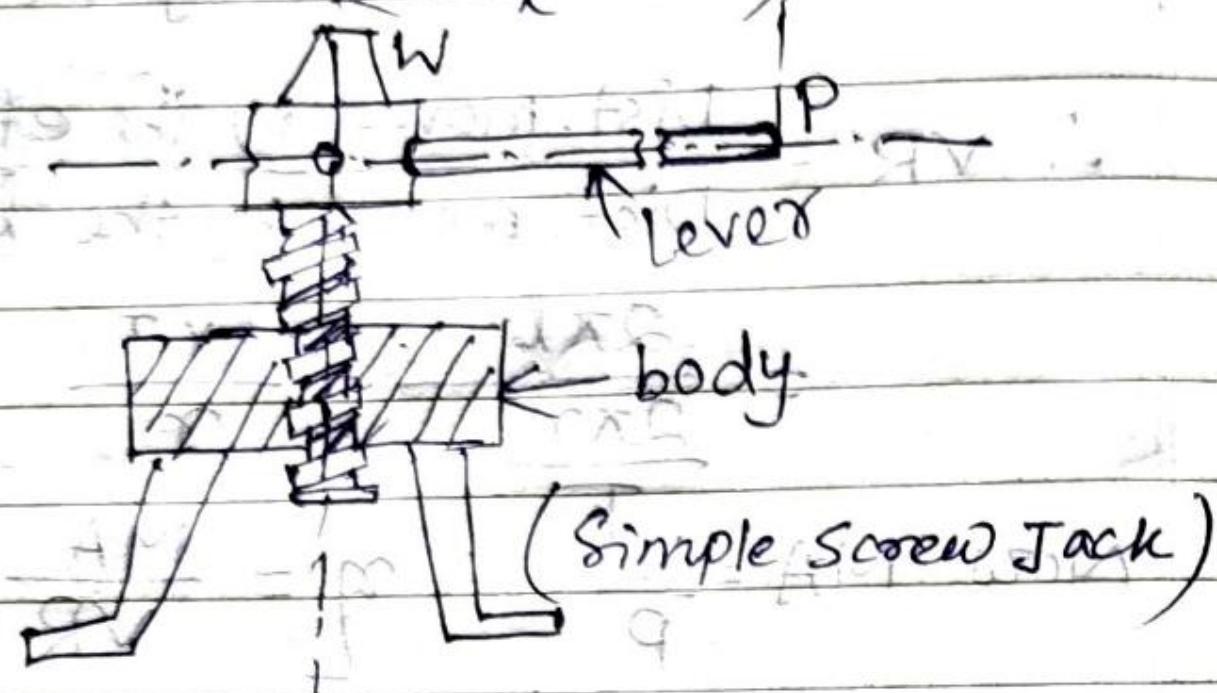
$$\therefore \frac{W}{P} = 0.3 \therefore 0.3 = \frac{300}{P}$$

$$\therefore P = \frac{300}{0.3} = 1000\text{N. (Ans)}$$

$$l \times T = 100 \times 50 = 500$$

\therefore Final effort = $500 \times 0.3 = 150\text{N.}$

Screw Jack



Screw jack consists of a nut act as a body in which a screw is there. The working principle of screw jack is similar to that of an inclined plane. Screw Jack is rotated by the application of an effort at the end of the lever for load lifting.

Consider a single threaded simple screw jack.

Let, l = length of the effort lever

p = pitch of the screw

W = load to be lifted

P = effort required to lift

the load, which is applied at the end of the lever.

\therefore In one revolution of the screw, distance moved by the effort is $\rightarrow = 2\pi l$

& distance moved by the load = P .

$$\therefore V.R = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}} = \frac{2\pi l}{P}$$

$$\therefore M.A = \frac{W}{P} \cdot \text{Now, } \eta = \frac{M.A}{V.R}$$

Problem — A screw jack has a thread of 2cm pitch. What effort applied at the handle of 1m long will be required to lift a load of 10KN, if η at this load is 40%.

Solution →

$$\text{Given, } p = 2\text{cm}, l = 1\text{m} = 100\text{cm}, W = 10\text{KN} = 10,000\text{N}, \eta = 40\% = 0.4.$$

For a screw Jack,

$$V.R = \frac{2\pi l}{P} = \frac{2\pi \times 100}{2} = 314.6$$

$$\text{Then } M.A = \frac{W}{P} = \frac{10000}{P}, \text{ (where, } P = \text{effort})$$

$$\text{But } \eta = \frac{M.A}{V.R}$$

$$\text{or, } 0.4 = \frac{10000}{P} = \frac{31.83}{314.6} \cdot P$$

$$\text{or, } 0.4 = \frac{31.83}{P}$$

$$\text{or, } P = \frac{31.83}{0.4} = 79.575\text{N.}$$

(Ans)