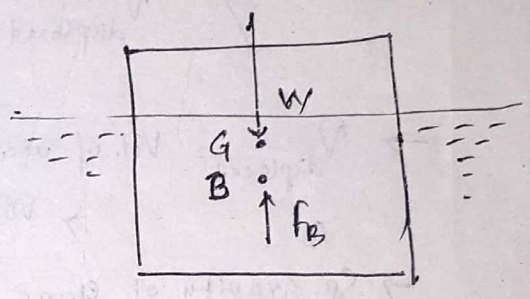
* Buoyancy and floatation :-

→ **Buoyancy** - (Archimede's Principle)

When a body is immersed in a fluid, an upward force is exerted by the fluid on the body. This upward force is equal to the weight of the fluid displaced by the body, and is called the force of buoyancy or simply buoyancy

→ **Centre of Buoyancy** -

It is the point through which the force of buoyancy is supposed to act. As the force of buoyancy is a vertical force and is equal to the weight of the fluid displaced by the body, the centre of buoyancy will be the centre of gravity of the fluid displaced.



W = weight of the body

G = C.G. of the body

F_B = Buoyant force or wt. of fluid displaced by the body

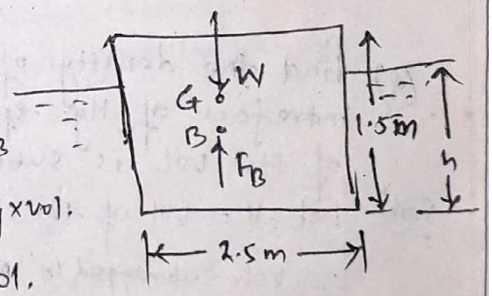
B = Centre of buoyancy or C.G of the fluid displaced.

→ **Example** (Q) Find the vol. of water displaced and position of centre of buoyancy for a wooden block of width 2.5m and of depth 1.5m when it floats horizontally in water. The density of wooden block is 650 kg/m³ and its length is 6m.

Sol^n: width = 2.5m $\quad \rho_{block} = 650\ kg/m^3$

depth = 1.5m

Length = 6m



Vol. of the block = $2.5 \times 1.5 \times 6 = 22.5\ m^3$

weight of block = ~~density × vol.~~ wt. density × vol.

= ~~650 × 22.5~~ = $\rho g \times$ vol.

= $650 \times 9.81 \times 22.5 = 143471\ N$.

for equilibrium ⇒ Weight of water displaced = wt. of the block

⇒ wt. density of water × vol. displaced = 143471 N

⇒ $\rho_{H_2O} \times g \times$ vol. of water displaced = 143471

⇒ $1000 \times 9.81 \times V = 143471$

⇒ $V = \dfrac{143471}{9810} = 14.625\ m^3$

Position of centre of Buoyancy $= h/2$

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$\therefore \; 2.5 \times h \times 6 = 14.625 \; m^3$

$\rightarrow \; h = \dfrac{14.625}{2.5 \times 6} = 0.975 \; m$

$\therefore$ Centre of Buoyancy $= \dfrac{h}{2} = \dfrac{0.975}{2} = 0.4875 \; m$ from bas

(Q) A stone weighs 392.4 N in air and 196.2 N in water. Compute the vol. of stone and its sp. gravity.

Solⁿ. → for equilibrium $W_{in\ air} - W_{in\ water} = W_{water\ displaced}$

$\Rightarrow \; 392.4 - 196.2 = 1000 \times 9.81 \times V_{displaced}$

$\Rightarrow \; V_{displaced} = \dfrac{196.2}{9810} = \dfrac{1}{50} \; m^3 = \dfrac{1}{50} \times 10^6 \; cm^3$

$= 2 \times 10^4 \; cm^3$.

→ $V_{displaced}$ = Vol. of water displaced = Vol. of stone

→ Vol. of stone $= 2 \times 10^4 \; cm^3$

→ Sp. gravity of stone :-

Mass of stone $= \dfrac{W_{in\ air}}{g} = \dfrac{392.4}{9.81} = 40 \; kg$

Density of stone $= \dfrac{\text{Mass of stone}}{\text{Vol. of stone}} = \dfrac{40 \; kg}{1/50 \; m^3}$

$\Rightarrow \; \rho_{stone} = 40 \times 50 = 2000 \; kg/m^3$

$\therefore$ Sp. gr. of stone $= \dfrac{\rho_{stone}}{\rho_{H_2O}} = \dfrac{2000}{1000} = 2$

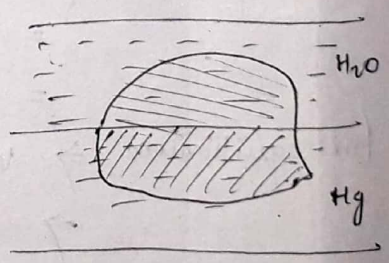
(Q) Find the density of metallic body which floats at the interface of Hg sp. gr. 13.6 and water such that 40% of its vol. is submerged in Hg and 60% in water.

Solⁿ. Let V = Vol. of the body

$\therefore$ Vol. submerged in mercury $= \dfrac{40}{100} \times V$

$= 0.4V \; m^3$

Vol. submerged in water $= 0.6V \; m^3$



for equilibrium :-

Total buoyant force = weight of the body

= force of buoyancy due to water + force of buoyancy due to mercury

= wt. of water displaced + wt. of mercury displaced

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→ Total Buoyant force $= \rho_{H_2O} \times g \times 0.6V + \rho_{Hg} \times g \times 0.4V$

→ $\rho_{body} \times g \times V = gV(\rho_{H_2O} \times 0.6 + \rho_{Hg} \times 0.4)$

→ $\rho_{body} = 1000 \times 0.6 + 13600 \times 0.4$

$= 600 + 5440$

$= 6040 \, kg/m^3$ (Ans)

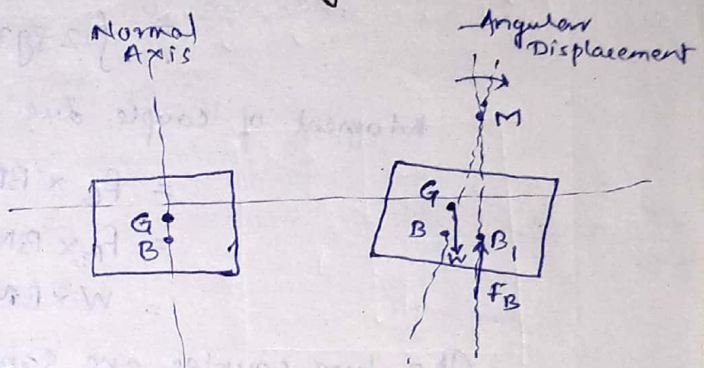
* Meta-centre and Meta-centric Height :-

→ **Meta-centre** -

The point about which a body starts oscillating when the body is tilted by a small angle.

OR

The point at which the line of action of the force of buoyancy will meet the normal axis of the body when the body is given a small angular displacement.



B = centre of buoyancy

B_1 = shifted centre of buoyancy

M = Meta-centre

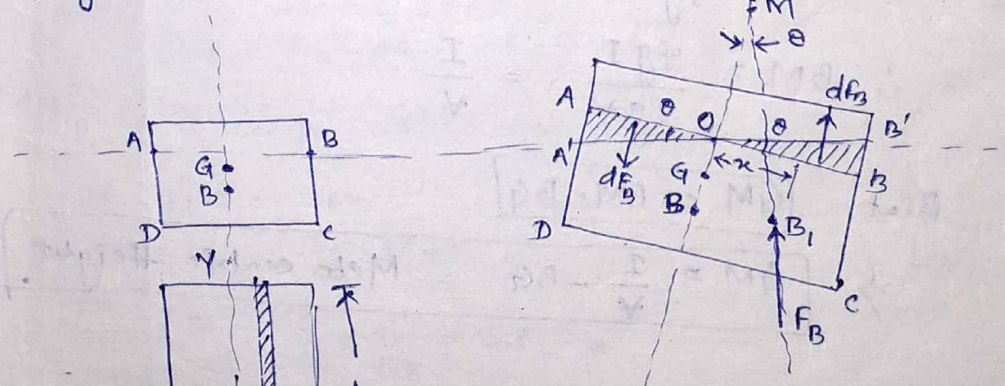
→ **Meta-centric Height** :-

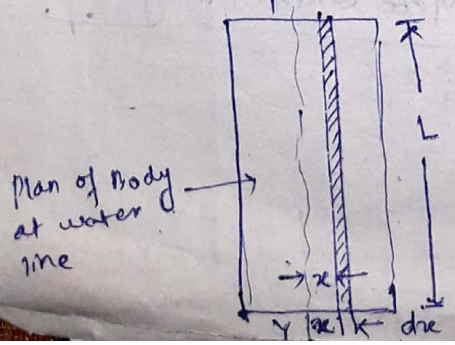
(a) Analytical Method

(b) Experimental Method

Metal-centric height (MG) is the distance between the meta-centre of a floating body and the centre of gravity of the body.

(a) **Analytical Method for Meta-centric Height**





θ = angular displacement

MG = Metacentric Height

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→ Couple due to wedges :-

vol. of the strip = $x\theta L\,dx$

weight of strip = $\rho g x\theta L\,dx$

Moment of couple = (weight of each strips) × (distance between these two weights)

$= \rho g x\theta L\,dx \times (x+x)$

$= 2\rho g x^2 \theta L\,dx$

∴ Moment of the ~~whole~~ couple for whole wedge

$= \int 2\rho g x^2 \theta L\,dx$ —— ①

Moment of couple due to shifting of B to B_1

$= F_B \times BB_1$

$= F_B \times BM \times \theta$

$= W \times BM \times \theta$ —— ②

$\left\{ \begin{array}{l} \because F_B = W \\ BB_1 = BM \times \theta \\ \text{for small } \theta \end{array} \right.$

The two couples are same, equating eq^n ① and ②

$W \times BM \times \theta = 2\rho g\theta \int x^2 L\,dx$

$\Rightarrow W \times BM = 2\rho g \int x^2 L\,dx$ $\quad (\because L\,dx = dA)$

$\Rightarrow W \times BM = 2\rho g \int x^2\,dA$

$\Rightarrow W \times BM = \rho g I$

$\Rightarrow BM = \dfrac{\rho g I}{W}$

$\left\{ \begin{array}{l} \because I = 2\int x^2\,dA \\ = \text{Second moment of area of the plan about axis Y-Y} \end{array} \right.$

But W = weight of the body

$= \rho g \times$ vol. of fluid displaced

$= \rho g \times$ vol. of body submerged in water

$= \rho g \times \forall$

$\therefore BM = \dfrac{\rho g I}{\rho g \forall} = \dfrac{I}{\forall}$

But $\boxed{GM = BM - BG}$

$\Rightarrow \boxed{GM = \dfrac{I}{\forall} - BG = \text{Meta centric Height}}$

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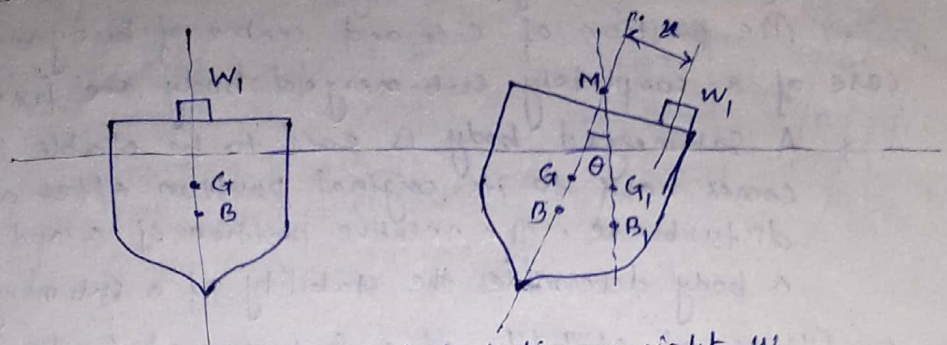
(b) Experimental Method for Meta-centric Height :-



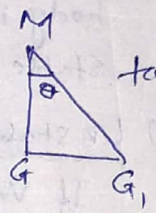
Let W = weight of vessel including weight w_1

G = C.G of the vessel

B = Centre of buoyancy of the vessel

Moment due to change of G to G_1

= Moment due to movement of w_1



$\tan\theta = \frac{GG_1}{GM}$

$\Rightarrow \quad GG_1 \times W = w_1 \times x$

$\Rightarrow \quad GM \tan\theta \times W = w_1 x$

$\Rightarrow \quad \boxed{GM = \frac{w_1 x}{W \tan\theta}}$

w_1 = Known weight

x = Known displacement

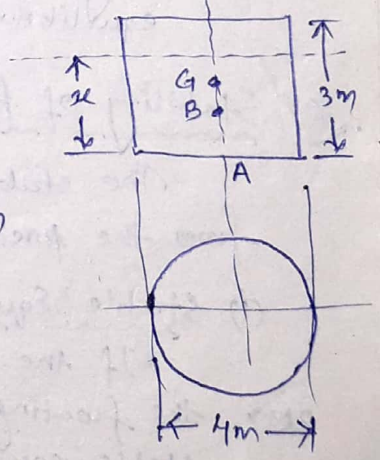
GM = Meta centric Height

Examples

(Q) A Solid cylinder of diameter 4m and height 3m. Find the meta centric height of the cylinder when it is floating in water with its axis vertical. The sp. gravity of the cylinder is 0.6. State the condition of stability in this case.

Sol^n: $D = 4m \quad h = 3m \quad S = 0.6$

x = depth of immersion $= 0.6 \times 3 = 1.8m$



$\therefore \quad AB = \frac{x}{2} = \frac{1.8}{2} = 0.9m$

$AG = \frac{3}{2} = 1.5m$

$BG = AG - AB = 1.5 - 0.9 = 0.6m$

Meta-centric Height $GM = \frac{I}{V} - BG$

$\Rightarrow \quad GM = \frac{\pi/64 \times D^4}{\pi/4 \times D^2 \times x} - BG$

$\Rightarrow \quad GM = \frac{1}{16} \times \frac{4^2}{1.8} - 0.6$

$= 0.55 - 0.6$

$= -0.05 \, m$

-ve sign means that meta centre (M) is below centre of gravity (G).

BM < BG, Unstable

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Stability of Sub-merged Body :-

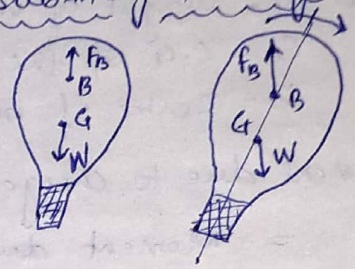
→ The position of C.G and centre of buoyancy in case of a completely sub-merged body are fixed.

→ A sub-merged body is said to be stable if it comes back to its original position after a slight disturbance. The relative position of G and B_1 of a body determines the stability of a sub-merged body.

→ Conditions of stability of a Sub-merged body (e.g. a balloon)

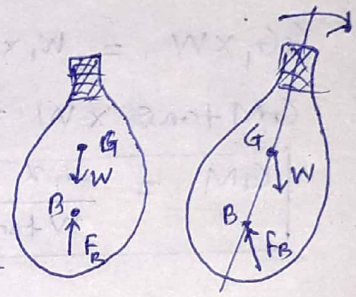
(i) Stable Equilibrium :

When $W = F_B$ and the point B is above G, the body is said to be in a stable equilibrium.



(ii) Unstable Equilibrium :-

If $W = F_B$, but B is below G, the body is in unstable equilibrium. A slight displacement to the body in clockwise dir^n gives the couple due to W and F_B also in clockwise dir^n. Thus the body doesn't return to its original position and hence it is in unstable equilibrium.



(iii) Neutral Equilibrium :-

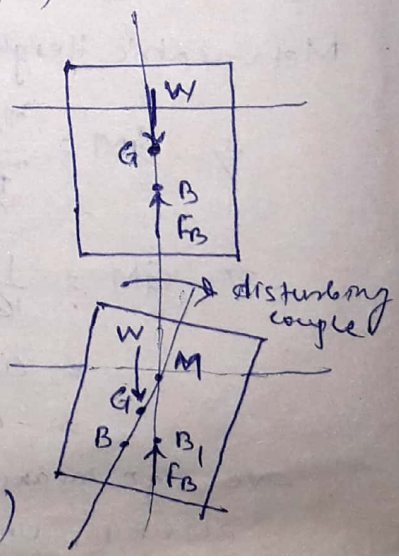
If $F_B = W$ and B and G are at the same point, the body is said to be in neutral equilibrium.

Stability of floating Body :-

The stability of a floating body is determined from the position of meta-centre (M)

(i) Stable Equilibrium :-

If the point M is above G, the floating body will be in stable equilibrium. F_B through B, and W through G constitute a couple acting in the anti-clockwise dir^n. Thus bring the body to its original position. (after angular displacement in clock-wise dir^n.) (BM > BG)



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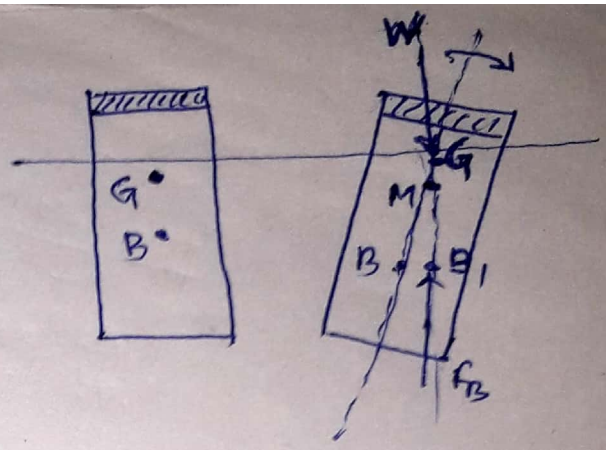
(ii) Unstable Equilibrium :-

If point M is below G, the floating body will be in unstable equilibrium. The disturbing couple is acting in clockwise dir^n. The couple due to F_B and W is also acting in clockwise direction; Thus overturning the floating body. (BM < BG)



(iii) Neutral Equilibrium :- If the point M is at the centre of gravity G of the body, the floating body will be in neutral equilibrium. (BM = BG)

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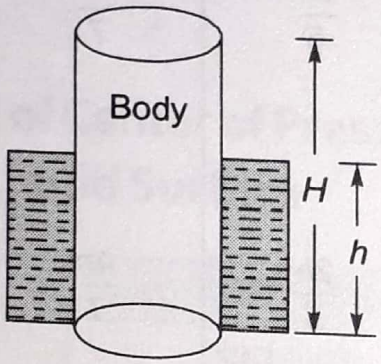
1. Archimedes Principle

When a body is submerged either fully of partially then it is acted upon by a force of buoyancy vertically up which is equal to weight of liquid displaced by the body.

Remember:

- ☑ This force of buoyancy always acts through the centroid of liquid displaced.
- ☑ Centre of Buoyancy is that point through which buoyant force act.

2. Principal of Flotation



At equilibrium

$$F_B = W$$

$$\rho_{Body} \times V = \rho_{fluid} \times \forall$$

$$H\rho_{Body} = h \cdot \rho_{fluid}$$

(for constant area)

Here, H = Height of body

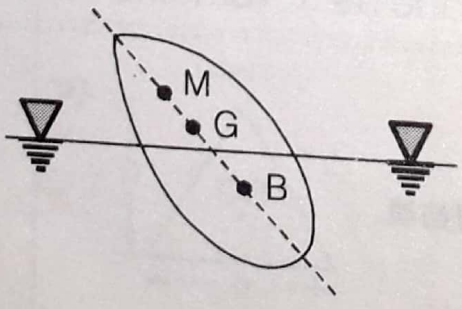
h = Height of body that is submerges in fluid

3. Condition for Equilibrium for Floating/Submerged Body

For stable equilibrium

- In case of **floating body**, metacenter should be above centre of gravity.

GM Positive $\Rightarrow$ M above G, $BM > BG$



Stable Equilibrium

$$GM = \frac{I_{min}}{\forall_{immersed}} - BG$$

- In case of **submerged body**, center of buoyancy should be above centre of gravity.

B above G

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- Distance between metacenter and centre of buoyancy (B.M.) =

$$\frac{I_{min}}{V_{immersed}} \text{ (Metacentric radius)}$$

Here,

I_{min} = Moment of inertia of top view of floating body about longitudinal axis

V = Volume of body immersed in liquid

Remember:

☑ Metacentric height for rolling condition will be less then metacentric height for pitching condition.

☑ For Neutral equilibrium $\boxed{M = G}$.

4. Time Period of Oscillation

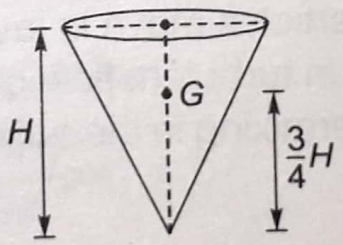
If a floating body oscillates then its time period of transverse oscillation is given by

$$T = 2\pi\sqrt{\frac{K_G^2}{g \cdot GM}}$$

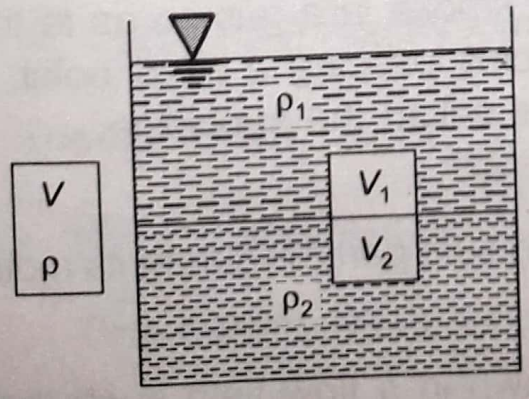
Here, K_G = Least radius of gyration

GM = Meta-centric height

5. For cone the center of gravity lies at $\frac{3}{4}H$ from the pointed end.



6.



$$\boxed{\frac{V_2}{V} = \frac{\rho - \rho_1}{\rho_2 - \rho_1}}$$

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