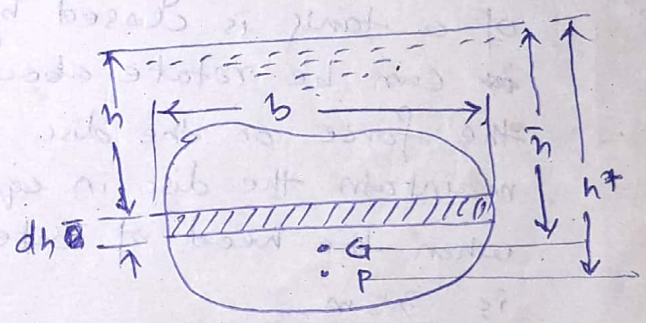
* Hydrostatic forces on Surfaces *

→ The forces acting on fluid particles will be.
1. due to pressure of fluid normal to the surface
2. due to gravity (or self-weight of fluid particles)

→ Total Pressure = The force exerted by a static fluid on a surface either plane or curved when the fluid comes in contact with the surfaces. This force always acts normal to the surface.

→ Centre of Pressure - The point of application of the total pressure on the surfaces. There are 4 cases of submerged surfaces on which the total pressure force and centre of pressure is to be determined :-
1. Vertical plane surface
2. Horizontal plane surface
3. Inclined plane surface
4. Curved surface

(1) Vertical plane surface



Total pressure force

$$\boxed{F = \rho g A \bar{h}}$$

Centre of pressure

$$h^* = \frac{I_o}{A\bar{h}}$$

$$= \frac{I_G + A\bar{h}^2}{A\bar{h}}$$

$$\Rightarrow \boxed{h^* = \frac{I_G}{A\bar{h}} + \bar{h}}$$

I_G = moment of inertia ~~about~~ of area about ~~CG~~, an axis passing through CG and parallel to the free surface of liq.

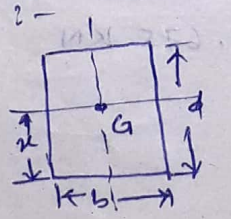
I_o = Moment of inertia of the surface about free surface of liq.

$\Rightarrow \int h^2 dA = \int b h^2 dh$

or $I_o = I_G + A\bar{h}^2$ (by parallel Axes theorem)

Cases:-
Shape of plane surfaces may be :-

(1) Rectangle



$I_G = \frac{bd^3}{12}$ $\because x = d/2$, Area $= bd$

$I_o = \frac{bd^3}{3}$ = moment of inertia about base.

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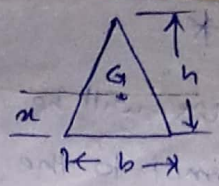
(2) Triangle



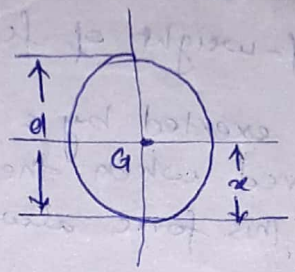
$I_G = \frac{bh^3}{36}$

$I_0 = \frac{bh^3}{12}$

$x = h/3$

$A = \frac{bh}{2}$

(3) Circle

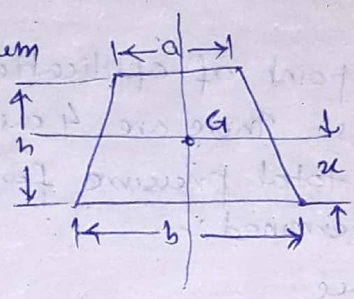


$I_G = \frac{\pi d^4}{64}$

$x = \frac{d}{2}$

$A = \frac{\pi d^2}{4}$

(4) Trapezium



$x = \left(\frac{2a+b}{a+b}\right)\frac{h}{3}$

$A = \left(\frac{a+b}{2}\right) \times h$

$I_G = \left[\frac{a^2+4ab+b^2}{36(a+b)}\right] \times h^3$

Examples

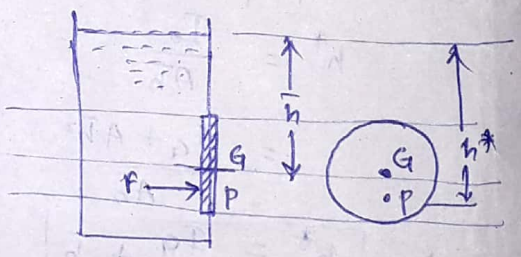
2015 (5m) (Q) A circular opening 2.5 m diameter, in a vertical side of a tank is closed by a disc of 2.5 m dia. which can be rotate about horizontal diameter. Find the force on the disc and the torque required to maintain the disc in equilibrium in vertical position when the head of water above the horizontal dia. is 3.5 m.

Sol^n: $\bar{h} = 3.5\,m$ $\quad d = 2.5\,m$

$A = \frac{\pi}{4} d^2$

$= \frac{\pi}{4} \times (2.5)^2 = 4.9\,m^2$

$\approx 5\,m^2$



(i) force on the disc $= F = wA\bar{h}$

$= \rho g A\bar{h}$

$= 9.81 \times 1000 \times 5 \times 3.5$

$= 171.675\ KN$

(ii) Depth of centre of pressure

$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$

$= \frac{\pi/64 \times d^4}{A\bar{h}} + \bar{h}$

$= \frac{\pi}{64} \times \frac{(2.5)^4}{5 \times 3.5} + 3.5 = 3.71\,m$

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∴ force F is acting at a distance 3.71 m from the free surface

So, the torque required to maintain the disc in equilibrium in vertical position (about axis C.G. horizontal dia.)

$$\Rightarrow T = F(h^* - \bar{h})$$

$$= 171.675 \times (3.71 - 3.5)$$

$$= 34.335 \text{ kN-m} \quad \text{(Ans)}$$

2013 (4 marks)

(Q) A rectangular surface 3m (width) × 6m (height) lies in a vertical plane. Determine force and centre of pressure on plate when its upper edge is 10m below the water surface.

Sol^n

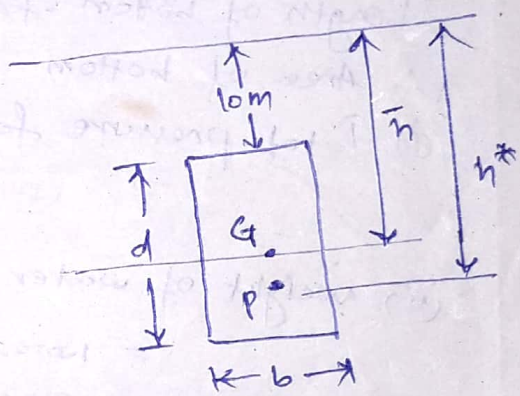
$b = 3m$

$d = 6m$

$A = bd = 18 m^2$

$\bar{h} = 10m + \frac{d}{2}$

$= 10m + 3m = 13m$



$F = \rho g A \bar{h}$

$= 9.81 \times 1000 \times 18 \times 13$

$= 2295540 \text{ N}$

$h^* = \frac{I_G}{A\bar{h}} + \bar{h}$

$= \frac{54}{18 \times 13} + 13 = 13.23 \text{ m}$

$I_G = \frac{bd^3}{12}$

$= \frac{3 \times 6^3}{12}$

$= 54 \text{ m}^4$

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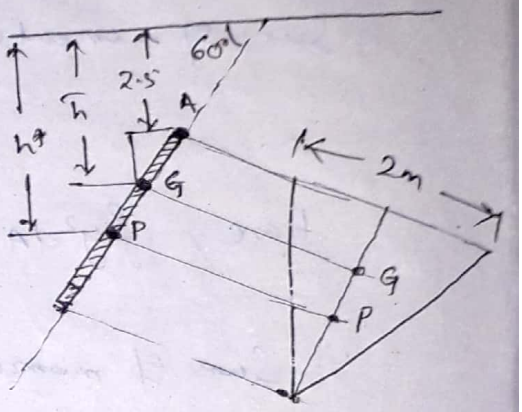
(Q) Find the total pressure and position of centre of pressure on a triangular plate of base 2m and height 3m is immersed in water in such a way that the plane of the plate makes an angle 60° with the free surface of the water. The base of the plate is parallel to water surface and at a depth of 2.5m from water surface.

Sol^n: $h = 3m$ $b = 2m$

$A = \frac{1}{2} b \times h = 3m^2$

$\theta = 60°$



$\bar{h} = 2.5 + AG \sin\theta$ $\left[\because AG = \frac{1}{3} \times h\right]$

$= 2.5 + \frac{1}{3} \times 3 \times \frac{\sqrt{3}}{2}$

$= 2.5 + 0.866 = 3.366\,m$

(i) Total pressure force $F = \rho g A \bar{h}$

$= 1000 \times 9.81 \times 3 \times 3.366$

$= 99061.38\ N$

(ii) Centre of Pressure $= h^* = \frac{I_G \sin^2\theta}{A\bar{h}} + \bar{h}$

But $I_G = \frac{bh^3}{36} = \frac{2 \times 3^3}{36} = 1.5\,m^4$

$\therefore h^* = \frac{1.5 \times \sin^2 60°}{3 \times 3.366} + 3.366 = 3.477\,m$

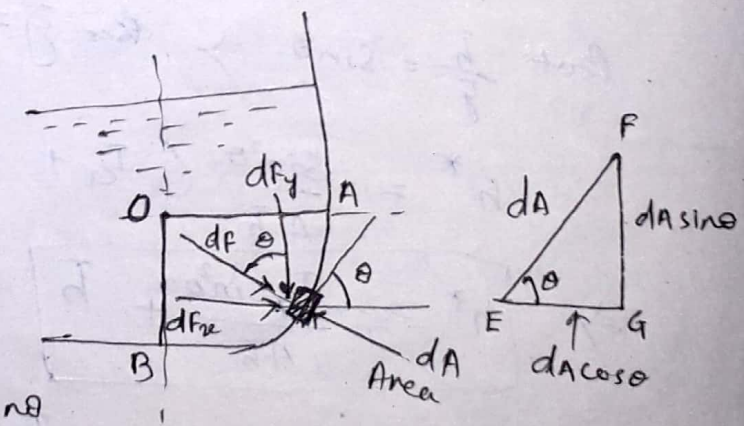
(4) Curved Surface Submerged on liquid :-

$dF = \rho g h\, dA$

$F = \int \rho g h\, dA$

$F = \sqrt{F_x^2 + F_y^2}$

$\tan\theta = \frac{F_y}{F_x}$



$dF_x = dF \sin\theta = \rho g h\, dA \sin\theta$

$dF_y = dF \cos\theta = \rho g h\, dA \cos\theta$

$\therefore F_x = \int dF_x = \rho g \int h\, dA \sin\theta$

$F_y = \int dF_y = \rho g \int h\, dA \cos\theta$

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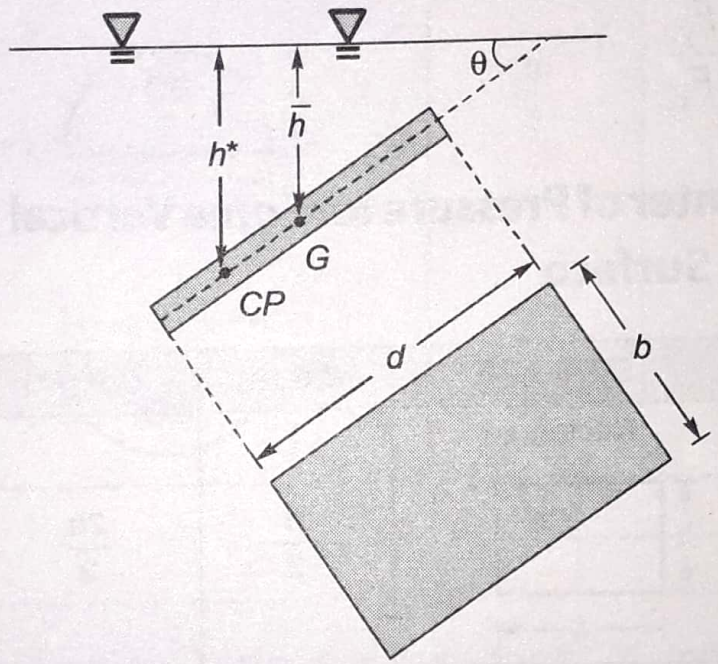
1. Hydrostatic Force on Submerged Surface

Case	Force	Center of pressure (h^*)
Horizontal position	$\rho g A\bar{h}$	$h^* = \bar{h}$
Vertical position	$\rho g A\bar{h}$	$h^* = \bar{h} + \frac{I_G}{A\bar{h}}$
Inclined position	$\rho g A\bar{h}$	$h^* = \bar{h} + \frac{I_G}{A\bar{h}} \sin^2\theta$



Note:

☑ Depth of centre of pressure is independent of density of the fluid.

$$I_G = \frac{bd^3}{12} \quad \text{(For rectangular plate)}$$

$$I_G = \frac{\pi}{64}(d^4) \quad \text{(For circular plate)}$$

where, A = Area of surface touching fluid

I_G = Area moment of inertia about centroidal axis and parallel to free axis.

$\bar{h}$ = Vertical distance of CG body from free surface.

w = Specific weight

θ = Angle at which the surface is inclined with horizontal

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2. Pressure Force on Curved Surface

- **Horizontal Force (F_H)**

  Horizontal component of the resultant hydrostatic force 'F_x' of curved surface may be computed by projecting the surface upon a vertical plane and multiplying the projected area by the pressure at its own centre of area.

- **Vertical Force (F_V)**

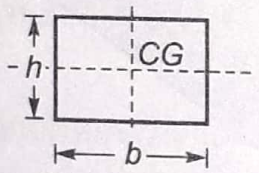
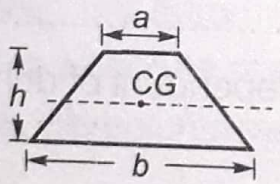
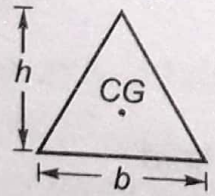
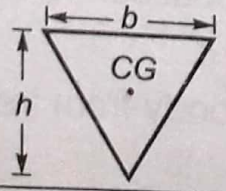
  Vertical component of force 'F_y' is equal to the weight of the liquid block lying above the curved surface upto free surface.

- **Resultant Force (F)**

$$F = \sqrt{(F_H)^2 + (F_V)^2}$$

Angle of line of action of resultant force with the horizontal is given by $\tan\theta = \dfrac{F_y}{F_x}$

3. Depth of Center of Pressure for Some Vertical Plane Surfaces from Liquid Surface

SURFACE	C.G.($\bar{h}$)	C.P.(h^*)
Rectangle 	$\dfrac{h}{2}$	$\dfrac{2h}{3}$
Trapezium 	$\dfrac{a+2b}{a+b}\cdot\dfrac{h}{3}$	$\dfrac{a+3b}{a+2b}\cdot\dfrac{h}{2}$
Triangle (a) 	$\dfrac{2h}{3}$	$\dfrac{3h}{4}$
(b) 	$\dfrac{h}{3}$	$\dfrac{h}{2}$

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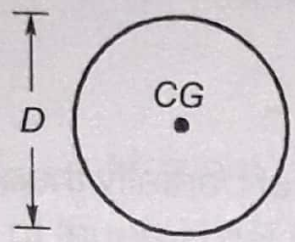
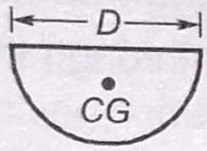
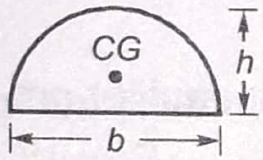
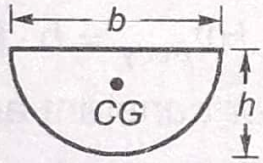
Circle 	$\frac{D}{2}$	$\frac{5D}{8}$
Semi Circle 	$\frac{2D}{3\pi}$	$\frac{3\pi D}{32}$
Parabola (a) 	$\frac{3h}{5}$	$\frac{5h}{7}$
(b) 	$\frac{2h}{5}$	$\frac{4h}{7}$

Remember:

- ☑ In case of vertical surface, when depth of immertion $(\bar{h})$ is very large then centre of pressure = centre of gravity or $\boxed{h^* = \bar{h}}$.
- ☑ Magnitude of hydrostatic forces on a plane surface does not change with rotation in a horizontal plane as $\bar{x}$ remains same.

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